This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

A TEXTBOOK

ON

CIVIL ENGINEERING

INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

STRENGTH OF MATERIALS
ANALYSIS OF STRESSES
PROPORTIONING THE MATERIAL
DETAILS OF CONSTRUCTION
DETAILS, BILLS, AND ESTIMATES

5780

SCRANTON
INTERNATIONAL TEXTBOOK COMPANY
B-3

Kurt F. Wendt Library
University of Wisconsin - Madison
215 N. Randall Avenue
Madison, WI 53706-1688

Copyright, 1897, 1898, 1899, by THE COLLIERY ENGINEER COMPANY.

Strength of Materials: Copyright, 1894, 1899, by THE COLLIERY ENGINEER COMPANY.

Analysis of Stresses: Copyright, 1895, by THE COLLIERY ENGINEER COMPANY. Copyright, 1901, by INTERNATIONAL TEXTBOOK COMPANY.

Proportioning the Material: Copyright, 1895, by THE COLLIERY ENGINEER COMPANY.

Details of Construction: Copyright, 1896, by THE COLLIERY ENGINEER COMPANY.

Details, Bills, and Estimates: Copyright, 1896, by THE COLLIERY ENGINEER COMPANY.

All rights reserved.

d 11

BURR PRINTING HOUSE,
FRANKFORT AND JACOB STREETS,
NEW YORK.

CONTENTS.

STRENGTH OF MATERIALS.

PAGE
Materials Used in Construction, 625
Stresses and Strains, 632
Tension, 633
Compression, 637
Shear, 639
Factors of Safety, 640
Pipes and Cylinders, 643
Beams, 647
Columns, 672
Torsion and Shafts, 678
Ropes, 681
Chains, 683

ANALYSIS OF STRESSES.

The Truss, 685
Loads, 690
Maximum Live Load Chord Stresses, 694
Maximum Live Load Web Stresses, 699
Dead Load, 704
Counter Stresses, 709
Wind Stresses, 711
Stresses in Latticed Portal, 718
Floor Beam Stresses, 726
Stress Sheets, 732
Method by the Moment and Shear Diagrams, 733
Short Method for Web Stresses, 737
The Howe Truss, 741
The Warren Girder, 744
Trusses with Inclined Chords, 749

ANALYSIS OF STRESSES—*continued.* PAGE

The Whipple Truss, - - - - - - - - 773
The Baltimore Truss, - - - - - - - - 778
The Petit Truss, - - - - - - - - - 786
Stresses in Braced Portal, - - - - - - 795
Floor Beams with Sidewalk Cantilevers, - - 798
Concentrated Wheel Loads, - - - - - - 801

PROPORTIONING THE MATERIAL.

Materials Used for Bridge Superstructures, - - 807
The Use of Wrought Iron, - - - - - - 812
Unit Stresses Allowed for Tension Members, - 814
Proportioning the Material for Tension Members, 817
Compression Members: Radii of Gyration, - - 819
Weight of Wrought Iron, - - - - - - 828
Proportioning the Material for Compression Members, - - - - - - - - - - 833
Effect of Wind Stress upon Chords and End Posts, 850
Proportioning the Material for Floor Beams, - 859
The Floor, - - - - - - - - - - 861
Completing the Stress Sheet, - - - - - 866
The Use of Steel, - - - - - - - - 868
The Fatigue of Metals, - - - - - - 872
Items from Thacher's and Lewis's Specifications, 875
Weight and Specific Gravity of Wrought Iron and Steel, - - - - - - - - - - - 878
Eccentricity; Position of Pins; Bending Stresses due to Weight of Member, - - - - - 880

DETAILS OF CONSTRUCTION.

General Requirements, - - - - - - - 895
Specifications, - - - - - - - - - 897
Camber, - - - - - - - - - - 903
Exact Length of Diagonal Members, - - - 905
Positions of Pins in Chords and End Posts, - - 908
General Dimensions of Upper Chord and End Post, - - - - - - - - - - 910

DETAILS OF CONSTRUCTION—*continued.* PAGE

Clearance, 911
Size of Pins, 912
Proportioning Pin Plates, 914
Moments on Pins, 925
Constructive Details of the End Post, 941
Details of the Upper Chord, 948
Constructive Details of the Intermediate Post, 951
Constructive Details of the Portal Bracing, 958
Constructive Details of the Lateral Struts and Knee Braces, 963
Constructive Details of the Floor Beam, 966
Eye-Bar Heads, 975
Constructive Details of the Lower Chord Members, 979
Constructive Details of the Tension Web Members, 986
Constructive Details of the Lateral Rods, 992
Constructive Details of the Beam Hanger, 998

DETAILS, BILLS, AND ESTIMATES.

Shoes, Rollers, and Bed-Plates, 1005
Chord Pins, Pin Nuts, Cotter Pins, and Pin Washers, 1017
Shop Lists, 1027
Iron Order, 1034
Lumber Bill, 1044
Shipping Bills, 1048
The Erection Diagram, 1053
The Approximate, or Preliminary Estimates, 1054
The Close Estimate, 1060
Metal Joists and Stringers, 1062
Positions and Connections of Floor-Beams, 1066
Name Plates and Railings, 1071
Paint and Painting, 1075
Imperfect Design and Inconsistent Requirements, 1082

STRENGTH OF MATERIALS.

MATERIALS USED FOR CONSTRUCTION.

1184. The principal materials used in engineering construction are *timber*, *brick*, *stone*, *cast iron*, *wrought iron*, and *steel*. Table 19 gives their average weights per cubic foot.

TABLE 19.

Material.	Average Weight.	Approximate Weight of Piece 1" Square and 1' Long in Lb.
	Pounds per Cubic Foot.	
Timber	40	.278
Brick	125	
Stone	160	
Cast Iron	450	3.125
Wrought Iron	480	3.333⅓
Steel	490	3.403

CAST IRON.

1185. Cast iron is a combination of pure iron with from 2% to 6% of carbon.

Pig iron is the result of the first smelting, and is obtained directly from the ore. Pig iron is rarely, if ever, used for anything except to be remelted and made into cast iron or wrought iron.

1186. Cast iron is of two kinds, *white cast iron* and *gray cast iron*. The first is a chemical compound of iron with from 2% to 6% of carbon, nearly all the carbon being chemically combined with the iron. The second, or gray cast iron, contains a part of the carbon in chemical combination, and the rest in the state of graphite mechanically mixed with the iron. When a piece of gray cast iron is

For notice of copyright, see page immediately following the title page.

broken, a large number of black specks are seen on the broken ends; these specks are pure carbon in the form of graphite.

1187. White cast iron contains hardly any free carbon. It is of two kinds, granular and crystalline. Both are very hard and brittle, and are only used for conversion into wrought iron or steel.

1188. Gray cast iron is divided into three classes, known as Nos. 1, 2, and 3.

No. 1 contains the largest amount of carbon in mechanical mixture, which makes it soft and fusible, though not as strong as Nos. 2 and 3. It is very suitable for making castings where precision in form is required, as, owing to its fluidity, it fills the mold well; but it is not suitable for castings requiring strength.

No. 2 is most suitable for use in constructions, as it is stronger than No. 1, and not so soft.

No. 3 contains the smallest amount of carbon in the graphitic (uncombined) form, and is, in consequence, harder and more brittle. It is fit only for the massive and heavy parts of machinery.

1189. Cast iron has certain advantages and disadvantages as a material for engineering construction. It is easy to give it any desired form. A pattern of the piece desired is made; a mold is made in the sand, the pattern is removed, and the melted iron poured in. Cast iron resists oxidation (rust) better than either wrought iron or steel. Its compressive (crushing) strength is very high, but its tensile (stretching) strength is comparatively low. It cannot be riveted, or welded by forging. It is brittle, breaking off without giving much warning, and stretching but little before giving away. It is liable to have hidden and small surface defects and air bubbles, and this makes its strength uncertain.

Another serious drawback in the use of cast iron is its liability to initial stresses from inequality in cooling after having been poured into the molds. Thus, if one part of the casting is very thin and another very thick, the thin part cools first, and, in cooling, contracts; the thick part, cooling afterwards, causes stresses in the thin part, which may be sufficient to break it, or, if not, there may be so great a stress in the thin part that a small additional force will break it.

Cast iron is not well adapted to a tensile stress, nor to resist shocks. It is used for columns and posts in buildings, on account of its high compressive strength. In machinery, it is used in all those parts where weight, mass, or form is of more importance than strength, as in frames and bed plates of machines, and for hangers, pulleys, gear wheels, etc. It is also used for water mains where the pressure to be resisted is not too great.

1190. Malleable cast iron is made by heating the casting in an annealing oven, in powdered hematite ore. It can be hammered into any desired shape when cold, but is very brittle when hot.

WROUGHT IRON.

1191. Wrought iron is the product resulting from the reduction of the carbon in cast iron. It is obtained by melting white cast iron, and passing an oxidizing flame over it. When the carbon is burned out, the mass of iron is left in a pasty condition, full of holes. It is then taken out, and hammered or rolled in order to unite it into one mass. The result of this first process is not suitable for use in any construction of importance, and has to be reheated and rerolled a number of times, in order to make it homogeneous, and to remove flaws from within the iron.

At best, therefore, wrought iron is a series of welds; if a piece is broken, the separate layers of which it is composed can be seen plainly. It cannot be melted and run into molds, as can be done with cast iron; but it can be easily welded by forging; that is, two pieces of wrought iron can be united by raising them to the proper temperature and hammering them together. Wrought iron is much more capable of bearing a tensile or transverse stress than cast iron; it is tougher, stretches more, and gives more warning before fracture.

It withstands shocks far better than cast iron. Two pieces may be punched or drilled and riveted together. The entire process weakens the iron, and cast iron would not withstand it. It has also to withstand flanging and the stresses due to changes of temperature.

Wrought iron cannot be hardened like steel by heating and then dipping in water, but may be *case-hardened* by rubbing the surface with potassium cyanide or potassium ferrocyanide while at a cherry-red heat and then dipping in water. The cyanide causes the iron to be carbonized to a slight depth; that is, through a depth of about $\frac{1}{32}$ of an inch the iron is converted into steel which can be hardened. Cast iron may be hardened in the same way.

The quality of wrought iron varies considerably, and the terms by which it is known in the market refer to the amount of working which the iron has received. We thus have common bar iron, best iron, double best, and triple best. These terms are only rough indications of quality. When wrought iron is rolled cold under great pressure, it has a smooth polished surface, and its strength is greatly increased.

When the word iron is used alone, wrought iron is meant.

STEEL.

1192. Steel is a chemical compound of iron and carbon; that is, it contains no carbon in a free state, as cast iron does. Its tensile strength is greater than that of wrought iron, and its compressive strength greater than that of cast iron. It is by far the strongest material used in the mechanic arts. Its strength varies greatly with its purity and the amount of carbon it contains. According to the amount of carbon in it, steel is divided into high grade, medium grade, and low grade, the high grades having the most carbon. Steel, unlike wrought iron, is fusible; unlike cast iron, it can be forged; and, with the exception of the higher grades, it can be welded by heating and hammering, although care must be exercised in so doing.

1193. The special characteristic of steel (the very lowest grades excepted) is that, when it is raised to a cherry red heat and suddenly cooled, it becomes brittle and exceedingly hard, and that by subsequent heating and slow cooling, the hardness may be reduced to any desired degree down to the point of least hardness that steel possessing that amount of carbon can have. The first process is called *hardening*, and the second *tempering*.

If the surface of a piece of steel that has been hardened is polished slightly so as to remove the dark scale or soot which covers it, and is then reheated, it will be found that, as the temperature increases, a series of colors appear. These colors are always the same for the same temperature, and, if the steel is suddenly cooled when one of the colors appears, it acquires a degree of hardness which is always the same for the same color and for the same quality of steel.

In Table 20 are given the colors with the corresponding temperatures that occur in tempering different kinds of tools.

TABLE 20.

Tools.	Color.	Temperature, Fahr.
Lancets	Pale yellow	430°
Razors and scalpels	Pale straw	450°
Penknives, chisels for cast iron, and screw taps	Yellow	470°
Scissors and chisels for wrought iron	Brown	490°
For carpenters' tools in general	Red	510°
Fine watch springs and table knives	Purple	530°
Swords and lock springs	Blue, bright	550°
Daggers, fine saws and needles	Blue, full	560°
Common saws and springs	Blue, dark	600°

Steel that has been hardened may be softened by heating it to a hardening temperature and then cooling it very slowly; this process is called **annealing.**

1194. Steel is made in one of the three following ways:

1. By adding carbon to wrought iron.
2. By removing carbon from cast iron.
3. By melting together cast and wrought iron in suitable proportions.

Several processes, varying with the quality of the product desired and the use for which it is intended, are used in making steel. The processes in general use are the following:

(*a*) **Cementation,** in which bars of very pure wrought iron are heated to a high temperature in contact with carbon. The product, known as **blister steel,** is used for cutlery, tools, etc.

(*b*) **Crucible steel,** also called **cast steel,** is made by melting pure wrought iron in a crucible with enough charcoal and cast iron to introduce the required amount of carbon. It is used for making springs, cutlery, tools, etc.

(*c*) **Bessemer steel** is made by decarbonizing cast iron by forcing a powerful blast of air through a melted mass of the iron. This removes the greater part of its carbon. A small quantity of very pure cast iron, rich in carbon, is then added, bringing up the percentage of carbon to the required amount.

(*d*) **Open-hearth steel** is made by fusing a charge consisting of the suitable proportions of cast iron with wrought iron scrap, or with Bessemer steel scrap.

1195. Bessemer and open-hearth steel contain more impurities than blister and crucible steel do; but they are much cheaper, and are just as suitable for many purposes. It is only in consequence of the introduction of these two cheap varieties that steel can be extensively used, as blister and crucible steels would, in the majority of cases, be too expensive.

Bessemer and open-hearth steels contain from .05% to 1½% of carbon. The proportion of carbon in the best kinds of tool and cutlery steels is as follows:

Razor steel,	1½%.	Very difficult to forge, and easily burnt.
Saw file,	1⅜%.	Bears heat not above cherry redness.
Tool steel,	1¼%.	Ordinary cutting tools. Welds with difficulty.
" "	1⅛%.	For mandrils and heavy cutting tools.
" "	1%.	For chisels, gravers, etc.

At the Imperial Works at Neuberg, Austria, the following percentages of carbon are present in the different grades of Bessemer steel:

1.58% to 1.38%.	Cannot be welded, and is rarely used.
1.38% " 1.12%.	Great care must be used in working.
1.12% " .88%.	Welds easily; used for bits, chisels, etc.
.88% " .62%.	Used for cutting tools, files, etc.
.62% " .38%.	Mild steel; for tires, etc.
.38% " .15%.	Tempers slightly; for boiler plates and axles.
.15% " .05%.	Does not temper; steel for pieces of machinery.

1196. It will be noticed from what precedes that the hardness of steel depends upon the amount of carbon it contains.

Some kinds of crucible cast steel can be hardened by heating to a low red heat and then allowing them to cool slowly in the air without dipping in water. They are called self-hardening steels, the best known being Mushet's special tool steel. This contains about 2% carbon with 7% to 12% tungsten in alloy with the iron. The same property is characteristic of Hadfield's manganese steel, which contains between .8% and 1.2% of carbon and 7% to 20% of manganese.

As a rule, when a piece of steel is broken across the grain, the finer the grain and the whiter and cleaner the fracture the more carbon it contains.

STRESSES AND STRAINS.

1197. The molecules of a solid or rigid body being held together by the force of cohesion, this force must be overcome to a greater or less degree in order to change the form and size of the body, or to break it into parts. The internal resistance which a body offers to any force tending to overcome the force of cohesion is called a **stress.** If a weight of 1,000 pounds is held in suspension by a rod, there will be a stress of 1,000 pounds in the rod. In this country and England, stresses are measured in pounds or tons; in nearly all other civilized countries, in kilograms. Whenever a body is subjected to a stress, the total stress induced by the acting force at any section of the body is the same as the total stress at any other section.

1198. The **unit stress** (called also the **intensity** of stress) is the stress per unit of area; or, it is the total stress divided by the area of the cross-section. In the above illustration, if the area had been 4 sq. in. the unit stress would have been $\frac{1,000}{4} = 250$ lb. per sq. in. Had the area been $\frac{1}{2}$ sq. in., the unit stress would have been $\frac{1,000}{.5} = 2,000$ lb. per sq. in.

Let $P =$ the total stress in pounds;
$A =$ area of cross-section in square inches;
$S =$ unit stress in pounds per square inch.

Then, $S = \frac{P}{A}$, or $P = A S$. **(65.)**

That is, *the total stress equals the area of the section, multiplied by the unit stress.*

When a body is stretched, shortened, or in any way deformed through the action of a force, the amount of deformation is called a **strain.** Thus, if the rod before mentioned had been elongated $\frac{1}{10}''$ by the load of 1,000 pounds, the strain would have been $\frac{1}{10}''$. Within certain limits, to be given hereafter, strains are proportional to the stresses producing them.

1199. The **unit strain** is the strain per unit of length or of area, but is usually taken per unit of length and called the *elongation* per unit of length. In this paper, the unit of length will be considered as one inch. The unit strain, then, equals the total strain divided by the length of the body in inches.

Let $l =$ length of body in inches;
$e =$ elongation in inches;
$s =$ unit strain.

Then, $$s = \frac{e}{l}, \text{ or } e = ls. \qquad \textbf{(66.)}$$

1200. Whenever a force, no matter how small, acts upon a body, it produces a stress and a corresponding strain.

According to the manner in which forces act upon a body, the stresses are divided into the following classes:

1. *Tension*, which produces a tensile or pulling stress.
2. *Compression*, which produces a compressive or crushing stress.
3. *Shear*, which produces a shearing or cutting stress.
4. *Torsion*, which produces a torsional or twisting stress.
5. *Flexure*, which produces a transverse or bending stress.

TENSION.

1201. When two forces act upon a body in opposite directions (away from each other) the body is said to be in tension. The two forces tend to elongate the body and thus produce a tensile stress and strain. A weight supported by a rope affords a good example. The weight acts downwards, and the reaction of the support to which the upper end of the rope is fastened acts upwards; the result being that the rope is stretched more or less, and a tensile stress is produced in it. Another familiar example is the connecting rod of a steam engine on the return stroke. The cross-head then exerts a pull on one end of the rod, which is resisted by the crank-pin on the other.

Example.—An iron rod, 2 inches in diameter, sustains a load of 90,000 pounds; what is the unit stress?

Solution.—Using formula **65**,

$$P = A\,S, \text{ or } S = \frac{P}{A} = \frac{90{,}000}{2^2 \times .7854} = 28{,}647.82 \text{ lb. per sq. in. Ans.}$$

EXPERIMENTAL LAWS.

1202. The following laws have been established by experiment:

1. *When a body is subjected to a small stress, a small strain is produced, and when the stress is removed the body springs back to its original shape. This leads to the conclusion that, for small stresses, bodies are perfectly elastic.*

2. *Within certain limits, the change of shape (strain) is directly proportional to the applied force.*

3. *When the stress is sufficiently great, a strain is produced which is partly permanent; that is, the body does not spring back entirely to its original form when the stress is removed. This lasting part of the strain is called a* **set,** *and in such cases the strain is not proportional to the stress.*

4. *Under a still greater stress, the strain rapidly increases, and the body is finally ruptured or broken.*

5. *A force acting suddenly, as a shock, causes greater injury than a force gradually applied.*

According to the first law, the body will resume its original form when the force is removed, provided the stress is not too great. This property is called *elasticity.* According to the second law, the strain is proportional to the stress within certain limits. Thus, if a pull of 1,000 pounds elongates a body .1″, a pull of 2,000 pounds would elongate it .2″. This is true up to a certain limit, beyond which the body will not resume its original form upon the removal of the stress, but will be permanently strained more or less, according to the amount of stress. The stress at the point where the set begins is called the **elastic limit.** All strains produced by stresses within (less than) the elastic limit are directly proportional to the stresses.

1203. Stresses are equal but opposed to the external forces producing them, and are, therefore, measured and represented by these forces. Thus, as we have explained before, a force of 1,000 pounds produces a stress of 1,000 pounds. The external force is the force applied to a fixed body; the stress is the resistance offered by the body to a change of form; and when the body ceases to change (as when a rod ceases to elongate), the stress just balances the external force.

COEFFICIENT OF ELASTICITY.

1204. Amongst engineers, the term *elasticity* means the *resistance* which a body offers to a permanent change of form; and by *strength*, the resistance which a body offers to division or separation into parts.

It follows from this that those bodies which have the highest elastic limit are the most elastic.

1205. The **coefficient of elasticity** is the ratio of the unit stress to the unit strain, provided the elastic limit is not exceeded. Let S be the unit stress, s the unit strain, and E the coefficient of elasticity; then, by definition, $E = \frac{S}{s}$. Substituting the values of S and s obtained from formulas **65** and **66**,

$$E = \frac{S}{s} = \frac{P}{A} \div \frac{e}{l} = \frac{Pl}{Ae}. \qquad \textbf{(67.)}$$

1206. If in this formula we assume $e = l$, and $A = 1$ (1 square inch), then $E = P$. That is, the coefficient of elasticity is that force which, *if stress and strain continued proportional to each other*, would produce in a bar of unit area a strain equal to the original length of the bar ($l = e$). This, however, is never the case, as the elastic limit and the ultimate strength are reached before the applied force reaches the value E.

EXAMPLE.—A wrought iron bar 2 inches square and 10 feet long is stretched .0528 inches by a stress of 44,000 pounds; what is the coefficient of elasticity?

SOLUTION.—Using formula **67**,

$$E = \frac{Pl}{Ae} = \frac{44{,}000 \times 10 \times 12}{2^2 \times .0528} = 25{,}000{,}000 \text{ lb. per sq. in. Ans.}$$

1207. The **ultimate strength** of any material is that unit stress which is just sufficient to break it.

1208. The **ultimate elongation** is the total elongation produced in a unit of length of the material having a unit of area, by a stress equal to the ultimate strength of the material.

1209. For the same size, quality, and kind of material, the ultimate strength, ultimate elongation, coefficient of elasticity, and elastic limit are the same for different pieces. Table 21 gives the *average* values of the coefficient of elasticity (E_1), elastic limit (L_1), ultimate strength (S_1), and ultimate elongation (s_1), of different materials, the quantities given being for *tension* only. As brick and stone are never used in tension, their values are not given.

TABLE 21.

Material.	Coefficient of Elasticity. E_1.	Elastic Limit. L_1.	Ultimate Tensile Strength. S_1.	Ultimate Elongation. s_1.
	Lb. per Sq. In.	Lb. per Sq. In.	Lb. per Sq. In.	In. per Linear Inch.
Timber	1,500,000	3,000	10,000	0.015
Cast Iron	15,000,000	6,000	20,000	0.005
Wrought Iron	25,000,000	25,000	55,000	0.20
Steel	30,000,000	50,000	100,000	0.10

The values in this table are subject to great variation, and cannot be depended upon in designing machine parts. Thus, the ultimate tensile strength of steel varies from less than 60,000 to more than 180,000 pounds per square inch, according to its purity and the amount of carbon it contains; that of cast iron from 12,000, or 13,000, to over 40,000; wrought iron varies from 40,000 to 72,000, according to quality, the latter value being for iron wire. Timber varies fully as much as, if not more than, any of the three preceding materials, its properties depending upon the kind of wood, its degree of dryness, the manner of drying, etc.

All the problems in this section will be solved by using the average values given in the preceding and following tables; the designer, however, should not use them, but either test the materials himself or state in the specifications what strength the material must have. For example, mild steel, for boiler shells, should have a tensile strength of not less than 60,000 or 65,000 pounds per square inch; Bessemer steel, for steel rails, not less than 110,000; open-hearth steel, for locomotive tires, not less than 125,000, and crucible cast steel, for tools, cutlery, etc., not less than 150,000. It is also customary to specify the amount of elongation. This is necessary because, as a rule, the elongation decreases as the tensile strength increases. Having tested the material about to be used, or having specified the lowest limits, the designer can ascertain the strength and stiffness of construction by means of the formulas and rules which are to follow.

Example.—How much will a piece of steel 1 inch in diameter and 1 foot long elongate under a steady load of 15,000 pounds?

Solution.—$E_1 = \frac{P l}{A e}$, or $e = \frac{P l}{A E_1}$.

From Table 21, $E_1 = 30{,}000.000$ for steel; hence,

$$e = \frac{15{,}000 \times 12}{1^2 \times .7854 \times 30{,}000{,}000} = .00764''. \quad \text{Ans.}$$

Note.—All lengths given in this treatise on Strength of Materials must be reduced to inches before substituting in the formulas.

Example.—A piece of timber has a cross-section 2″ × 4″ and is 6 feet long. A certain stress produces an elongation of .144 inch; what is the value of the stress in pounds?

Solution.—

$$E_1 = \frac{P l}{A e}, \text{ or } P = \frac{E_1 A e}{l} = \frac{1{,}500{,}000 \times 2 \times 4 \times .144}{6 \times 12} = 24{,}000 \text{ lb.} \quad \text{Ans.}$$

COMPRESSION.

1210. If the length of the piece is not more than five times its least transverse dimension (its diameter, when round; its shorter side, when rectangular, etc.), the laws of compression are similar to those of tension. The strain is proportional to the stress until the elastic limit has been reached; after that, it increases more rapidly than the stress, as in the case of tension. The area of the cross section is slightly enlarged under compression. In Table 22 are given the average compression values of E, L, and S for wood, brick, stone, cast iron, wrought iron, and steel. (See also Table 21.) E is not given for brick, nor L for cast iron, brick, or stone, because these values are not known. To distinguish between tension and compression when applying a formula, E_2, L_2, and S_2 will be used instead of E_1, L_1, and S_1.

TABLE 22.

Material.	Coefficient of Elasticity. E_2.	Elastic Limit. L_2.	Ultimate Compressive Strength. S_2.
	Lb. per Sq. In.	Lb. per Sq. In.	Lb. per Sq. In.
Timber............	1,500,000	3,000	8,000
Brick	....	......	2,500
Stone.............	6,000,000	......	6,000
Cast Iron.........	15,000,000	......	90,000
Wrought Iron.....	25,000,000	25,000	55,000
Steel..............	30,000,000	50,000	150,000

1211. When the length of a piece subjected to compression is greater than ten times its least transverse dimension, it is called a *column*, and the material fails by a sidewise bending or flexure. The preceding table is to be used only for pieces whose length does not exceed *five* times the least dimension of the cross-section. (See Art. **1266***a*.)

Example.—How much will a wrought iron bar 4 inches square and 15 inches long shorten under a load of 100,000 pounds?

Solution.—$E_2 = \frac{P\,l}{A\,e}$, or $e = \frac{P\,l}{A\,E_2}$.

Hence, $e = \frac{100,000 \times 15}{16 \times 25,000,000} = .00375''$. Ans.

SHEAR.

1212. When two surfaces move in opposite directions very near together in such a manner as to cut a piece of material, or to pull part of a piece through the remainder, the piece is said to be *sheared.* A good example of a shearing stress is a punch; the two surfaces in this case are the bottom of the punch and the top of the die. Another example is a bolt with a thin head; if the pull on the bolt is great enough, it will be pulled through the head and leave a hole in it, instead of the bolt breaking by pulling apart, as would be the case with a thick head. In this case, the two surfaces are the under side of the head and the surface pressed against. Other examples are a knife cutting a piece of wood, and the ordinary shears from which this kind of stress takes its name.

TABLE 23.

Material.	Coefficient of Elasticity. E_s.	Ultimate Shearing Strength. S_s.
Timber (across the grain).....	....	3,000
Timber (with the grain)......	400,000	600
Cast Iron.....................	6,000,000	20,000
Wrought Iron................	15,000,000	50,000
Steel..........................	....	70,000

1213. Formula **65** applies in cases of shearing stress, but formulas **66** and **67** are never used for shearing. In the preceding table, E_s and S_s are used to represent, respectively, the coefficient of shearing elasticity and ultimate shearing strength.

Example.—What force is necessary to punch a one-inch hole in a wrought-iron plate $\frac{3}{8}$ of an inch thick?

Solution.—$1'' \times 3.1416 \times \frac{3}{8}'' = 1.1781$ sq. in. = area of punched surface = area of a cylinder 1" in diameter and $\frac{3}{8}$" high. Using formula **65**,

$$P = A\,S_s = 1.1781 \times 50,000 = 58,905 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE.—A wooden rod 4 inches in diameter and 2 feet long is turned down to 2 inches diameter in the middle so as to leave the enlarged ends each 6 inches long. Will a steady stress pull the rod apart in the middle, or shear the ends?

SOLUTION.—$P = A\ S_s = 2 \times 3.1416 \times 6 \times 600 = 22{,}620$ lb. to shear off the ends.

The force required to rupture by tension is

$$P = A\ S_1 = 2^2 \times .7854 \times 10{,}000 = 31{,}416 \text{ lb.}$$

Since the former is only about $\frac{2}{3}$ of the latter, the piece will fail through the shearing off of the end.

Had a transverse stress been used, the force necessary to shear off a section of the end would have been

$$4^2 \times .7854 \times 3{,}000 = 37{,}700 \text{ lb.}$$

FACTORS OF SAFETY.

1214. It was previously stated that no stress should ever be applied to a machine part that would strain it beyond the elastic limit. The usual practice is to divide the ultimate strength of the material by some number depending upon the kind and quality of the material, and upon the nature of the stress; this number is called a **factor of safety.**

The factor of safety for any material is the ratio of its ultimate strength to the actual stress to which it is subjected, or for which it is intended.

In Table 23, 70,000 pounds per square inch is given as the ultimate shearing strength for steel. Now, suppose that the actual stress on a piece of steel is 10,000 pounds per square inch; then, the factor of safety for this piece would be $\frac{70{,}000}{10{,}000} = 7$.

1215. To find the proper allowable working strength of a material, divide the ultimate strength for tension, compression, or shearing, as the case may be, by the proper factor of safety.

Table 24 gives the factors of safety generally used in American practice. Factors of safety will always be denoted by the letter f in the formulas to follow.

TABLE 24.

Material.	For Steady Stress. (Buildings.)	For Varying Stress. (Bridges.)	For Shocks. (Machines.)
Timber...........	8	10	15
Brick and Stone...	15	25	30
Cast Iron.........	6	10	15
Wrought Iron.....	4	6	10
Steel.............	5	7	10

1216. *Twice as much strain is caused by a suddenly applied stress as by one that is gradually applied.* For this reason a larger safety factor is used for shocks than for steady stresses. In general, the factor of safety for a given material must be chosen according to the nature of the stress.

The designer usually chooses his own factors of safety. If the material has been tested, or the specifications call for a certain strength, then the factor of safety can be chosen accordingly.

Example.—Assuming the mortar and brick to be of the same strength, how many tons could be safely laid upon a brick column 2 feet square and 8 feet high?

Solution.—$P = A\ S_s = 2 \times 2 \times 144 \times 2{,}500 = 1{,}440{,}000$ lb. = 720 tons. The factor of safety for this case is 15 (see Art. **1215** and Table **24**); hence, $720 \div 15 = 48$ tons. Ans.

Example.—What must be the diameter of the journals of a wrought iron locomotive axle to resist shearing safely, the weight on the axle being 40,000 pounds?

Solution.—Let f be the factor of safety; then, $P = \frac{A\ S_s}{f}$, or $A = \frac{P f}{S_s}$. Since the axle has two journals, the stress on each journal is 20,000 lb. Owing to inequalities in the track, the load is not a steady one, but varies; for this reason, the factor of safety will be taken as 6. Then, $A = \frac{20{,}000 \times 6}{50{,}000} = 2.4$ sq. in. Therefore, $d = \sqrt{\frac{2.4}{.7854}} = 1\frac{3}{4}''$. Ans.

Example.—Considering the piston rod of a steam engine as if its length were less than ten times its diameter, what must be the diameter of a steel rod, if the piston is 18 inches in diameter and the steam pressure is 110 pounds per square inch ?

Solution.—Area of piston is $18^2 \times .7854 = 254.47$ sq. in. $254.47 \times 110 =$ 27,991.7 lb., or, say, 28,000 lb. = stress in the rod. $A = \frac{P f}{S_2} = \frac{28,000 \times 10}{150,000}$ $= 1.87''$, nearly. Hence, diameter $= \sqrt{\frac{1.87}{.7854}} = 1.543''$, say $1\frac{9}{16}''$. Ans.

1217. When designing a machine, care should be taken (1) *to make every part strong enough to resist any stress likely to be applied to it; and* (2) *to make all parts of equal strength.* The reason for the first statement is obvious, and the second should be equally clear, since no machine can be stronger than its weakest part (proportioned, of course, for the stress it is to bear), and those parts of the machine which are stronger than others contain an excess of material which is wasted. In actual practice, however, this second rule is frequently modified. Some machines are intended to be massive and rigid, and need an excess of material to make them so; in others, there are difficulties in casting that modify the rule, etc., etc. In most cases, the designer must rely on his own judgment.

EXAMPLES FOR PRACTICE.

1. A cast iron bar is subjected to a steady tensile stress of 120,000 pounds. The cross-section is an ellipse whose axes are 6 and 4 inches. (*a*) What is the stress per square inch ? (*b*) What load will the bar carry with safety ?
Ans. { (*a*) 6,366.18 lb. per sq. in.
{ (*b*) 62,832 lb.

2. How much will a piece of steel 2 inches square and 10 inches long shorten under a load of 300,000 pounds ? Ans. .025".

3. A cylindrical wooden pin $1\frac{3}{4}$ inches in diameter is subjected to a double shearing stress. If the stress is suddenly applied, what total force is necessary to shear the pin ? Ans. 7,216 lb.

4. A wrought iron tie rod is $\frac{3}{4}$ inch diameter; how long must it be to lengthen $\frac{3}{8}$ inch under a steady pull of 5,000 pounds ? Ans. 69 ft.

5. A steel bar having a cross-section of 5" × 4" and 14 feet long is lengthened .036 inch by a steady pull of 120,000 pounds; what is its coefficient of elasticity ? Ans. 28,000,000 lb. per sq. in.

6. Which is the stronger, weight for weight, a bar of chestnut wood whose tensile strength is 12,000 pounds per square inch and specific gravity .61, or a bar of steel whose tensile strength is 125,000 pounds per square inch?

7. What should be the diameter of a cast-iron pin subjected to a suddenly applied double shearing stress of 40,000 pounds to withstand the shocks with safety? Ans. $4\frac{3}{8}''$, nearly.

8. What safe steady load may be placed upon a brick column 2 feet square and 9 feet high? Ans. 96,000 lb.

PIPES AND CYLINDERS.

1218. A pipe or cylinder subjected to a pressure of steam or water is strained equally in all its parts, and, when rupture occurs, it is in the direction of its length.

Let $d =$ inside diameter of pipe in inches;
$l =$ length of pipe in inches;
$p =$ pressure in pounds per square inch;
$P =$ total pressure.

Then, $P = p\,l\,d$.

This formula is derived from a principle of hydrostatics that the pressure of water in any direction is equal to the pressure on a plane perpendicular to that direction. In Fig. 254, suppose the direction of pressure to be as shown by the arrows; $A\ B$ would then be the plane perpendicular to this direction, the width of the plane being equal to the diameter, and the length equal to the length of the pipe. The area of the plane would then be $l \times d$, and the total pressure $P = p \times l \times d$, as above.

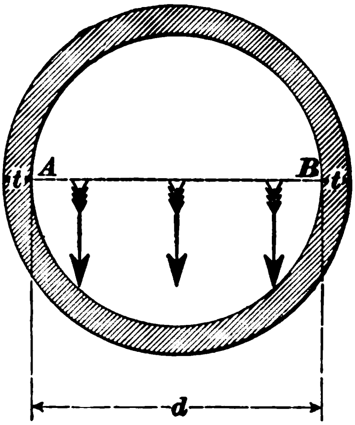


FIG. 254.

Suppose the pipe to have a thickness t, and let S be the working strength of the material; then, the resistance of the pipe on *each* side is $t\,l\,S$. Resistance must equal pressure; therefore, $p\,l\,d = 2\,t\,l\,S$, or

$$p\,d = 2\,t\,S. \qquad (68.)$$

Also, $p = \frac{2tS}{d}$, which is the maximum pressure a pipe of a given material and of given dimensions can stand.

The pressure of water per square inch may be found by the formula $p = .434\,h$, where h is the head in feet. In pipes where shocks are likely to occur, the factor of safety should be high. The thickness of a pipe to resist a given pressure varies directly as its diameter, the pressure remaining constant.

EXAMPLE.—Find the factor of safety for a cast-iron water pipe 12 inches in diameter and ¾ inch thick, under a head of 350 feet.

SOLUTION.—Here p, pressure per square inch, equals $.434\,h = .434 \times 350 = 151.9$ lb. Substituting, in formula **68**, the values given,

$$151.9 \times 12 = 2 \times \tfrac{3}{4} \times S, \text{ or } S = 1{,}215.2 \text{ lb. per sq. in.}$$

In Table 21, Art. **1209**, the ultimate tensile strength of cast iron is given as 20,000 pounds per square inch; then, the factor of safety is $f = \frac{20{,}000}{1{,}215.2} = 16 +$. The pipe would, therefore, be secure against shocks.

EXAMPLE.—Find the proper thickness for a wrought-iron steam pipe 18 inches in diameter to resist a pressure of 140 pounds per square inch.

SOLUTION.—Using a factor of safety of 10, the working strength $S = \frac{55{,}000}{10} = 5{,}500$ lb. per sq. in. From formula **68**, $t = \frac{pd}{2S} = \frac{140 \times 18}{2 \times 5{,}500} = 0.23$ in. In practice, however, the thickness is made somewhat greater than the formula requires.

CYLINDERS.

1219. The tendency of a cylinder subjected to internal pressure is to fail or rupture in the direction of its length, the same as a pipe.

Let π (pronounced *pi*) be the ratio of the circumference of a circle to its diameter $= 3.1416$; then, $\frac{1}{4}\pi = .7854 =$ ratio of area to the square of the diameter.

The total pressure on the cylinder head $= \frac{1}{4}\pi d^2 p$. Let $S =$ working unit stress $= \frac{S_1}{f}$, then $\pi\, d\, t\, S =$ the resistance to rupture caused by the pressure acting on the opposite cylinder heads and tending to elongate the cylinder. Since the resistance must equal the force, or pressure, $\frac{1}{4}\pi d^2 p = \pi d t S$, or

$$p d = 4 t S. \qquad \textbf{(69.)}$$

$$\text{Also, } p = \frac{4 t S}{d}.$$

Since, for longitudinal rupture, $p = \frac{2 t S}{d}$, it is seen that a cylinder is twice as strong against transverse rupture as against longitudinal rupture. Hence, other things being equal, the cylinder will always fail by longitudinal rupture.

1220. The foregoing formulas are for comparatively thin pipes and cylinders, in which the thickness is less than about $\frac{1}{10}$ inside radius. For pipes and cylinders whose thickness is greater than $\frac{1}{10}$ radius, use the following formula, in which r = the inner radius, and the other letters have the same meaning as before:

$$p = \frac{S t}{r + t}. \qquad \textbf{(70.)}$$

Substituting the values given in the example in Art. **1218**, in formula **70**, instead of formula **68**,

$$p = \frac{S t}{r + t}, \text{ or } S = \frac{p(r + t)}{t} = \frac{151.9 \times (6 + \frac{3}{4})}{\frac{3}{4}} =$$
$$151.9 \times 6\tfrac{3}{4} \times \tfrac{4}{3} = 1{,}367.1 \text{ lb.}$$

When formula **68** was used, $S = 1{,}215.2$ lb.; hence, formula **70** gives, for this case, a value $12\frac{1}{2}\%$, or $\frac{1}{8}$ greater.

The formula for spheres is the same as that for transverse rupture of cylinders, or $p d = 4 t S$.

1221. A cylinder under external pressure is theoretically in a similar condition to one under internal pressure, so long as its cross-section remains a true circle. A uniform internal pressure tends to preserve the true circular form, but an external pressure tends to increase the slightest variation from the circle, and to render the cross-section elliptical. The distortion, when once begun, increases rapidly, and failure occurs by the collapsing of the tube rather than by the crushing of the material. The flues of a steam boiler are the most common instances of cylinders subjected to external pressure.

The letters having the same meaning as before, the following formula gives the collapsing pressure in pounds per square inch for wrought-iron pipe:

$$p = 9{,}600{,}000 \frac{t^{2.18}}{l\,d}. \qquad \textbf{(71.)}$$

Example.—What must be the thickness of a boiler tube 2 inches in diameter and 11 feet long, if the steam pressure is to be not over 160 pounds per square inch?

Solution.—Using formula **71**, with a factor of safety of 10, and solving for t,

$$t = \sqrt[2.18]{\frac{10\,p\,l\,d}{9{,}600{,}000}} = \sqrt[2.18]{\frac{10 \times 160 \times 11 \times 12 \times 2}{9{,}600{,}000}} = \sqrt[2.18]{\frac{11}{250}}.$$

Hence, $\log t = \dfrac{\log 11 - \log 250}{2.18} = \bar{1}.37773$, or $t = .2386''$, say $\frac{1}{4}''$.

EXAMPLES FOR PRACTICE.

1. What must be the thickness of a 16-inch cast-iron stand pipe which is subjected to a head of water of 250 feet? Assume that the stress is steady. Ans. .26″.

2. What should be the thickness of a wrought-iron boiler flue 15 feet long, 4 inches in diameter, and subjected to an external pressure of 200 pounds per square inch? Ans. .42″.

3. What pressure per square inch can be safely sustained by a cast-iron cylinder 12 inches in diameter and 3 inches thick?
Ans. 1,111⅑ lb. per sq. in.

4. What external pressure per square inch can a wrought-iron pipe 20 feet long, 3 inches in diameter, and ⅜ inch thick, safely sustain and be secure against shocks? Ans. 157.2 lb. per sq. in.

5. A cast-iron cylinder 14 inches in diameter sustains a total pressure of 125 tons; what is the necessary thickness, assuming that the pressure is gradually applied, and that the cylinder is not subjected to shocks? Ans. 6.65″.

6. A cylindrical boiler shell 3 feet in diameter is subjected to a steady hydrostatic pressure of 180 pounds per square inch. What should its thickness be if made of steel having a tensile strength of 60,000 pounds per square inch? Ans. .27″.

BEAMS.

1222. Any bar resting upon supports in a horizontal position is called a **beam.**

1223. A **simple beam** is a beam resting upon two supports very near its ends.

1224. A **cantilever** is a beam resting upon one support in its middle, or which has one end fixed (as in a wall) and the other end free.

1225. A **restrained beam** is one which has both ends fixed (as a plate riveted to its supports at both ends).

1226. A **continuous beam** is one which rests upon more than two supports.

In this Course, the continuous beam will not be discussed, as the subject requires a knowledge of higher mathematics.

THE VERTICAL SHEAR.

1227. In Fig. 241, Art. **1157,** imagine that part of the beam at a minute distance to the left of a vertical line passing through the point of support *A*, to be acted upon by the reaction $R_1 = 160$ pounds, and that part to the right of the line to be acted upon by the equal downward force due to the load. The two forces acting in opposite directions tend to shear the beam.

Suppose the line had been situated at the point marked *3* instead of at *A*; the reaction upwards would then be partly counterbalanced by the 50-pound weight, and the total reaction at this point would be $160 - 50 = 110$ pounds. Since the upward reaction must equal the downward load at the same point, the downward force at *3* also equals 110 pounds, and the shear at this point is 110 pounds. At the point *6*, or any point between *5* and *8*, the downward force due to the weight at the left is $50 + 80 = 130$ pounds, and the upward reaction is 160 pounds. The resultant shear is therefore, 160 − 130 = 30 pounds. At any point between *8* and *10*, the shear is 160 − (50 + 80 + 100) = − 70 pounds. The negative sign means nothing more than that the weights exceed the reaction of the left-hand support.

The vertical shear equals the reaction of the left-hand support, minus all the loads on the beam to the left of the point considered.

For a simple beam, the greatest positive shear is at the left-hand support, and the greatest negative shear is at the right-hand support, and both shears are equal to the reactions at those points.

1228. The vertical shear may be represented graphically, as shown in Fig. 255, which is Fig. 241 repeated. Draw the force diagram, continue the lines of action of R_1 and

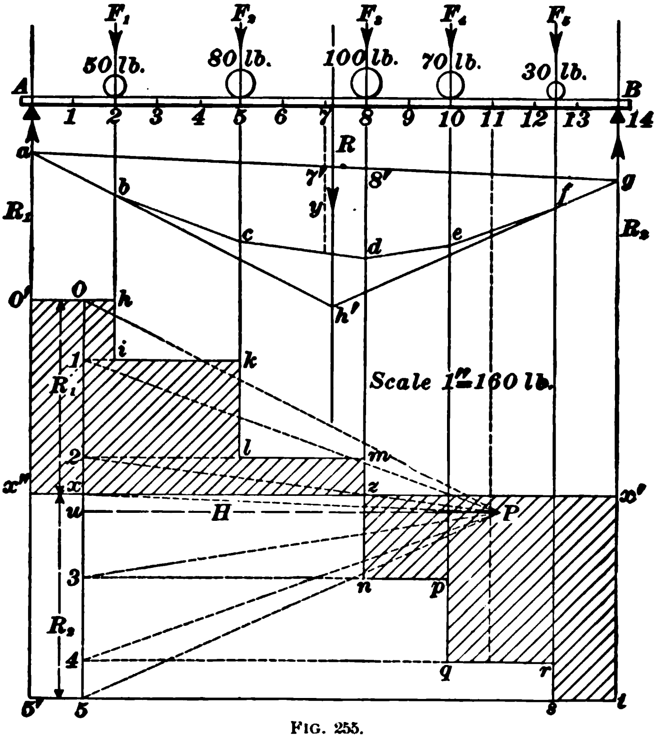


FIG. 255.

R_2 downwards and make $O'\,5' = O\,5$. Through x draw the horizontal line $x\,x'$, called the **shear axis.** The vertical shear is the same for any point between A and 2 and $= O\,x = O'\,x'' = 160$; hence, draw $O'\,h$ parallel to $x\,x'$, and any ordinate measured from $x\,x'$ to $O'\,h$, between O' and $h = 160$ pounds = the vertical shear at any point between O' and h when $O'\,h = A\,2$. Through 1, draw $1\,k$ and project the points 2 and 5 upon it, in i and k. Then, the length of the ordinate between $x\,x'$ and $i\,k =$ the vertical shear between 2 and 5. In the same way, find the remaining points l, m, etc. The broken line $O'\,h\,i \ldots\ldots t$ is called the **shear line;** and the figure $O'\,h\,i$ ______ $r\,s\,t\,x'\,x''$ is called the **shear diagram.** To find the shear at any point, as 11, project the point upon the shear axis and measure the ordinate to the shear line, drawn through the projected point. If the ordinate is measured from the shear axis upwards, the vertical shear is positive; if downwards, it is negative. For the point 11, the vertical shear $= -140$ lb. The maximum negative vertical shear is -170 lb. $= x'\,t = x\,5$. The greatest shear, whether positive or negative, is the one which the beam must be designed to withstand.

1229. A beam seldom fails through shearing, but generally breaks by reason of the load bending and breaking it; that is, through flexure. In order to design a beam to resist flexure, the greatest (maximum) bending moment must be known.

The ***bending moment*** *at any point of a beam is the algebraic sum of the moments of all of the forces (the reaction included) acting upon the beam, on either side of that point, the point being considered as the center of moments.*

The expression "algebraic sum" refers to the fact that, when considering the forces acting at the left of the point taken, the moments of all the forces acting upwards are considered positive, and the moments of all the forces acting downwards are considered negative. Hence, the algebraic sum is the moment of the left reaction about the given point, minus the sum of the moments about the same point of all the downward forces between the reaction and the given point. Should there be any force or forces acting upwards, their moments must be *added*, since they are positive. If the forces on the right of the point are considered, all lever arms are negative, distances to the left of the point being +, to the right, – (see Art. **1127**). Hence, the downward forces give positive moments, and the right reaction gives a negative moment. This is as it should be, for the downward forces on the right and the upward forces on the left tend to rotate the beam in the direction of the hands of a watch, while the downward forces on the left and the upward forces on the right tend to rotate the beam in the opposite direction.

1230. To find the bending moment for any point of a beam, as *7* in Fig. 255, by the graphical method, draw the vertical line *7-7'* through the point. Let $y =$ that part of the line included between $a\ g$ and $a\ c\ f\ g$ of the equilibrium polygon (= vertical intercept). Then, $H \times y =$ the bending moment. H, of course, = the pole distance $= P\ u$. For any other point on the beam, the bending moment is found in the same manner—i. e., by drawing a vertical line through the point and measuring that part of it included between the upper and lower lines of the equilibrium polygon. The scale to which the intercept y is measured is the same as that used in drawing the length $A\ B$ of the beam. The pole distance H is measured to the same scale as $O\ 5$. In the present case, $y = 2.05$ feet, and $H = 349$ pounds; hence, the bending moment for the point *7* is $H\ y = 349 \times 2.05 = 715.45$ foot-pounds.

NOTE.—The expression "foot-pounds," used in stating the value of a moment, must not be confounded with foot-pounds of work. The former means simply that a force has been multiplied by a distance, while the latter means that a resistance has been overcome through a distance. In expressing the value of a moment, the force is usually measured in pounds or tons, and the distance in inches or feet; hence, the moment may be inch-pounds or inch-tons and foot-pounds or foot-tons. Unless otherwise stated, the bending moment will always be expressed in inch-pounds, the length of the beam being always measured in inches, and, consequently, also the length of the intercept y.

1231. If expressed in inch-pounds, the value of the moment just found is $715.45 \times 12 = 8{,}585.4$ inch-pounds.

It will be noticed that after the force diagram and equilibrium polygon have been drawn the value of the bending moment depends solely upon the value of y, since the length $P u = H$ is fixed. At the points a and g, directly under the points of support of the beam, $y = 0$; hence, for these two points, bending moment $= Hy = H \times 0 = 0$; that is, for any simple beam, the bending moment at either support is zero. The greatest value for the bending moment will evidently be at the point *8*, since $d\,8'$ is the longest vertical line which can be included between *a g* and *a c f g*.

The figure *a c e g a* is called the **diagram of bending moments.**

1232. Consider now the case of a simple beam uniformly loaded. Let the distance between the supports in Fig. 256 be 12 feet, and let the total load uniformly distributed over the beam be 216 pounds. Divide the load into a convenient number of equal parts, the more the better, say 12, in this case. The load which each part represents is $216 \div 12 = 18$ pounds. For convenience, lay off *O C* on the vertical through the left-hand support, equal to 216 pounds to the scale chosen, and divide it into 12 equal parts, *O a*, *a b*, etc.; each part will represent 18 pounds to the same scale. Choose a pole *P*, and draw the rays *P O*, *P a*, *P b*, etc. Through the points *d*, *e*, etc., the centers of gravity of the equal subdivisions of the load, draw the verticals *d 1*, *e 3*, *f 5*, etc., intersecting the horizontals through *O*, *a*, *b*, etc., in *1*, *3*, *5*, etc. Draw *O 1*, *1-2*, *2-3*, *3-4*, *4-5*, etc., and the broken line thus found will be the shear line. In drawing the shear line for a uniform load in this manner, it is assumed that each part of the total load is concentrated at its center of gravity, or, in other words, that a force equal to each small load (18 lb.) acts upon the beam at each of the points *d*, *e*, *f*, etc.

Construct the diagram of bending moments in the ordinary manner by drawing *g i* parallel to *P O*, *i k* parallel to *P a*, etc. Draw *P M* parallel to *g h*, and *M q* horizontal; *M q* is the shear axis. When the load is uniform and the work has been done correctly, *O M* should equal *M C*—that is, the reactions of the two supports are equal.

1233. The shear line is not a broken line in reality, as shown, since the load is distributed evenly over the entire beam, and not divided into small loads concentrated at *d*, *e*, *f*, etc., as was assumed. The points *1*, *3*, etc., are evidently too high, and the points *2*, *4*, etc., too low. To find the real shear line, bisect the lines *1-2*, *3-4*, etc., locating the points *6*, *7*, *8*, etc. Trace a line through *O*, *6*, *7*, *8*..................*N*, and it will be the real shear line. For all cases of a uniform load, the shear line will be straight and may be drawn from *O* to *N* directly.

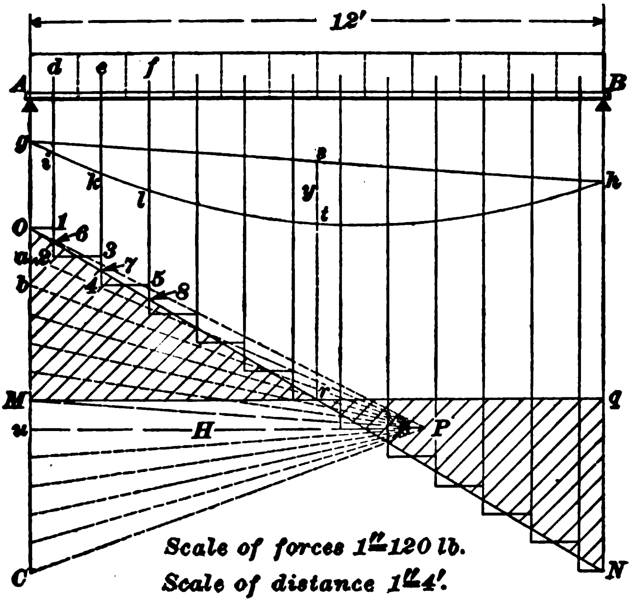


FIG. 256.

The diagram of bending moments is also not quite exact, but may be corrected by tracing a curve through the points *g* and *h*, which shall be tangent to *g i*, *i k*, *k l*, etc., at their middle points, as shown in the figure.

1234. To find the maximum bending moment for any beam having two supports, draw a vertical line through r, where the shear line cuts the shear axis, and the intercept, $s\,t = y$, on the diagram of bending moments, will be the greatest value of y, and, consequently, the greatest bending moment $= H \times y$. In the present example, $H = P\,u = 247.5$ pounds and maximum $y = s\,t = 15.72$ inches; hence, the maximum bending moment $= H\,y = 247.5 \times 15.72 = 3{,}890.7$ inch-pounds. The above is true, no matter how the beam may be loaded. In Fig. 255, the shear line cuts the shear axis at z, and $d\,8'$, on the vertical through z, was shown previously to be the maximum intercept.

1235. If there is a uniform load on the beam and one or more concentrated loads, as in Fig. 257, the method of finding the moment diagram and shear line is similar to that used in the last case. In Fig. 257, let the length $A\,B$ of the beam be 15 feet; the uniformly distributed load 180 pounds, with two concentrated loads, one of 24 pounds, 5 feet from A, and the other of 30 pounds, 11 feet from A. Draw the beam and loads as shown, the length of the beam and the distances of the weights from A being drawn to scale. Divide the uniform load into a convenient number of equal parts, say 10 in this case; each part will then represent $\frac{180}{10} = 18$ pounds. Draw $A\,O\,C$, as usual, and lay off 3 of the 18-pound subdivisions from O downwards; then lay off 24 pounds, to represent the first weight. Lay off four more of the equal subdivisions, and then the 30-pound weight. Finally, lay off the remaining three equal subdivisions, the point C being the end of the last 18-pound subdivision. $O\,C$ should then equal $180 + 24 + 30 = 234$ pounds to the scale to which the weights were laid off. It will be noticed in the above that the equal subdivisions of the load were laid off on $O\,C$ until that one was reached on which the concentrated loads rested, and that the concentrated loads were laid off before the equal subdivision on which the concentrated load rested was laid off. Had one of the concentrated loads been to the right of the center of gravity of the subdivision on which it rests, the weight of the subdivision would have been laid off first. Locate the centers of gravity of the equal subdivisions and draw the verticals through them as in the previous case. Choose a pole *P*, draw the rays, the diagram of bending moments, and the shear axis *M q*, as previously described.

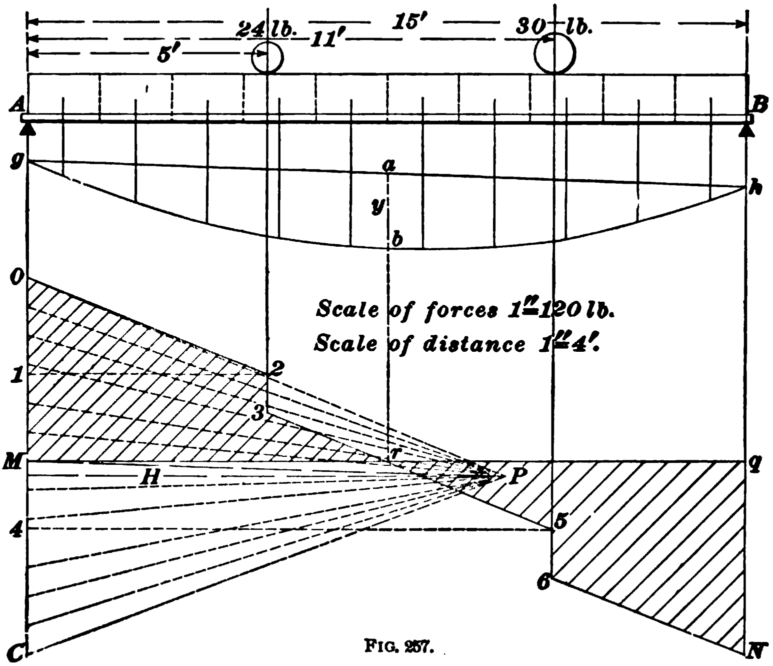


FIG. 257.

To find the maximum bending moment, the shear line must be drawn and its intersection with the shear axis determined.

The weight of the uniform load per foot of length is $\frac{180}{15} = 12$ lb. The weight of that part between *A* and the center of the 24-lb. weight is $12 \times 5 = 60$ lb. Lay off *O 1* = 60 lb. and draw *1-2* horizontal, cutting the vertical through the center of the 24-lb. weight in *2*. Draw *O 2* and it will be a part of the shear line. Lay off *2-3* vertically downwards, equal to 24 lb., locating the point *3*. Lay off *O 4* equal to $12 \times 11 + 24 = 156$ lb., and draw the horizontal *4-5*, cutting the vertical through the center of the 30-lb.

weight in *5*. Join *3* and *5* by the straight line *3-5*. Lay off *5-6* vertically downwards equal to 30 lb. Draw the horizontal *C N*, intersecting the vertical *B q N* in *N* and join *6* and *N* by the straight line *6 N*. The broken line *O 2-3-5-6 N* is the shear line and cuts the shear axis in the point *r*. Drawing the vertical *r b a*, through *r*, it intersects the moment diagram in *a* and *b*; hence, *a b* is the maximum *y*. For this case, the maximum bending moment $= H \times y = 300 \times 18.36 = 5{,}508$ in.-lb.

It is better, in ordinary practice, to choose the pole *P* on a line perpendicular to *O C* and at some distance from *O C* easily measured with the scale used to lay off *O C*. Thus, suppose *O C* to be laid off to a scale of 1″ = 60 lb. At some convenient point, as *M*, on *O C*, draw a perpendicular line and choose a point on this line whose distance from *O C* shall be easily measurable, say 3½″. Then, *H* is known to be exactly $60 \times 3\frac{1}{2} = 210$ lb. and will not have to be measured when finding the bending moment *H y*.

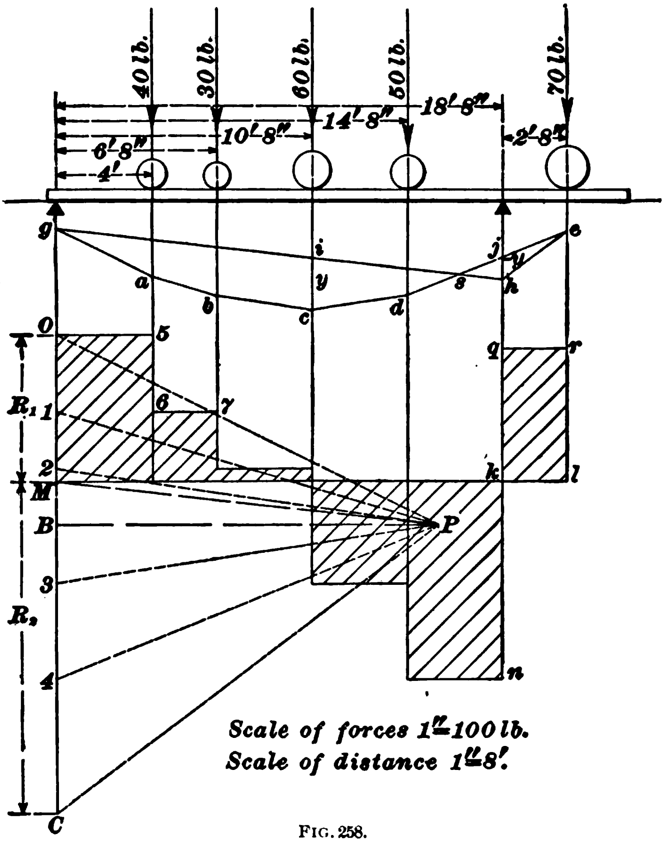
As stated in Art. **1171,** the preceding operations may be reduced to a very simple rule, which, for convenience, will be here repeated. It must, however, be remembered that this rule gives only approximate results: to obtain exact results it is necessary to use the analytical instead of the graphical method.

Rule.—*Divide the beam into an even number of parts* (the more the better), *and the uniform load into half as many. Consider the divisions of the load as concentrated loads applied, alternately, at the various points of division of the beam* (the ends included); *that is, the first point of division* (*the support*) *carries no load, the next one does, the following one does not*, etc. *Then proceed as in the ordinary case of concentrated loads.*

EXAMPLE.—Find the reactions of the supports, the maximum bending moments, and the maximum vertical shear of the beam shown in Fig. 258, which has one overhanging end.

SOLUTION.—Draw *O C* and the force diagram in the usual manner. Construct the bottom curve of the moment diagram in the same manner as in the preceding cases. The side *d e* is parallel to *P 4*; *e h* is parallel to *P C*, and cuts the vertical through the right reaction in *h*.

Join *h* and *g* by the straight line *g h*, and draw *P M* parallel to *g h*. Then, $O M = 77$ lb. = left reaction and $M C = 173$ lb. = right reaction. The shear line is drawn as in the previous cases until the point *n*, on the vertical *h n*, is reached; *k n* here denotes the vertical shear for any point between the 50-lb. weight and the right support, and this shear is negative. The point *k* denotes the intersection of the shear axis and the vertical through the right support. For any point to the right of *k*, between *k* and *l*, the vertical shear is positive, and is equal to 70 lb.; hence, lay off *k q* upwards equal to 70 lb., and draw *q r* horizontal. The line, *O* 5-6-7 .... *n q r* is the shear line. Measuring *O M*, *k n*, and *k q*, it is found that $O M = 77$ lb., $k n = -103$ lb., and $k q = 70$ lb.; therefore, $k n = -103$ lb. = maximum vertical shear; *c i* is evidently the maximum *y*; hence, the maximum bending moment =



Scale of forces 1" = 100 lb.
Scale of distance 1" = 8'.

FIG. 258.

$Hy = PB \times ci = 200 \times 26'' = 5{,}200$ in.-lb. Any value of y measured in the polygon $gabcdsg$ is positive, and any value measured in the triangle ehs is negative. Consequently, the bending moment for any point between s and the vertical, through the center of the 70-lb. weight, is negative, since $H \times (-y) = -Hy$. In all cases, when designing beams with overhanging studs, the maximum bending moment, whether positive or negative, should be used. In the present case, the maximum negative y, or hj, is less than the maximum positive y, or ci; therefore, the maximum negative bending moment is also less than the maximum positive moment.

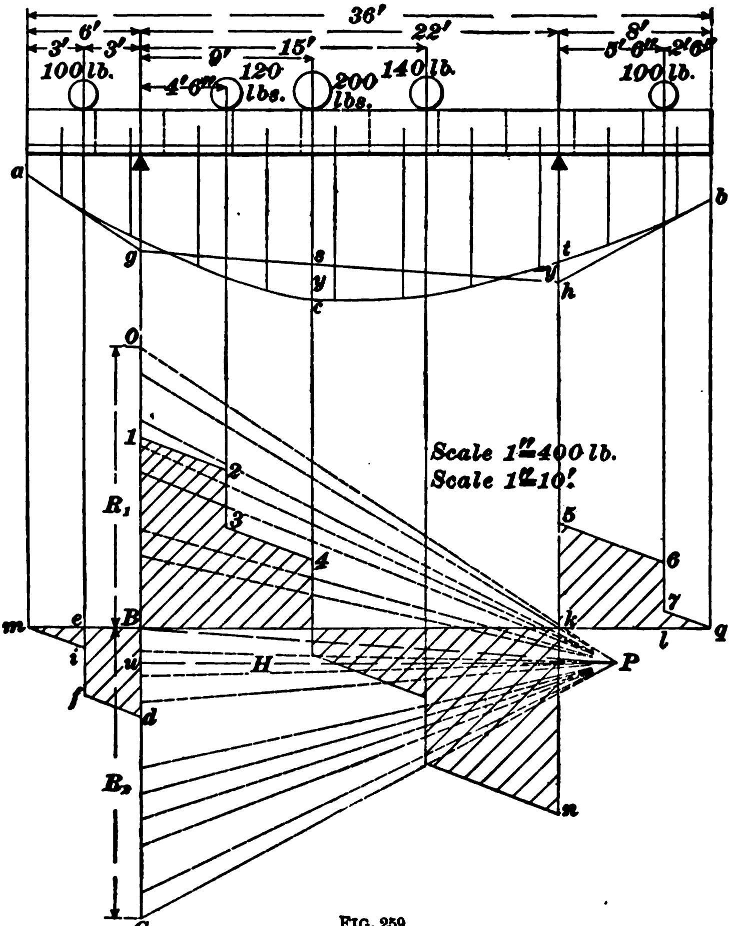


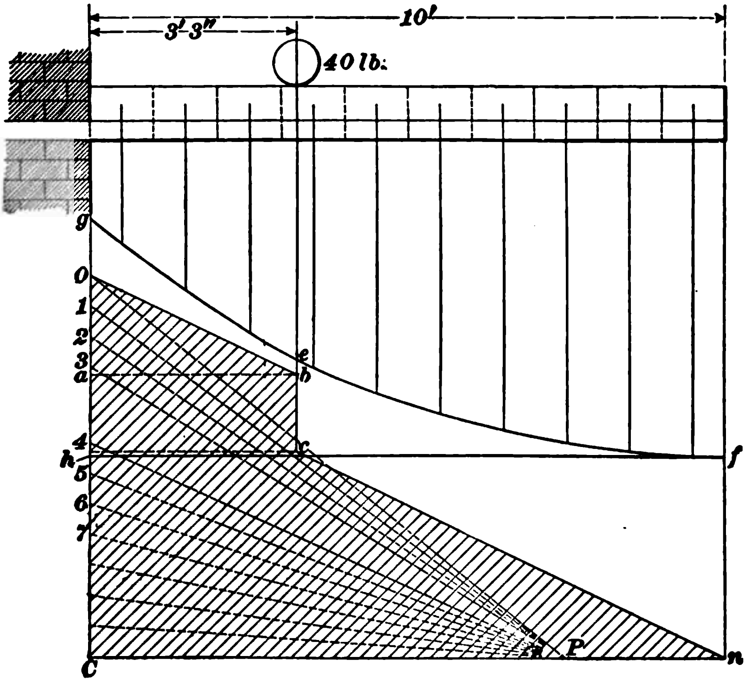
FIG. 259.

EXAMPLE.—Fig. 259. shows a beam overhanging both supports, which carries a uniform load of 15 pounds per foot of length, and has five concentrated loads at distances from the supports as marked in the figure. Required the reactions of the supports, the maximum positive and negative bending moments, and the maximum vertical shear.

SOLUTION.—Construct the force diagram and equilibrium polygon in the ordinary manner, continuing the latter to b and a, points on the verticals passing through the ends of the beam. Draw $b\,h$ and $a\,g$ parallel to $P\,C$ and $P\,O$ respectively, intersecting the verticals through the points of support in h and g. Join g and h, and draw $P\,B$ parallel to $g\,h$. Then, $O\,B = 588$ lb. $= R_1$, and $B\,C = 612$ lb. $= R_2$. Through B, draw the shear axis $m\,q$. To draw the shear line, proceed as follows: The shear for any point to the left of the left support is negative, and for any point to the right of the right support is positive; between the two supports it is positive or negative, according to the manner of loading, and the point considered. The negative shear at the left support $= 15 \times 6 + 100 = 190$ lb.; hence, lay off $B\,d$ downwards equal to 190 lb. For a point to a minute distance to the right of e, the shear is $15 \times 3 + 100 = 145$ lb. $= e\,f$, and for a minute distance to the left, it is $15 \times 3 = 45$ lb. $= e\,i$; at m, it is 0. Consequently, $m\,i\,f\,d$ is the shear line between the end of the beam and the left support. Lay off $O\,1 = B\,d = 190$ lb. and draw the shear line *1-2-3-4*..................*n* in the usual manner. Draw the shear line *5-6-7 q*, laying off $k\,5 = 15 \times 8 + 100 = 220$ lb.; $l\,6 = 100 + 15 \times 2\frac{1}{2} = 137\frac{1}{2}$ lb., and $6\text{-}7 = 100$ lb. At q, the vertical shear is again 0. The broken line *m i f d 1-2-3*————*n 5-6-7 q* is the shear line. The maximum positive bending moment is $H \times y = P\,u \times s\,c = 1{,}000 \times 22.5 = 22{,}500$ in.-lb. The greatest maximum negative moment is $H \times (-y) = P\,u \times -t\,h = 1{,}000 \times -11.6 = -11{,}600$ in.-lb. It will be noticed that there are two negative and one positive maximum bending moments.

1236. The student should now be able to find the bending moment for any beam having but two supports, whatever the character of the loading. The bending moment plays a very important part in the flexure of beams, which is the next subject to be considered. In all cases of loading heretofore considered, no other forces than the loads themselves have been considered. Should forces act upon the beam which are not vertical, the force polygon will be no longer a straight line, but a broken line somewhat similar in character to *O 1-2-3-4-5* in Fig. 240, Art. **1156.**

In Fig. 260 is shown a cantilever beam projecting 10 ft. from the wall. It carries a uniform load of 16 pounds per foot of length, and a concentrated load of 40 pounds at a distance of 3¼ feet from the wall. The maximum bending moment is required. The method is similar to the last, except that, as there is but one support, there can be but one reaction. Since the beam is 10 feet long, the total weight of the uniform load is 16 × 10 = 160 pounds. Hence, the reaction = 160 + 40 = 200 pounds.



Scale of forces 1" = 100 lb.
Scale of distance 1" = 3'.
FIG. 260.

Draw *O C* equal to 200 lb., to some convenient scale. Draw *P C* perpendicular to *O C* at *C* and choose the pole *P* at a convenient distance from *O C*. For convenience, divide the uniform load into 10 equal parts, as shown; then, each part will represent 16 lb. Lay off *O 1*, *1-2*, *2-3*, each equal to 16 lb., and *3-4* equal to 40 lb. Also, *4-5*, *5-6*, *6-7*, etc., equal to 16 lb. each. 3 ft. 3 in. = $3\frac{1}{4}$ ft., and $16 \times 3\frac{1}{4} = 52$ lb. Lay off $O\,a = 52$ lb., and draw *a b*, meeting the vertical through the center of the weight in *b*. Draw *O b*; lay off *b c* equal to 40 lb. and draw *C n*. *O b c n* is the shear line. The perpendicular through the point *C* coincides with *P C*; hence, *n P C* is the shear axis. Draw the line *g c f* of the moment diagram as in the previous cases. In Fig. 257, and in the preceding figures, the line *g h* was drawn connecting the extreme ends of the bottom line; in other words, it joined the points where the equilibrium polygon cut the lines of direction of the reactions of the supports. This cannot be done in this case, because there is no right reaction; therefore, *g h* must be drawn by means of some other property of the polygon. In the previous cases, the shear axis was drawn perpendicular to *O C* at the point where a line through the pole *P* parallel to *g h* cut *O C*, or, in other words, the shear axis was drawn through the point which marked the end *M* of the left reaction *O M*. In the present case, Fig. 260, the point *C* is the end of the left reaction; hence, *f h* is parallel to *P C*, *h g* is the maximum *y*, and the bending moment $= H\,y = P\,C \times h\,g = 250 \times 45 =$ 11,250 inch-pounds.

It will also be noticed that the distance $y = h\,g$ is measured from the line *h f* upwards, while, for all points between the supports in the previous examples, this distance was measured downwards. The same observation is true for any point between *h* and *f*; hence, for a cantilever, all bending moments are negative.

1237. All the cases of beams heretofore given might have been solved by analytical methods—that is, by algebraic processes and formulas; but the graphical method is to be preferred, as it is usually shorter, very nearly as accurate, and less liable to error. Moreover, when the diagram has once been drawn, both the bending moment and the shear for any point can be instantly determined. Except in special cases, the graphical method will be employed to determine the maximum bending moment, but in some particular cases formulas will be used, by which the result can be obtained more quickly than by the graphical method.

EXAMPLES FOR PRACTICE.

1. A simple beam 24 feet long carries 4 concentrated loads of 160, 180, 240, and 120 pounds at distances from the left support of 4, 10, 16, and 21 feet, respectively. (*a*) What are the values of the reactions? (*b*) What is the maximum bending moment in inch-pounds?

$$\text{Ans.}\begin{cases}(a)\ R_1 = 333\tfrac{1}{3}\text{ lb.};\ R_2 = 366\tfrac{2}{3}\text{ lb.}\\(b)\ 28{,}480\text{ in.-lb.}\end{cases}$$

2. A simple beam carries a uniform load of 40 pounds per foot, and supports two concentrated loads of 500 and 400 pounds at distances from the left support of 5 and 12 feet, respectively. The length of the beam is 18 feet. What are (*a*) the reactions? (*b*) the maximum bending moment in inch-pounds?

$$\text{Ans.}\begin{cases}(a)\ R_1 = 854\tfrac{4}{9}\text{ lb.};\ R_2 = 765\tfrac{5}{9}\text{ lb.}\\(b)\ 48{,}846\text{ in.-lb.}\end{cases}$$

3. A cantilever projects 10 feet from a wall and carries a uniform load of 60 pounds per foot; it also supports three concentrated loads of 100, 300, and 500 pounds at distances from the wall of 2, 5, and 9 feet, respectively. Required, (*a*) the maximum vertical shear, and (*b*) the maximum bending moment in inch-pounds.

$$\text{Ans.}\begin{cases}(a)\ -1{,}500\text{ lb.}\\(b)\ -110{,}400\text{ in.-lb.}\end{cases}$$

4. A beam which overhangs one support sustains six concentrated loads of 160 lb. each at distances from the left support of 4 ft. 9 in., 7 ft., 9 ft. 6 in., 12 ft., 15 ft., and 18 ft. 3 in., respectively, the distance between the supports being 16 ft. What are (*a*) the reactions? (*b*) the maximum bending moment?

$$\text{Ans.}\begin{cases}(a)\ R_1 = 295\text{ lb.};\ R_2 = 665\text{ lb.}\\(b)\ 20{,}460\text{ in.-lb.}\end{cases}$$

5. A beam which overhangs both supports equally carries a uniform load of 80 pounds per foot, and has a load of 1,000 pounds in the middle, the length of the beam being 15 feet, and the distance between the supports 8 feet. What is (*a*) the vertical shear? (*b*) the maximum bending moment?

$$\text{Ans.}\begin{cases}(a)\ 820\text{ lb.}\\(b)\ 25{,}800\text{ in.-lb.}\end{cases}$$

NOTE.—The student may not obtain the exact answers given above, but if his results do not differ by more than 1%, he will know that his method is right.

NEUTRAL AXIS.

1238. In Fig. 261, let *A B C D* represent a cantilever. Suppose that a force *F* acts upon it at its extremity *A*. The beam will then be bent into the shape shown by *A' B C D'*.

It is evident from the cut that the upper part $A'\,B$ is now longer than it was before the force was applied; i. e., $A'\,B$ is longer than $A\,B$. It is also evident that $D'\,C$ is shorter than $D\,C$. Hence, the effect of the force F in bending the beam is to lengthen the upper fibers and to shorten the lower ones. In other words, when a cantilever is bent through the action of a load, the upper fibers are in tension and the lower fibers in compression. The reverse is the case in a simple beam in which the upper fibers are in compression and the lower fibers in tension. Further consideration will show that there must be a fiber, $S\,S''$, which is neither lengthened nor shortened when the beam is bent, i. e., $S'\,S'' = S\,S''$. When the beam is straight the fiber $S\,S''$, which is neither lengthened nor shortened when the beam is bent, is called the **neutral line.** There may be

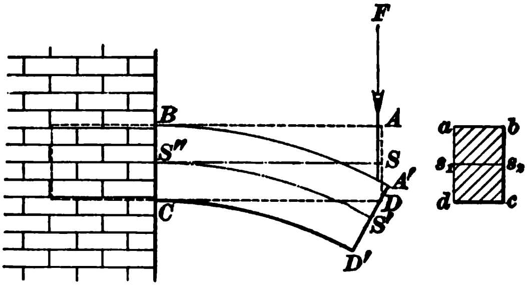


FIG. 261.

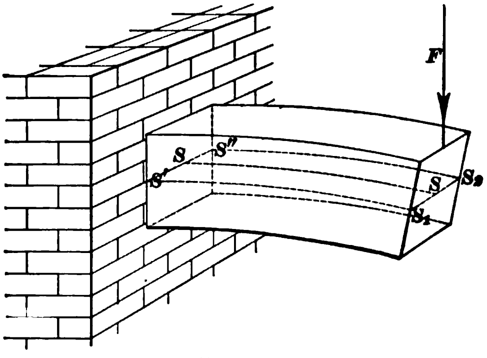


FIG. 262.

any number of neutral lines dependent only on the width of the beam. For, let $b\,a\,d\,c$, Fig. 261, be a cross-section of the beam. Project s upon it in s_1. Make $b\,s_2 = a\,s_1$ and draw $s_1\,s_2$; then, any line in the beam which touches $s_1\,s_2$ and is parallel to $S\,S''$ is a neutral line. Thus, in Fig. 262, $S_1\,S'$, $S\,S$, $S_2\,S''$, etc., are all neutral lines. The line $S_1\,S_2$ is called the **neutral axis,** and the surface $S_1\,S'\,S''\,S_2$ is called the **neutral surface.** The neutral axis, then, is the line of intersection of a cross-section with the neutral surface. It is shown in works on mechanics that *the neutral axis always passes through the center of gravity of the cross-section of the beam.*

1239. Experimental Law.—*When a beam is bent, the horizontal elongation (or compression) of any fiber is directly proportional to its distance from the neutral surface, and, since the strains are directly proportional to the horizontal stresses in each fiber, they are also directly proportional to their distances from the neutral surface, provided the elastic limit is not exceeded.*

1240. Suppose the beam to be a rectangular prism, then every cross-section will be a rectangle, and the neutral axis will pass through the center o. See Fig. 263.

Let the perpendicular distance from the neutral axis $M\,N$ to the *outermost fiber* be denoted by c, and the horizontal unit stress (stress per square inch) at the distance c from the axis by S. If a is the area of a fiber, the stress on the outermost fibers will be $a\,S$. The stress on a fiber at the distance unity (1 inch) from $M\,N$ is $\frac{a\,S}{c}$; and the stress on a fiber at the distance r_1 is $\frac{a\,S}{c} \times r_1 = \frac{a\,S\,r_1}{c}$. The moment of this stress about the axis $M\,N$ is $\frac{a\,S\,r_1}{c} \times r_1 = \frac{a\,S\,r_1^2}{c} = \frac{S}{c}\,a\,r_1^2$. The moment of the

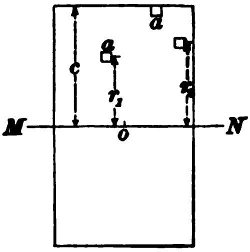


FIG. 263.

stress on any other fiber at a distance r_2 from MN is evidently $\frac{S}{c} a r_2^2$, and for a distance r_3, $\frac{S}{c} a r_3^2$, etc. If n is the number of fibers, the sum M of the moments of the horizontal stresses on all the fibers is

$$M = \frac{S}{c} a r_1^2 + \frac{S}{c} a r_2^2 + \frac{S}{c} a r_3^2 +, \text{ etc.}, =$$

$$\frac{S}{c}(a r_1^2 + a r_2^2 + a r_3^2 + \ldots\ldots a r_n^2) = \frac{S}{c} a (r_1^2 + r_2^2 + r_3^2 + \ldots\ldots r_n^2).$$

Now, let r be a quantity whose square equals the mean of the squares of $r_1, r_2, r_3, \ldots\ldots r_n$. Then, $r^2 = \frac{r_1^2 + r_2^2 + \ldots\ldots + r_n^2}{n}$; and, therefore, $r_1^2 + r_2^2 + r_3^2 + \ldots\ldots + r_n^2 = n r^2$. Substituting above, we get $M = \frac{S}{c} n a r^2$. But, since a is the area of one fiber, $n a$ is the area of all the fibers—that is, the area A of the cross-section; hence, the sum of the moments of all the horizontal stresses $= \frac{S}{c} A r^2$.

1241. The expression $A r^2$, which is found by dividing a section into a large number of minute areas (a, a, etc.), multiplying each area by the square of its distance from an axis (r_1^2, r_2^2, r_3^2, etc.), and then adding the products thus obtained, is called the **moment of inertia** of the section with respect to that axis, and is usually denoted by the letter I. Hence,

$$I = A r^2. \qquad \textbf{(72.)}$$

1242. The quantity r, whose square is the mean of the squares of all the distances of the minute areas from the axis, is called the **radius of gyration.**

1243. The sum of the moments of all the horizontal stresses may then be written as $\frac{S}{c} A r^2 = \frac{S}{c} I$, or $S \frac{I}{c}$. This expression is called the **resisting moment,** since it is the measure of the resistance of the beam to bending (and, consequently, to breaking) when loaded. The resisting moment must equal the bending moment when the beam is in equilibrium; hence; denoting the bending moment by M,

$$M = S_4 \frac{I}{c}. \qquad (73.)$$

1244. The values of I and c depend wholly upon the size and form of the cross-section of the beam, and S_4 is the ultimate strength of flexure of the material.

In Table 25, the average ultimate strength of flexure S_4 is given for a number of different materials.

TABLE 25.

Material.	Ultimate Strength of Flexure in Lb. per Sq. In. S_4.
Cast Iron	38,000
Wrought Iron	45,000
Steel	120,000
Brass	17,000
Ash	14,000
Brick	1,000
Stone	2,000
Hemlock	7,200
Oak, white	12,500
Pine, white	9,000
Pine, yellow	11,000
Hickory	16,000

1245. Exact values of I for most cross-sections can only be determined by the aid of the calculus. The least value of I occurs when the axis passes through the center of gravity of the cross-section—that is, when I is found with reference to the neutral axis.

The least moments of inertia for a number of different sections are given in the Table of Moments of Inertia; also, the area of the sections and the values of c. The dotted line indicates the position of the neutral axis, about which the moment of inertia is taken.

In the Table of Moments of Inertia, A is the area of the section, and π = ratio of the circumference of a circle to its diameter = 3.1416. It will be noticed that d is always taken vertically.

1246. To use formula **73,** find the bending moment in inch-pounds by the graphical method previously described, or calculate it by means of the Table of Bending Moments. If it is desired to find the size of a beam that will safely resist a given bending moment, take S_4 from Table 25, Art. **1244,** and divide it by the proper factor of safety taken from Table 24. Then, formula **73** becomes

$$M = \frac{S_4 I}{f c}. \qquad \textbf{(74.)}$$

From this $\frac{I}{c} = \frac{M f}{S_4}$. Substituting the values of M, f, and S_4, the value of $\frac{I}{c}$ is found. The kind and shape of beam having been decided upon, the size can be so proportioned that $\frac{I}{c}$ for the section shall not be less than the value calculated above. An example will make this clear.

EXAMPLE.—What should be the size of an ash girder to resist safely a bending moment of 28,000 inch-pounds, the cross-section to be rectangular and the load steady?

SOLUTION.—$M = \frac{S_4 I}{f c}$, or $\frac{I}{c} = \frac{M f}{S_4}$.

$M = 28{,}000$; from Table 25, Art. **1244,** $S_4 = 14{,}000$; from Table 24, Art. **1215,** $f = 8$.

Then, $\frac{I}{c} = \frac{28{,}000 \times 8}{14{,}000} = 16$.

From the Table of Moments of Inertia, $I = \frac{b d^3}{12}$ and $c = \frac{d}{2}$ for a rectangle; hence, $\frac{I}{c} = \frac{b d^3}{12} \times \frac{2}{d} = \frac{b d^2}{6} = 16$, or $b d^2 = 96$. Any number of values of b and d can be found that will satisfy this equation. If b is taken as 6 inches, $d^2 = \frac{96}{6} = 16$ and $d = \sqrt{16} = 4$. Hence, the beam may be a 6 × 4, with the short side vertical. When possible, it is always better to have the longer side vertical. If b is taken as 2 inches, $d^2 = 48$ and $d = \sqrt{48} = 7$ inches, nearly; hence, a 2 × 7 will also answer the purpose. The advantage of using a 2 × 7 instead of a 6 × 4 is evident, since the 6 × 4 contains nearly twice as much material as the 2 × 7. Thus, the area of the cross-section of a 6 × 4 is 24 square inches, and of a 2 × 7, 14 square inches. Moreover, the 2 × 7, with its long side vertical, is slightly stronger than the 6 × 4, with its short side vertical, since $\frac{I}{c} = \frac{2 \times 7^2}{6} = 16\frac{1}{3}$ for the former, and $\frac{6 \times 4^2}{6} = 16$ for the latter. If the 6 × 4 had its longer side vertical, thus making it a 4 × 6, $\frac{I}{c}$ would then equal $\frac{4 \times 6^2}{6} = 24$, and the safe bending moment could be increased to $M = \frac{S_4 I}{f c} = \frac{14{,}000 \times 24}{8} = 42{,}000$ in.-lb.

1247. If the breaking bending moment, form, and size of the cross-section of the beam are known, the ultimate strength of flexure S_4 can be readily found from formula **73,** by substituting the values of M, I, and c, and solving for S_4.

Example.—A cast iron bar, 2 inches square, breaks when the maximum bending moment = 63,360 inch-pounds; what is its ultimate strength of flexure?

Solution.—$M = S_4 \frac{I}{c}$, or $S_4 = \frac{M c}{I}$. $c = \frac{d}{2} = 1''$.

$I = \frac{d^4}{12} = \frac{2^4}{12}$; therefore, $S_4 = \frac{63{,}360 \times 1}{\frac{1}{12} \times 2^4} = 47{,}520$ lb. per sq. in.

1248. In order to save time in calculating, the bending moments for cases of simple loading are given in the Table of Bending Moments. W denotes a concentrated load, and w the uniform load per inch of length. All dimensions are to be taken in inches when using the formulas.

For any other manner of loading than is described in the Table of Bending Moments, the maximum bending moment must be found by the graphical method.

Example.—A wrought iron cantilever, 6 feet long, carries a uniform load of 50 pounds per inch. The cross-section of the beam is an equilateral triangle, with the vertex downwards; what should be the length of a side?

Solution.—$M = \frac{w l^2}{2}$, from the Table of Bending Moments, $= \frac{50 \times (6 \times 12)^2}{2} = 129{,}600$ in.-lb. $I = \frac{b d^3}{36}$ and $c = \frac{2}{3} d$, from the Table of Moments of Inertia; hence, $\frac{I}{c} = \frac{b\,d^2}{24}$. $S_4 = 45{,}000$, from Table 25, Art. **1244**, and $f = 4$, from Table 24, Art. **1215**. Therefore, $129{,}600 = \frac{45{,}000}{4} \times \frac{b\,d^2}{24}$, or $b\,d^2 = \frac{129{,}600 \times 4 \times 24}{45{,}000} = 276.48$. Since an equilateral triangle has been specified, b can not be given *any* convenient value in order to find d. For an equilateral triangle, $d = b \sin 60^\circ = .866\,b$. Hence, $b\,d^2 = b\,(.866\,b)^2 = .75\,b^3$. Therefore, $b\,d^2 = .75\,b^3 = 276.48$ or $b = \sqrt[3]{\frac{276.48}{.75}}$ $\log b = \frac{\log 276.48 - \log .75}{3} = .85553$, or $b = 7.17''$, nearly.

Example.—What weight would be required to break a round steel bar 4 inches in diameter, 16 feet long, fixed at both ends and loaded in the middle?

Solution.—Use formula **73**; $M = \frac{S_4\,I}{c}$. Here $M = \frac{W\,l}{8}$, from the Table of Bending Moments; $S_4 = 120{,}000$; $\frac{I}{c} = \frac{\frac{1}{64}\pi\,d^4}{\frac{1}{2}d} = \frac{\pi\,d^3}{32}$. Hence, $\frac{W\,l}{8} = \frac{W \times (16 \times 12)}{8} = \frac{120{,}000 \times 3.1416 \times 4^3}{32}$, or $W = \frac{120{,}000 \times 3.1416 \times 64 \times 8}{16 \times 12 \times 32} = 31{,}416$ lb.

DEFLECTION OF BEAMS.

1249. The deflection, or amount of bending, produced in a beam by one or more loads is given by certain general formulas, whose derivation is too complicated to be given here. We shall give the formulas only, illustrating their application by examples.

In the third column of the Table of Bending Moments are given expressions for the value of the greatest deflection of a beam when loaded as shown in the first column. From this it is seen that the deflection s equals a constant (depending upon the manner of loading the beam and upon the condition of the ends—whether fixed or free), multiplied by $\frac{W\,l^3}{E\,I}$. Let a represent the constant and s the deflection; then,

$$s = a\frac{W\,l^3}{E\,I}. \qquad (75.)$$

In the above formula, E = coefficient of elasticity, and is to be taken from Table 25, Art. **1356,** l = length in inches, W = concentrated load in pounds, W' = total uniform load in pounds, and I is the moment of inertia about the neutral axis; a has values varying from $\frac{1}{384}$ to $\frac{1}{3}$.

It will be noticed that the deflection is given for only nine cases; for any other manner of loading a beam than those here given, it is necessary to use the calculus to obtain the deflection.

EXAMPLE.—What will be the maximum deflection of a simple wooden beam 9 feet long, whose cross-section is an ellipse, having axes of 6 inches and 4 inches (short axis vertical), under a concentrated load of 1,000 pounds?

SOLUTION.—Use formula **75,** $s = a\frac{Wl^3}{EI}$. From the Table of Bending Moments,

$$a = \frac{1}{48},\ W = 1{,}000 \text{ lb.},\ l = 9 \times 12 = 108 \text{ in.},$$

$$E = 1{,}500{,}000 \text{ and } I = \frac{\pi b d^3}{64} = \frac{\pi \times 6 \times 4^3}{64}.$$

$$\text{Hence, } s = \frac{1{,}000 \times 108^3 \times 64}{48 \times 1{,}500{,}000 \times \pi \times 6 \times 64} = .9282''.$$

1250. The principal use of the formula for deflection is to determine by its means the stiffness of a beam or shaft. In designing machinery, it frequently occurs that a piece may be strong enough to sustain the load with perfect safety, but the deflection may be more than circumstances will permit; in this case, the piece must be made larger than is really necessary for mere strength. An example of this occurs in the case of locomotive guides, and the upper guides of a steam engine when the engine runs under. It is obvious that they must be very stiff. In such cases it is usual to allow a certain deflection, and then proportion the piece so that the deflection shall not exceed the amount decided upon.

EXAMPLE.—The breadth of the guides of a certain locomotive is not to exceed $2\frac{1}{4}$ inches. Regarding the guides as fixed at both ends, (*a*) what must be their depth to resist a load of 10,000 pounds at the middle? The guides are made of cast iron and are 38 inches long between the points of support. (*b*) What weight would these guides be able to support with safety? The deflection must not exceed $\frac{1}{100}$ of an inch. The cross-section is, of course, rectangular.

SOLUTION.—Since the load comes on two guides, each piece must support $10{,}000 \div 2 = 5{,}000$ lb. In the formula,

$$s = \frac{1''}{200} = a\frac{Wl^3}{EI},\ a = \frac{1}{192} \text{ for this case, } W = 5{,}000,\ l = 38,$$

$$E = 15{,}000{,}000, \text{ and } I = \frac{bd^3}{12} = \frac{2\frac{1}{4}d^3}{12} = \frac{3}{16}d^3. \text{ Substituting,}$$

$$s = \frac{5{,}000 \times 38^3 \times 16}{192 \times 15{,}000{,}000 \times 3d^3} = \frac{1}{200},$$

$$\text{or } d = \sqrt[3]{\frac{5{,}000 \times 38^3 \times 16 \times 200}{192 \times 15{,}000{,}000 \times 3}} = 4.67'', \text{ nearly, say } 4\tfrac{11}{16}''.$$

To find the weight which these guides could support with safety, use formula **74**, $M = \frac{S_4 I}{fc}$, in which $M = \frac{Wl}{8}$, $S_4 = 38{,}000, f = 10, \frac{I}{c} = \frac{bd^2}{6} = \frac{2\frac{1}{4} \times (4\frac{11}{16})^2}{6} = \frac{50{,}625}{6{,}144}$. Substituting, $W = \frac{38{,}000 \times 50{,}625 \times 8}{10 \times 6{,}144 \times 38} = 6{,}592$ lb.

Hence, the beam is over 30% stronger than necessary, the extra depth being required for stiffness.

EXAMPLES FOR PRACTICE.

1. How much will a simple wooden beam 16 ft. long, 2 in. wide and 4 in. deep deflect under a load in the middle of 120 lb.? Ans. 1.106".

2. What should be the size of a rectangular yellow pine girder 20 ft. long, to sustain a uniformly distributed load of 1,800 lb.? Assume a factor of safety for a varying stress. Ans. 5" × 8".

3. A hollow cylindrical beam, fixed at both ends, has diameters of 8 in. and 10 in. If the beam is 30 ft. long and is made of cast iron, (*a*) what steady load will it safely support at 15 ft. from one of the supports? (*b*) What force will be required to rupture the beam if applied at this point?

Ans. $\begin{cases} (a)\ 8{,}158 \text{ lb.} \\ (b)\ 48{,}946 \text{ lb.} \end{cases}$

4. A simple cylindrical wrought-iron beam, resting upon supports 24 ft. apart, sustains three concentrated loads of 350 lb. each, at distances from one of the supports of 5, 12, and 19 ft.; what should be the diameter of the beam to withstand shocks safely? Ans. 4.71", say 4¾".

5. Find the value of $\frac{I}{c}$ for a hollow rectangle whose outside dimensions are 10 in. and 13 in., and inside dimensions are 8 in. and 10 in.; (*a*) when the long side is vertical; (*b*) when the short side is vertical.

Ans. $\begin{cases} (a)\ 179.103 \\ (b)\ 131\tfrac{1}{3}. \end{cases}$

6. What is the deflection of a steel bar 1 in. square and 6 ft. long, which supports a load of 100 lb. at the center? Ans. .31104".

7. Which will be the stronger, a beam whose cross-section is an equilateral triangle, one side measuring 15 in., or one whose cross-section is a square, one side measuring 9 in.? Both beams are of the same length. Ans. The one having the square cross-section.

8. A wooden beam of rectangular cross-section sustains a uniform load of 50 lb. per foot. If the beam is 8" × 14" and 16 ft. long, how much more will it deflect when the short side is vertical than when the long side is vertical? Ans. .055417".

COMPARISON OF STRENGTH AND STIFFNESS OF BEAMS.

1251. Consider two rectangular beams, loaded in the same manner, having the same lengths and bending moments, but different breadths and depths. Then, $M = S\frac{I}{c} = S\frac{b d^2}{6}$ (1) and $M = S\frac{b_1 d_1^2}{6}$ (2). Dividing (1) by (2), $\frac{M}{M} = \frac{6 S b d^2}{6 S b_1 d_1^2} = \frac{b d^2}{b_1 d_1^2} = 1$, or $b d^2 = b_1 d_1^2$ (3).

Equation 3 shows that, if both beams have the same depth, their strengths will vary directly as their breadths, i. e., if the breadths are increased 2, 3, 4, etc., times, their strengths will also be increased 2, 3, 4, etc., times. It also shows that, if the breadths are the same and the depths are increased, the strengths will vary as the square of the depth, i. e., if the depths are increased 2, 3, 4, etc., times, the strengths will be increased 4, 9, 16, etc., times. Hence, it is always best, when possible, to have the long side of a beam vertical. If the bending moments are the same, but the weights and lengths are different, $M = g\,W l$ (1) and $M = g\,W_1 l_1$ (2), when g denotes the fraction $\frac{1}{2}$, $\frac{1}{4}$, etc., according to the manner in which the ends are secured, and the manner of loading. Dividing (1) by (2) $\frac{M}{M} = \frac{g\,W l}{g\,W_1 l_1}$, or $W l = W_1 l_1$ (3).

Equation 3 shows that if the load W or W_1 be increased, the length l or l_1 must be decreased; consequently, the strength of a beam loaded with a given weight varies inversely as its length, i. e., if the load be increased 2, 3, 4, etc., times, the length must be shortened 2, 3, 4, etc., times, the breadth and depth remaining the same.

EXAMPLE.—If a simple beam, loaded in the middle, has its breadth and depth reduced one-half, what proportion of the original load could it carry ?

SOLUTION.—In the preceding paragraphs, it was shown that the strength varied as the product of the breadth and the square of the depth, or $b_1 d_1^2 = \frac{1}{2} \times (\frac{1}{2})^2 = \frac{1}{8}$. Consequently, the beam can support only $\frac{1}{8}$ of the original load. Had the breadth remained the same, $(\frac{1}{2})^2 = \frac{1}{4}$ of the original load could have been supported. Had the depth remained the same, $\frac{1}{2}$ of the original load could have been supported.

EXAMPLE.—A beam 10 ft. long, loaded in the middle, has a breadth of 4 in. and a depth of 6 in. The length is increased to 12 ft., the breadth to 6 in., and the depth to 8 in.; how many times the original load can it now support ?

SOLUTION.—The strength varies directly as the product of the breadth and square of the depth and inversely as the length, or as $\frac{b\,d^2}{l}$. If b, d, and l denote the original sizes, the strength of the two beams will be to each other as $\frac{b_1 d_1^2}{l} : \frac{b\,d^2}{l}$, or as $\frac{6 \times 8^2}{12} : \frac{4 \times 6^2}{10}$; $\frac{6 \times 8^2}{12} = 32$ and $\frac{4 \times 6^2}{10} = 14.4$. $\frac{32}{14.4} = 2\frac{2}{9}$. Consequently, the beam will support a load $2\frac{2}{9}$ times as great as the original beam.

1252. By a process of reasoning similar to that employed above, it can be shown that the maximum deflection of a beam varies inversely as the cube of the depth and directly as the cube of the length. In other words, if the depth be increased 2, 3, 4, etc., times, the deflection will be decreased 8, 27, 64, etc., times; and, if the length be increased 2, 3, 4, etc., times, the deflection will be increased 8, 27, 64, etc., times. Hence, if a beam is required to be very stiff, the length should be made as short and the depth as great as circumstances will permit.

COLUMNS.

1253. When a piece ten or more times as long as its least diameter or side (in general, its least transverse dimension) is subjected to compression, it is called a **column** or **pillar.**

The ordinary rules for compression do not apply to columns, for the reason that when a long piece is loaded beyond a certain amount, it buckles and tends to fail by flexure. This combination of flexure and compression causes the column to break under a load considerably less than that required to merely crush the material. It is likewise evident that the strength of a column is principally dependent upon its diameter, since that part having the least thickness is the part that buckles, or bends. A column free to turn in any direction, having a cross-section of 3″ × 8″, is not nearly so strong as one whose cross-section is 4″ × 6″. The strength of a very long column varies, practically, inversely as the square of the length; i. e., if a column *b* is twice as long as a column *a*, the strength of $b = (\frac{1}{2})^2 = \frac{1}{4}$ the strength of *a*, the cross-sections being equal.

1254. The conditions of the ends of a column play a very important part in determining their strength, and must always be taken into consideration. In Fig. 264, are shown three classes of columns. The column marked *a* is used in architecture, while the columns similar to *b* and *c* are used in bridge and machine construction.

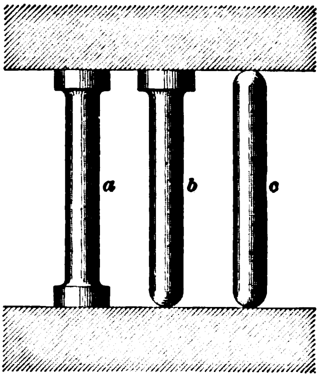


FIG. 264.

According to theory, which is confirmed by experiment, a column having one end flat and the other rounded, like *b*, is $2\frac{1}{4}$ times as strong as a column having both ends rounded, like *c*.

One having both ends flat, like *a*, is 4 times as strong as *c*, which has both ends rounded, the three columns being of the same length. If the length of *c* be taken as 1, the length of *b* may be $1\frac{1}{2}$, and that of *a* may be 2 for equal strength, the cross-sections all being the same; for, since the strengths vary inversely as the squares of the lengths, the strength of *c* is to that of *b* as $1 : \frac{1}{(1\frac{1}{2})^2}$ or as $1 : \frac{4}{9}$. But, since *b* is $2\frac{1}{4} = \frac{9}{4}$ times as strong as *c*, $\frac{4}{9} \times \frac{9}{4} = 1$, or, the length of *b* being $1\frac{1}{2}$ times that of *c*, its strength is the same. Similarly, when *a* is twice as long as *c* its strength is the same.

1255. Columns like *b* and *c* do not actually occur in practice, an eye being formed at the end of the column and a pin inserted, forming what may be termed a ***hinged end.*** A steam engine connecting-rod is a good example of a column having two hinged ends, and a piston rod of a column having one end hinged and one end flat.

1256. There are numerous formulas for calculating the strength of columns, but the one that gives the most satisfactory results for columns of all lengths is the following:

$$W = \frac{S_2 A}{f\left(1 + \frac{A l^2}{g I}\right)}. \qquad \textbf{(76.)}$$

In this formula, W = load, S_2 = ultimate strength for compression, taken from Table 22, Art. **1210,** A = area of section of a column in square inches, f = factor of safety, l = length in inches, g = constant, to be taken from Table 26, and I = least moment of inertia of the cross-section—that is, the moment of inertia about an axis passing through the center of gravity of the cross-section and parallel to the longest side. In other words, if the column has a rectangular cross-section, whose longer side is *b* and shorter side *d*, the least moment of inertia is $\frac{b\,d^3}{12}$, the axis in this case being parallel to the long sides *b*. The values of *g* are given in the following table:

TABLE 26.

Material.	Both Ends Fixed.	One End Hinged.	Both Ends Hinged.
Timber..................	3,000	1,690	750
Cast Iron................	5,000	2,810	1,250
Wrought Iron............	36,000	20,250	9,000
Steel....................	25,000	14,060	6,250

EXAMPLE.—The section of a hollow rectangular cast-iron column has the following dimensions (see Table of Moments of Inertia): $d = 8''$, $d_2 = 6''$, $b = 6''$, and $b_2 = 3\frac{1}{2}''$. If the length is 10 feet and the ends are fixed, what steady load will the column sustain with safety?

SOLUTION.—From the Table of Moments of Inertia, least $I = \frac{1}{12}(d b^3 - d_1 b_1^3) = \frac{8 \times 6^3 - 6 \times 3.5^3}{12} = 122.5625$. $A = b d - b_1 d_1 = 6 \times 8 - 3.5 \times 6 = 27$ sq. in. $S_2 = 90{,}000$, $f = 6$, $l = 10 \times 12 = 120''$, and $g = 5{,}000$. Therefore, $W = \frac{90{,}000 \times 27}{6\left(1 + \frac{27 \times 120^2}{5{,}000 \times 122.5625}\right)} = \frac{2{,}430{,}000}{9.806} = 247{,}800$ lb., nearly. Ans.

Had the column been less than $10 \times 6 = 60$ in. $= 5$ ft. long, the safe load would have been $\frac{90{,}000 \times 27}{6} = 405{,}000$ lb. Had it been twice as long, it would have supported a safe load of only $\frac{90{,}000 \times 27}{6\left(1 + \frac{27 \times 240^2}{5{,}000 \times 122.5625}\right)} = 114{,}500$ lb., nearly.

1257. In the actual designing of a column, the size of the cross-section is not known, but the form (square, round, etc.) is known, also the length, material, condition of ends and load it is to carry. To find the size of the cross-section, substitute $\frac{S_2}{f}$ in formula **65,** for S, and solve for A, obtaining $A = \frac{Pf}{S_2}$. Substituting in this equation the values of $P (= W)$, f, and S_2, this gives the value of A for a short piece less than 10 times the length of the shortest side, or diameter. Assume a value of A somewhat larger than that just found, and dimension a cross-section of the form chosen so that its area shall equal that assumed. Calculate the moment of inertia and substitute the values of W, A, I, l, and g in formula **76,** and solve for $\frac{S_2}{f}$. If the result last found equals the value of $\frac{S_2}{f}$ taken from Tables 22 and 24, the assumed dimensions are correct; if larger, the assumed dimensions must be increased; if smaller, they should be diminished, and in both cases the value of $\frac{S_2}{f}$ should be recalculated. An example will serve to illustrate the process.

Example.—What should be the diameter of a steel piston rod 5 feet long, the diameter of the piston being 18 inches and the greatest pressure 130 pounds per square inch?

Solution.—S_2 for this case $= 150{,}000$ lb. Since the piston rod is liable to shocks, a factor of safety of 10 should be used; hence, $\frac{S_2}{f} = \frac{150{,}000}{10} = 15{,}000$ lb. The load $W = 18^2 \times .7854 \times 130 = 33{,}081$ lb. $A = \frac{Pf}{S_2} = \frac{33{,}081}{15{,}000} = 2.2$ sq. in., nearly.

Assume that 3 sq. in. are needed. The diameter of a circle corresponding to an area of 3 sq. in. is $\sqrt{\frac{3}{.7854}} = 1.9544''$. Assume the diameter to be $1\frac{15}{16}'' = 1.9375$; the area will be $1.9375^2 \times .7854 = 2.9483$ sq. in. The value of $I = \frac{\pi d^4}{64} = \frac{3.1416 \times (1\frac{15}{16})^4}{64} = .69173$. $W = \frac{S_2 A}{f\left(1 + \frac{A l^2}{g I}\right)}$.

Consequently, $\frac{S_2}{f} = \frac{W}{A}\left(1 + \frac{A l^2}{g I}\right) = \frac{33{,}081}{2.9483}\left(1 + \frac{2.9483 \times (5 \times 12)^2}{14{,}060 \times .69173}\right)$ $= 23{,}465$ lb.

As this value exceeds 15,000 lb., the diameter of the rod must be increased. Trying $2\frac{1}{8}''$ as the diameter, the area is 3.5466 sq. in. and $I = 1.00093$. Substituting these values as before, $\frac{S^2}{f} = \frac{33{,}081}{3.5466}\left(1 + \frac{3.5466 \times 60^2}{14{,}060 \times 1.00093}\right) = 17{,}790$ lb. This is still too large; hence, trying $2\frac{1}{4}''$, the area $= 3.976$ sq. in. $I = 1.258$ and

$$\frac{S_2}{f} = \frac{33{,}081}{3.976}\left(1 + \frac{3.976 \times 60^2}{14{,}060 \times 1.258}\right) = 15{,}052 \text{ lb.}$$

Consequently, the diameter should be $2\frac{1}{4}''$.

EXAMPLES FOR PRACTICE.

1. What safe steady load will a hollow cylindrical cast-iron column support, which is 14 feet long, outside diameter 10 inches, inside diameter 8 inches, and which has flat ends? Ans. 273,500 lb.

2. A hollow wooden column having a square cross-section is to support a steady load of 15,155 pounds. If the thickness of the side is 1¼ inches, length of column 20 feet, and the ends flat, what should be the length of the sides of the cross-section, outside and inside?

Ans. Outside, 9″; inside, 6″.

3. Suppose a wrought-iron connecting-rod to have a rectangular cross-section of uniform size throughout its length. If the diameter of the steam cylinder is 40 inches, steam pressure 110 pounds per square inch, and the length of the rod is 12¼ feet, what should be the dimensions of the cross-section of the rod? Ans. 5¼″ × 9″.

1258. The preceding method for determining the dimensions of the cross-section, when the load and length are given, is perfectly general, and can, therefore, be used in every case. It is, however, somewhat long and cumbersome. For the special cases of square, circular, and rectangular columns, the following formulas may be applied, if preferred. They seem complicated, but, when substitutions are made for the quantities given, the formulas will be found of relatively easy application.

For square columns, the side c of the square is given by the formula

$$c = \sqrt{\frac{Wf}{2S_2} + \sqrt{\frac{Wf}{S_2}\left(\frac{Wf}{4S_2} + \frac{12l^2}{g}\right)}}. \qquad (77.)$$

For circular columns, the diameter d of the circle is given by the formula

$$d = 1.4142\sqrt{\frac{.3183\,Wf}{S_2} + \sqrt{\frac{.3183\,Wf}{S_2}\left(\frac{.3183\,Wf}{S_2} + \frac{16l^2}{g}\right)}}. \qquad (78.)$$

For rectangular columns, assume the shorter dimension (depth $= d$). Then the longer dimension (breadth $= b$) is given by the formula

$$b = \frac{Wf\left(1 + \frac{12l^2}{d^2g}\right)}{dS_2}. \qquad (79.)$$

Should the dimensions given by the last formula be too much out of proportion, a new value may be assumed for d, and a new value found for b.

Example.—Required the section of a square timber pillar to stand a steady load of 20 tons, the length of the column being 30 feet, and its ends both flat.

Solution.—Here $S_2 = 8{,}000$ lb., $f = 8$, $g = 3{,}000$, $W = 40{,}000$ lb., $l = 30 \times 12 = 360$ in. These values, substituted in formula **77**, give

$$c = \sqrt{\frac{40{,}000 \times 8}{2 \times 8{,}000} + \sqrt{\frac{40{,}000 \times 8}{8{,}000}\left(\frac{40{,}000 \times 8}{4 \times 8{,}000} + \frac{12 \times 360^2}{3{,}000}\right)}}$$

$$= \sqrt{20 + \sqrt{40\left(10 + \frac{4 \times 36^2}{10}\right)}}$$

$$= \sqrt{20 + \sqrt{21{,}136}} = \sqrt{20 + 145.35}$$

$$= \sqrt{165.35} = 12.90 = 12\tfrac{7}{8}'', \text{ nearly, or say } 13''.$$

Example.—Let it be required to solve the problem worked out by the general method in Art. **1257.**

Solution.—Here $S_2 = 150{,}000$ lb., $f = 10$, $g = 14{,}060$, $W = 33{,}000$ lb., nearly, and $l = 5 \times 12 = 60$ in. From these data we have

$$\frac{.3183\,Wf}{S_2} = \frac{.3183 \times 33{,}000 \times 10}{150{,}000} = \frac{.3183 \times 11}{5} = .7003,$$

$$\frac{16\,l^2}{g} = \frac{16 \times 60^2}{14{,}060} = \frac{8 \times 360}{703} = 4.0967.$$

Then, formula **78,**

$$d = 1.4142\sqrt{.7003 + \sqrt{.7003 \times 4.7970}}$$

$$= 1.4142\sqrt{.7003 + \sqrt{3.3593}} = 1.4142\sqrt{.7003 + 1.8328}$$

$$= 1.4142 \times 1.5916 = 2.25 = 2\tfrac{1}{4}'',$$

as found by the general or trial method.

The student may apply formula **79** to the solution of example 3 in the preceding article.

TORSION AND SHAFTS.

1259. When a force is applied to a beam in such a manner that it tends to twist it, the stress thus produced is termed **torsion.** In Fig. 265, bc represents a beam fixed at one end; a load W is applied at the end of a lever arm on, which twists the beam. If a straight line cb is drawn parallel to the axis before the load is applied, it will be found, after the weight W has been hung from n, that the line cb will take a position ca, forming a spiral. If the load does not strain the material beyond its elastic limit, *c a* will return to its original position *c b* when *W* is removed. It will also be found that the angles *a c b* and *a o b* are directly proportional to the loads.

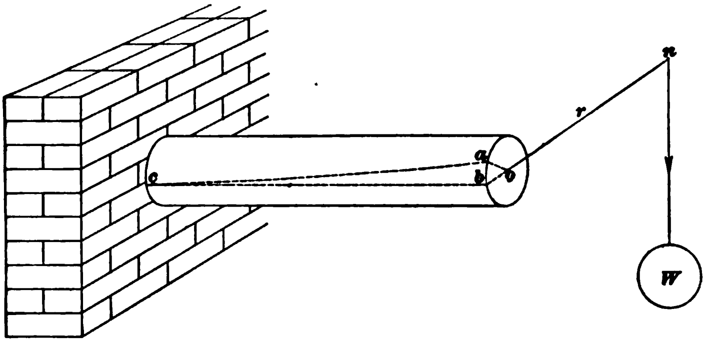


FIG. 265.

Torsion manifests itself in the case of rotating shafts. Instead of one end being fixed, as in the previous case, the reisistance which the shaft has to overcome takes the place of the force which before was necessary for fixing one end. Should the shaft be too small, the resistance will overcome the strength of the material and rupture it.

1260. The angle *a o b*, which may be called the angle of twist, plays an important part in the designing of shafts. For all solid shafts below 11 inches in diameter, the following formula may be used:

$$d = c\sqrt[4]{Pr} = c_1\sqrt[4]{\frac{H}{N}}, \qquad (80.)$$

in which d = diameter of round shaft or the side of a square shaft in inches; c = constant from Table 27; P = force or weight applied to the end of the lever arm, in pounds; r = length of lever arm in inches, from center of shaft to point of application of P; c_1 = constant from same table; H = horsepower transmitted, and N = number of revolutions per minute.

TABLE 27.

Material.	c.		c_1.	
	Round.	Square.	Round.	Square.
Wrought Iron......	.31	.272	4.92	4.31
Cast Iron	.353	.309	5.59	4.89
Steel	.297	.26	4.7	4.11

EXAMPLE.—What should be the diameter of a wrought-iron crank shaft for a 16″ × 20″ steam engine, if the greatest steam pressure is to be 90 lb. per sq. in. ? (Assume that the entire steam pressure is transmitted through the crank-pin at some point of the stroke).

SOLUTION.—Total pressure on piston $= 16^2 \times .7854 \times 90 = 18{,}095.616$, say 18,000 lb. $= P$ in formula **80.** $r = \frac{20}{2} = 10''$. Therefore,

$$d = c\sqrt[4]{Pr} = .31\sqrt[4]{18{,}000 \times 10} = 6.385''.$$

A 6½″ shaft would be sufficiently large.

EXAMPLE.—What horsepower could be safely transmitted by a 7-inch cast iron square shaft making 80 revolutions per minute ?

SOLUTION.—Formula **80** gives, $d = c_1\sqrt[4]{\frac{H}{N}}$, or $H = \frac{Nd^4}{c_1^4} = \frac{80 \times 7^4}{4.89^4} =$ 335.93, say 336 horsepower.

1261. If the diameter of a wrought-iron shaft is greater than 12.4 inches, of a cast-iron shaft greater than 10.3 inches, or of a steel shaft greater than 13.6 inches, the following formula should be used:

$$d = k\sqrt[3]{Pr} = k_1\sqrt[3]{\frac{H}{N}}, \qquad (81.)$$

k and k_1 being taken from Table 28. If the shaft is hollow (round), either of the two following formulas may be used:

$$P = q\left(\frac{d_1^4 - d_2^4}{d_1 r}\right), \qquad (82.)$$

$$\text{or} \qquad H = q_1 N\left(\frac{d_1^4 - d_2^4}{d_1}\right), \qquad (83.)$$

d_1 and d_2 being the outside and inside diameters respectively and q and q_1 constants to be taken from Table 28.

TABLE 28.

Material.	k	k_1	q	q_1
Wrought Iron.......	.0909	3.62	1,335	.0212
Cast Iron...........	.1145	4.56	669	.0106
Steel	.0828	3.3	1,767	.028

EXAMPLE.—What horsepower can be safely transmitted by a hollow wrought-iron shaft making 60 revolutions per minute, and whose diameters are 9½ and 12 inches?

SOLUTION.—

$$H = q_1 N\left(\frac{d_1^4 - d_2^4}{d_1}\right) = .0212 \times 60\left(\frac{12^4 - 9.5^4}{12}\right) = 1,334.65 \text{ H.P.}$$

EXAMPLES FOR PRACTICE.

1. What should be the diameter of a steel shaft to transmit 500 horsepower at 200 revolutions per minute? Ans. 5.91", say $5\frac{15}{16}$".

2. How many horsepower will an 8" round wrought-iron shaft transmit with safety, running at 150 R. P. M.? Ans. 1,048.5 H.P.

3. A hollow cast-iron shaft has an outside diameter of 10 inches and an inside diameter of 6 inches; at what speed should it be run to transmit 750 horsepower? Ans. 81.29 R. P. M.

4. A wrought-iron shaft 4 inches square runs at 110 revolutions per minute; what horsepower will it safely transmit? Ans. 81.6 H.P.

5. What should be the diameter of a wrought-iron shaft to transmit 6,000 horsepower at 100 revolutions per minute?
Ans. 14.172", say $14\frac{3}{16}$".

ROPES.

1262. The strength of hemp and manila ropes varies greatly, depending not so much upon the material and area of cross-section as upon the method of manufacture and the amount of twisting.

Hemp ropes are about 25% to 30% stronger than manila ropes or tarred hemp ropes. Ropes laid with tar wear better than those laid without tar, but their strength and flexibility are greatly reduced. For most purposes, the following formula may be used for the safe working load of any of the three ropes mentioned above:

$$P = 100\,C^2, \qquad (84.)$$

in which $P =$ working load in pounds and $C =$ circumference of rope in inches. This formula gives a factor of safety of from 7½ for manila or tarred hemp rope to about 11 for best three-strand hemp rope. When excessive wear is likely to occur, it is better to make the circumference of the rope considerably larger than that given by the formula.

1263. Wire rope is made by twisting a number of wires (usually 19) together into a strand and then twisting several strands (usually 7) together to form the rope. It is very much stronger than hemp rope, and may be much smaller in size to carry the same load.

For iron wire rope of 7 strands, 19 wires to the strand, the following formula may be used, the letters having the same meaning as in formula **84**:

$$P = 600\,C^2. \qquad \textbf{(85.)}$$

Steel wire ropes should be made of the best quality of steel wire; when so made they are superior to the best iron wire ropes. If made from an inferior quality of steel wire, the ropes are not as good as the better class of iron wire ropes. When substituting steel for iron ropes, the object in view should be to gain an increase of wear rather than to reduce the size. The following formula may be used in computing the size or working strength of the best steel wire rope, 7 strands, 19 wires to the strand:

$$P = 1{,}000\,C^2. \qquad \textbf{(86.)}$$

Formulas **85** and **86** are based on a factor of safety of 6.

1264. When using ropes for the purpose of raising loads to a considerable height, the weight of the rope itself must also be considered and added to the load. The weight of the rope per running foot, for different sizes, may be obtained from the manufacturer's catalogue.

EXAMPLE.—What should be the allowable working load of an iron wire rope whose circumference is 6¾ inches? Weight of rope not to be considered.

SOLUTION.—Using formula **85**,

$$P = 600 \times (6\tfrac{3}{4})^2 = 27{,}337.5 \text{ lb.}$$

EXAMPLE.—The working load, including weight, of a hemp rope is to be 900 pounds; what should be its circumference?

SOLUTION.—Using formula **84**,

$$C = \sqrt{\frac{P}{100}} = \sqrt{\frac{900}{100}} = 3''.$$

1265. In measuring ropes, the circumference is used instead of the diameter, because the ropes are not round and the circumference is not equal to 3.1416 times the diameter. For three strands the circumference is about 2.86 *d*, for seven strands about 3 *d*, *d* being the diameter.

CHAINS.

1266. The size of a chain is always specified by giving the diameter of the iron from which the link is made. The two kinds of chain most generally used are the **open link** chain and the **stud link** chain. The former is shown by (*a*), Fig. 266, and the latter by (*b*). The stud prevents the two sides of a link from coming together when under a heavy pull, and thus strengthens the chain.

It is a good practice to anneal old chains which have become brittle by overstraining. This renders them less liable to snap from sudden jerks. The annealing process reduces their tensile strength, but increases their toughness and ductility, two qualities which are sometimes more important than mere strength.

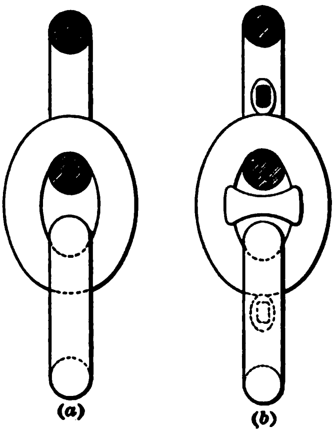


FIG. 266.

Let P = safe load in pounds;
d = diameter of link in inches.

Then, for open link chains, made from a good quality of wrought iron,

$$P = 12{,}000\ d^2, \qquad \textbf{(87.)}$$

and, for stud link chains,

$$P = 18{,}000\ d^2. \qquad \textbf{(88.)}$$

Example.—What load will be safely sustained by a ¾-inch open link chain?

Solution.—Using formula **87**,

$$P = 12{,}000\, d^2 = 12{,}000 \times (\tfrac{3}{4})^2 = 6{,}750 \text{ lb.}$$

Example.—What must be the diameter of a stud link chain to carry a load of 28,125 pounds?

Solution.—Using formula **88**, $P = 18{,}000\, d^2$. Hence,

$$d = \sqrt{\frac{P}{18{,}000}} = \sqrt{\frac{28{,}125}{18{,}000}} = 1\tfrac{1}{4}''.$$

1266*a*. The statement in the last sentence of Art. **1211** may be modified in practice to include pieces whose lengths are not greater than *ten* times their least transverse dimensions, without material error. Although the stress is pure compression only for pieces whose lengths do not exceed *five* times their least transverse dimensions, the results obtained by formula **76** agree so closely with those obtained by formula **67**, when the length does not exceed ten times the least transverse dimension, that the latter may be used in all such cases. When the length is greater than ten times the least transverse dimension, the piece becomes a column, and formula **76** must be used.

ANALYSIS OF STRESSES.

THE TRUSS.

1267. A **truss** is a simple framed structure composed of straight members so connected as to act as one rigid body. While the truss as a whole resists the effect of the external forces acting upon it in much the same manner as shear and bending moment are resisted by a solid beam, each individual member of the truss is subjected only to direct tensile or compressive stress in the direction of its length. In order that this may be the case, the external forces must be applied at the joints of the truss, through which they act upon the structure as a whole.

1268. The simplest possible truss is a triangle, and any truss is merely an assemblage of connected triangles. As the triangle is a rigid figure whose form can not change so long as the length of each of its sides remains the same, it is the primary and essential element of the truss. A short definition of a truss was given in Art. **1120.**

1269. The **external forces** are the loads, including the weight of the structure itself, and the supporting forces, or reactions, all of which tend to distort the structure or change its form.

1270. In bridge engineering any framed structure so designed that the reactions from the superimposed static loads are vertical is considered to be a truss. This distinguishes the **truss bridge** from the **arch** and the **suspension bridge,** in which the reactions are not vertical.

1271. A **symmetrical truss** is a truss having both ends alike; if it could be folded at the center upon itself in

For notice of copyright, see page immediately following the title page.

such manner that the two ends would come together, all corresponding members in the two halves of the truss would coincide. Nearly all trusses are symmetrical.

1272. A **simple truss** is a truss *simply supported;* that is, a truss whose *ends* simply rest on the points of support without being rigidly fixed to them. It is similar to a simple beam, and is distinguished from a **continuous truss** or a **cantilever truss** by the same characteristics that distinguish a simple beam from a continuous or a cantilever beam. (See Arts. **1222** to **1226.**)

1273. The theoretical **span** of a simple truss is the distance between the centers of its supports. The truss is divided into a certain number of parts or sections, usually of equal length, called **panels.** The **panel lengths** are the horizontal distances between the joints of the loaded chord. A truss that is divided into five panels is called a **five-panel truss;** a truss divided into six panels is called a **six-panel truss,** etc. Likewise, a bridge whose trusses are each divided into six panels is called a **six-panel bridge.**

The stresses in simple, symmetrical trusses only will be analyzed in the following pages.

THE MEMBERS OF A TRUSS.

1274. The names applied to the various members of a simple truss are given in Mechanical Drawing, Art. **54,** in connection with and preceding Plate, Highway Bridge: Details I, to which reference may be made for the name of any member.

When mentioned without reference to their positions in the truss, those members which resist compressive stresses are called **struts,** or **compression members,** and those which resist tensile stresses are called **ties,** or **tension members.** Each individual member, however, is usually designated with reference to its position in the structure.

When the diagonal members of a truss are compression members, they are called **braces;** the counters are called **counterbraces.** Inclined end posts are often called **batter braces.**

1275. A compression member can resist a certain amount of tension also; but a member designed to resist tension only is not usually capable of resisting compression. When it is desired that a tension member shall resist a small amount of compression also, the form of the member must usually be changed.

1276. When the loads upon a simple truss are *downwards*, as is usually the case, the upper chord is always in compression, and the lower chord always in tension. In the web system the struts and ties alternate.

1277. The extremities of each member are connected as nearly as possible on a line passing through the center of gravity of its section; the stress in a member is considered to act in a direct line between the centers of its connections, and, therefore, the member itself must be straight. For all purposes relating to the investigation of the stresses in the members of a truss, each member may be represented simply by a straight line indicating the line of action of its stress.

CLASSIFICATION OF BRIDGES.

1278. *Bridges are classified according to the positions in which their roadways are supported.*

Deck bridges are those that support their roadways or loads at or near the level of the upper chord. In this class of bridge all portions of the structure are entirely below the roadway, and are not visible from above it. Deck bridges require considerable space below the roadway, and, therefore, the locations to which they are adapted are not very common.

1279. **Through bridges** are those that support their roadways at or near the level of the lower chords; the loads pass between the trusses, or *through* the bridge. Bridges of this class are very common. If, in addition to the roadway, they also carry separate footways, the latter are usually supported outside of the trusses.

1. **High-truss bridges** are those bridges of this class in which the trusses are of sufficient height to require a system of overhead lateral bracing to be placed between the upper chords above the roadway. This class of trusses is not well adapted to spans of less than 80 feet.

2. **Low-truss bridges** are those in which the trusses are not of sufficient height to require a system of lateral bracing above the roadway; such trusses are also designated as *pony trusses.* This class of bridges is adapted to short spans only.

1280. Half-deck bridges support their roadways at some required elevation *between* the upper and lower chords. They are not very common.

1281. *A general classification of truss bridges may also be made according to the manner in which the members are connected.*

Pin-connected bridges are those in which the several members of the truss that meet at each joint are connected, and transmit their stresses, by means of an accurately turned pin, somewhat resembling a large bolt, which is made to fit very closely into holes drilled through the ends of the members. This affords a simple and convenient means for connecting the members. As the connection acts, to some extent, like a large hinge, it allows each member to readily adjust itself in the line of its stress without developing bending stresses. In this type of truss, each member can be practically finished at the shop; and, in erecting the bridge at its site, the members are assembled and connected, and the structure completed in the shortest possible time and with the minimum amount of field labor. Owing largely to this fact, the pin-connected type of bridge has become very popular in America.

1282. **Riveted girders** is the name commonly applied to those bridges in which all connections are riveted; certain forms of such bridges are also designated as **riveted trusses** and **latticed girders.** This type of structure is, by some engineers, very highly commended for spans of about 150 feet or less. For spans less than 100 feet, they undoubtedly possess some advantages over pin-connected bridges; but whether this be true for spans greater than 100 feet may be seriously questioned. It must, however, be conceded that the tendency of the best modern practice is towards such details as will connect the truss as a rigid whole.

1283. Under the preceding head may also be classified the type of structures known as **plate girders,** which are, within the limits of their availability, the best metal bridges known. But they are not trusses—rather a distinct class of girders.

1284. *Trusses may be further classified according to the form of their design;* that is, according to the arrangement of the members and the form of the truss as a whole. But such classification will not be further noticed at present. Several forms of trusses will be discussed in their proper place.

The truss represented in Mechanical Drawing Plate, Title: Highway Bridge: General Drawing, is of a design which, for bridges of ordinary span, is very common and popular in America. It is a *pin-connected, high-truss, through bridge,* of the type commonly known as the **Pratt truss;** though it is also known as a **single-intersection** and **single-quadrangular** truss. Substantially this form of truss was patented in 1844 by Thomas W. and Caleb Pratt as a combination wood and iron bridge. It is the favorite style of truss now used for moderate spans, and is usually constructed entirely of metal. As a metal structure it possesses advantages over all other forms of trusses. As this truss affords a simple and very practical example, all important phases of its design will be noticed in their proper connection.

LOADS.

1285. *Under ordinary conditions, a bridge must be designed to carry or resist three different characters of external forces or loads.*

I. The **live load** on a bridge is the load it must carry, exclusive of its own weight. Usually the live load moves over the bridge, and for this reason it is also called **moving load.** For highway bridges, the live load is assumed at a specified amount per square foot of roadway or a specified amount per lineal foot of structure. This load is assumed to be uniformly distributed over the roadway, and is known as a **uniform load.** For bridges so located as to be subjected to the passage of heavy loads concentrated upon wheels, the uniform load is augmented by certain assumptions of **concentrated loads** or **wheel loads.** For railroad bridges the live load usually consists of a system of concentrated wheel loads, or of certain assumed uniform loads that will give an approximately equivalent effect.

The amount of live load which a bridge is designed to carry is often called its **capacity.**

II. The **dead load** on a structure is the weight of the structure itself. The terms **bridge weight, fixed load,** and **static load** are also sometimes used in the same sense. The latter term is not so strictly applicable, as the live load also is treated as a static load. The dead load is generally considered as a uniform load; it is assumed at a uniform amount per lineal foot of structure.

III. The **wind load** is the load due to the force of the wind against the side of the structure. It must be resisted by the lateral systems. The wind load is usually assumed sufficiently large to also include and provide for the effect of such lateral vibrations in the trusses as are likely to be caused by the passage of heavy or rapidly moving loads across the structure. When specified, the amount of wind load per lineal foot assumed for each (loaded and unloaded) chord is usually stated.

PANEL CONCENTRATIONS.

1286. The amount of load which is transferred to and supported at each joint of the loaded chord is called a **panel concentration.** In the case of a uniform load, each panel concentration is equal to one-half of the load supported by one truss upon the two panels adjacent to the joint, and is called a **panel load.**

1287. *The live load per square foot multiplied by the clear width of roadway gives the live load per lineal foot.* This is the roadway load. When the bridge carries sidewalks, the sidewalk load per lineal foot, found by multiplying the sidewalk load per square foot by the combined clear width of both sidewalks, must be added to the roadway load.

1288. *One-half the live or dead load per lineal foot multiplied by the panel length gives the panel live load or panel dead load, as the case may be.* That is, this gives the amount of live or dead load to be carried by each joint of the loaded chord in *one truss;* it is the panel load to be used in finding the stresses.

1289. *The wind load per lineal foot, assumed or computed, for either chord, multiplied by the panel length, gives the panel wind load for that chord.*

Example.—A bridge 99 feet long is designed to sustain 100 pounds per square foot upon a roadway 16 feet wide, clear width. The trusses are divided into 6 panels. What is the live load per lineal foot, and the panel live load?

Solution.—The live load per lineal foot equals $100 \times 16 = 1{,}600$ lb. The panel length equals $\frac{99}{6} = 16.5$ ft. Therefore, the panel live load equals $\frac{1{,}600 \times 16.5}{2} = 13{,}200$ lb.

EXAMPLES FOR PRACTICE.

1. In the preceding example, if the wind load assumed for the lower chord is 300 pounds per lineal foot, and that assumed for the upper chord is 150 pounds per lineal foot, what is the panel wind load (*a*) for the lower chord, and (*b*) for the upper chord?

Ans. $\begin{cases} (a)\ 4{,}950 \text{ lb.} \\ (b)\ 2{,}475 \text{ lb.} \end{cases}$

2. An eight-panel bridge of 120 feet span carries a roadway 18 feet wide in the clear. The live load assumed for the trusses is 96 pounds per square foot of roadway. What is (*a*) the live load per lineal foot, and (*b*) the panel live load?

Ans. { (*a*) 1,728 lb.
(*b*) 12,960 lb.

3. If for the bridge of the preceding example the wind load per lineal foot is assumed at 300 pounds for the lower chord and 150 pounds for the upper chord, what is the panel wind load (*a*) for the lower chord, and (*b*) for the upper chord?

Ans. { (*a*) 4,500 lb.
(*b*) 2,250 lb.

4. Suppose for the same bridge the dead load were assumed to be 760 pounds per lineal foot, what would the panel dead load be?

Ans. 5,700 lb.

STRESSES FROM VARYING LOAD.

1290. In taking up this subject, a comparison of the stresses in the members of a simple truss under different conditions of load will first be made.

In solving Question 568, the student drew the stress diagram, and determined the stresses for a truss of three panels, carrying two loads, W_1 and W_2, equal to 4,200 pounds each. In Fig. 267 is represented the same truss loaded with W_2 only, the load W_1 having been removed.

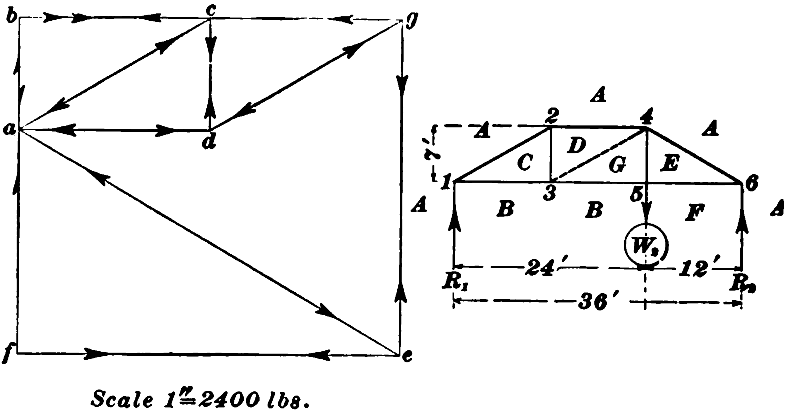


FIG. 267.

For this condition of load, the left reaction $R_1 = 4,200 \times \frac{12}{36} = 1,400$ pounds and $R_2 = 4,200 \times \frac{24}{36} = 2,800$ pounds.

At the left of the truss is shown the stress diagram. The line *a b f a* is the load line, or polygon, of the external forces R_1, W_2, and R_2, laid off in order. For joint *1*, the polygon *a b c a* is completed; and the polygon *a c d a* is drawn for joint *2*. For joint *3*, arrow-heads are marked on *d c* and *c b* in reversed positions and directions; but it is found impossible to return the pencil from *b* to the starting point *d* by a line parallel to *G B*, the only remaining member which connects at joint *3*. Therefore, another member must be introduced into the truss, connecting at joint *3*, in such manner that the polygon for this joint can be completed by lines drawn parallel to the additional member and to *G B*. A member *D G*, connecting joints *3* and *4*, as indicated by a dotted line, is found to fulfil this condition; and the polygon *d c b g d* may be completed for joint *3* by drawing the lines *b g* and *g d* parallel to *B G* and *G D* respectively. For joint *4*, the polygon is *a d g e a;* and *e g b f e* is the polygon for joint *5*. A check upon the correctness of the work is obtained in this polygon, as *b f* represents the load *B F*, or W_2.

The polygon for joint *6* is *a e f a*, in which *f a* represents R_2. The two arrow-heads on each line indicate in each case the sense of the force with reference to the joint for which each arrow-head was used.

Measuring with the scale used in laying off the load line, and designating compression by the + and tension by the − sign, the lines of the stress diagram are found to represent the following values:

$a\,c = +\,2{,}780$

$c\,d = -\,1{,}400$

$d\,g = +\,2{,}780$

$g\,e = -\,4{,}200$

$e\,a = +\,5{,}560$

$b\,c = -\,2{,}400$

$b\,g = -\,4{,}800$

$f\,e = -\,4{,}800$

$a\,d = +\,2{,}400$

1291. A comparison of the above stresses with those obtained for the corresponding members (notation slightly changed) in the solution of Question 568, is instructive.

The very prominent and noticeable difference between the two cases is that, while, with the two equal loads upon the truss, no diagonal member is required in the center panel, when one of the loads is removed, the diagonal member is required. It is evident that if the truss were loaded with W_1 only, instead of W_2, the diagonal *3-4* would be in tension, instead of in compression. As in most types of structures, such diagonals are usually designed to resist tension only or compression only, it would be the general practice to put two diagonals in this panel, one connecting joints *3* and *4*, and one connecting joints *2* and *5*.

When a truss is partly loaded in a certain manner, members are required which do not act when the truss is fully loaded.

But it is also noticed that, when the truss carried two loads, the stresses in the chords and end posts were greater than when it carried one load. These facts illustrate the following general principles:

The maximum stresses in the chords occur when the whole truss is loaded; but the maximum stresses in the other members occur when the truss is only partly loaded. What the condition of loading should be in the latter case will be explained further on.

THE MAXIMUM LIVE CHORD STRESSES.

1292. A live load will now be assumed for the bridge represented in Mechanical Drawing Plate, Title: Highway Bridge: General Drawing, and the maximum stress in each member due to the assumed load will be obtained by drawing the necessary stress diagrams. As has been stated, the theoretical length of the span of a bridge is the length from center to center of end pins, i. e., the pins in the shoe joints. In the present case, the distance between the centers of the pins in joints *a* and *a'* is 90 feet; it is divided into five equal panels of 18 feet each. The clear width of roadway, that is, the clear width between trusses, is 18 feet, and the height of the trusses, between centers of chord pins, is 18 feet.

The general dimensions of a bridge, such as have just been given, together with the loads and such other information as may be necessary to make a proper design of the same, are called the **data** for the bridge.

The live load will be assumed to be 100 pounds per square foot. The live load per lineal foot is, therefore, 100 × 18 = 1,800 pounds. (Art. **1287.**) This load is assumed to be applied upon any or all portions of the floor, in such manner as to cause the maximum stresses in the several members. By means of stringers the load upon the floor is transferred to the floor beams or cross girders, which are placed across and beneath the roadway at the lower chord joints b, c, c', and b'. The stringers and floor beams act independently of the truss proper, serving simply to concentrate the floor load upon the lower chord joints.

The amount of live load supported by one floor beam equals the load per lineal foot multiplied by the panel length, or 1,800 × 18 = 32,400 pounds. One-half this amount, or 16,200 pounds, constitutes the panel load for one truss. (Art. **1288.**) When the general dimensions of the truss are given, having ascertained the amount of live load that should be assumed for it, and the points at which the panel loads are concentrated, we have all the data required for determining the stresses due to live load.

The stresses are found for one truss, those for the other truss being usually identical. If the trusses have not the same form, the stresses must be found for each separately.

As previously stated, the greatest stresses are developed in the chords and end posts when the bridge is fully loaded. This condition of load will first be considered, the values of the reactions will be computed, and the stress diagram drawn to find the live load stresses in the chords and end posts.

1293. *That portion of the load upon each end panel which is carried directly by the abutment does not affect the truss.*

With the bridge fully loaded as above, one-half of each end panel load is carried by the corresponding abutment.

Therefore, in a truss having panels of equal length, fully loaded with a uniform load, the reactions *affecting the truss* have the following values:

$$R_1 = R_2 = \frac{W(n-1)}{2}, \qquad (89.)$$

in which W represents the panel load, and n the number of panels.

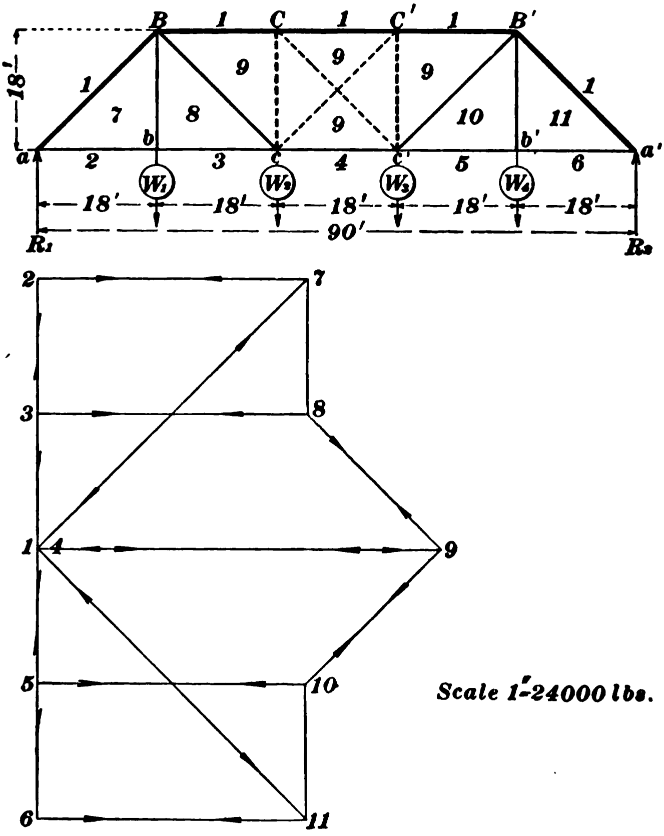


FIG. 268.

In the present case, therefore, $R_1 = R_2 = \frac{5-1}{2} \times 16{,}200 =$ 32,400 pounds. In Fig. 268 is represented a diagram of the truss, together with a stress diagram for this condition of load. Here W_1, W_2, etc., represent the panel loads, or 16,200 pounds each.

Bow's system of notation is used, but numerals are used instead of letters, because it is desired to use letters to designate the joints. Each member of the truss will be designated by the letters of the two joints at which it connects, as members *a B* (not member *1-7*); but the *stress* for *a B* will be designated as *1-7*, referring to the line of the stress diagram.

In drawing the stress diagram, it will be found that the condition of equilibrium can be fulfilled for all joints without requiring stress in any of the members represented by dotted lines. When the truss is fully loaded with a uniform load there is no stress in the members *C c*, *C c'*, *C' c'*, and *C' c*.

The load line *2-6* is laid off by taking the external forces in order, beginning with R_1 and passing to the right across the truss, or, in other words, beginning with R_1, and taking the forces in order, *passing to the left around the truss*, as though they were forces acting upon a single point. By acting through the medium of the truss the forces bear the same general relation *to each other* as though acting upon the same point. On the load line, therefore, *1-2* is laid off upwards equal to $R_1 = 32{,}400$ pounds, then *2-3*, *3-4*, *4-5*, and *5-6*, each equal to a panel load = 16,200 pounds, are laid off downwards, and finally *6-1*, equal to $R_2 = 32{,}400$ pounds, is laid off upwards to the starting point.

Commencing with R_1, the polygon *1-2-7-1* is completed for joint *a*, *7-2-3-8-7* for joint *b*, and *1-7-8-9-1* for joint *B*. For joint *c*, retrace *9-8* and *8-3* (that is, mark reversed arrow-heads, and consider them drawn from *9* to *8* and from *8* to *3*, respectively. [See Art. **1148**]); pass downward on the load line the amount *3-4*, equal to W_2; a line returning from *4* to the starting point *9* will represent the equilibrant of the forces *9-8*, *8-3*, and *3-4*, that is, will represent the remaining force which acts upon this joint. As the closing line *4-9* is a horizontal line, it is evident that it must represent the stress in a horizontal member, which must necessarily be the member *c c′*, this being the only remaining horizontal member which connects at this joint. As *4-9*, or the stress in *c c′*, fulfils the condition of equilibrium for this joint, the members *C c* and *C′ c*, represented by dotted lines, do not act with this condition of load. For joint *c′*, retrace *9-4* (that is, mark a reversed arrow-head, and consider it as drawn from *9* to *4*); pass downwards on the load line the amount *4-5*; it will be found that from *5* the pencil can be returned to the starting point *9* by the lines *5-10* and *10-9*, drawn parallel to *c′ b′* and *c′ B′*, respectively. The conditions at this joint are the same as at joint *c*, except that the positions of the forces are reversed; the condition of equilibrium is fulfilled without requiring the members *C c′* and *C′ c′* to act. The polygons for joints *B′ b′* and *a′* are substantially the same as for joints *B b* and *a*, respectively, and will be readily understood without special explanation. From an inspection of the stress diagram it is evident that the members represented by the dotted lines would not be required to act when the truss is fully loaded with a uniform load. For, in drawing the force polygon for joint *c*, the point *4* falls at the point *1*, showing that the entire vertical force of the left reaction R_1, represented by *1-2*, is entirely absorbed by the loads W_1 and W_2, represented by *2-3* and *3-4*. Likewise, the right reaction R_2 (= *6-1*) is absorbed by W_3 (= *4-5*), and W_4 (= *5-6*), and between joints *c* and *c′* no vertical forces are acting. To express this in the usual language, *there is no shear in the panel c c′*. The effect is the same as though W_1 and W_2 were supported by R_1, and W_3 and W_4 were supported by R_2.

By measuring the lines of the stress diagram, and using the conventional signs for tension and compression, the following values of the stresses are obtained :

Stress in *a B* = *1-7* = + 45,820 lb.
Stress in *a′ B′* = *1-11* = + 45,820 lb.
Stress in *B B′* = *1-9* = + 48,600 lb.
Stress in *a b* = *2-7* = − 32,400 lb.
Stress in *b c* = *3-8* = − 32,400 lb.

Stress in $c\,c'$ = *4-9* = − 48,600 lb.
Stress in $c'\,b'$ = *5-10* = − 32,400 lb.
Stress in $b'\,a'$ = *6-11* = − 32,400 lb.
Stress in $B\,c$ = *8-9* = − 22,910 lb.
Stress in $B'c'$ = *9-10* = − 22,910 lb.
Stress in $B\,b$ = *7-8* = − 16,200 lb.
Stress in $B'\,b'$ = *10-11* = − 16,200 lb.

NOTE.—The above stresses are given to the nearest ten pounds. In order that the student may better judge of the accuracy of his work, the stresses will be so given in what follows. But in actual practice it is never necessary to obtain the stress in any member of a bridge or similar structure closer than to the nearest hundred pounds.

It is noticed that the line *1-9* represents compression, while the line *4-9* represents tension. As these lines coincide, the student must be careful to distinguish the proper character of stress for each. The above stresses are the maximum live load stresses in the chords and end posts. As the office of the hip verticals $B\ b$ and $B'\ b'$ is simply to suspend the panel loads at b and b' from the joints B and B', respectively, it is evident that the greatest stress which can come upon either of these members is the panel load which it supports. Therefore, the above stresses for $B\,b$ and $B'\ b'$ are also maximum.

THE MAXIMUM LIVE LOAD WEB STRESSES.

1294. The stresses as found above for the web members $B\,c$ and $B'\ c'$, however, are not maximum. The maximum stresses in these and the remaining web members will occur under different conditions of load.

Since the two ends of the truss are alike and are acted upon in the same manner by external forces of the respective magnitudes, the stresses in the two halves of the truss are identical, and it is necessary to construct the stress diagram for those members only whose stresses are caused by one reaction.

In the present case, the upper half of the stress diagram is the only portion necessary to be drawn. Although the complete stress diagram is sometimes drawn for the purpose of checking the work, it is customary to obtain only those stresses due to the left reaction. This practice will generally be followed hereafter in obtaining the live load stresses; the stress diagram will usually be drawn for those stresses only which are produced by the left reaction and the corresponding loads. When stress in any member is mentioned, it will be understood to mean stress caused by the left reaction; the stress in the corresponding member in the opposite half of the truss, caused by the right reaction, will be of the same character and magnitude. The maximum stresses in the web members will now be found.

1295. *In a Pratt truss, the maximum stress will occur in the diagonal web member in any panel when all joints at the right of the panel are fully loaded and the joints at the left of it are not loaded. This condition will also give the maximum stress (of opposite character) in the vertical member which meets the diagonal in the unloaded chord.*

This rule, of course, does not apply to the hip vertical, whose maximum live load stress *is always equal to one panel load.*

The maximum stress in the tie $B\,c$ will occur when the joints c, c', and b' are loaded, and joint b is not loaded. In this condition $R_1 = 16{,}200 \times \frac{54 + 36 + 18}{90} = 16{,}200 \times \frac{6}{5} =$ 19,440 pounds.

In Fig. 269 is represented a diagram of the truss and that portion of the stress diagram necessary to determine the stress in $B\,c$. Those diagonal web members which transfer their load to the right reaction are omitted from the diagram of the truss. The load line *1-2* is laid off to scale equal to R_1, and the polygon *1-2-7-1* is completed for joint *a*. For joint *b*, the line *7-2* is first retraced; as there is no load at *b*, or, in other words, as the load at *b* is zero, the portion of the load line *2-3*, which represents this load, has no magnitude. In the stress diagram, therefore, the points *3* and *2* will coincide; the line *3-8* will coincide with the line *7-2*; and as the terminus of the last line of the polygon must close upon the starting point, it is evident that no stress can be obtained for the member *B b*. For joint *B*, retrace *1-7* (that is, mark a reversed arrow-head, and consider it as drawn from *1* to *7*), and, as there is no stress in *B b*, the pencil is returned to the starting point *1* by lines *7-9* and *9-1*, drawn parallel, respectively, to *B c* and *C B*. (These lines, of course, are actually drawn from *7* and *1*, respectively, their intersection locating the point *9*.) In other words, *1-7-8-9-1* is the polygon for joint *B*. The line *8-9* represents the maximum stress in *B c*. As this figure is not a complete stress diagram, the arrow-heads marked upon the lines but incompletely represent the characters of the stresses. However, if the arrow-heads have been marked in accordance with the instructions given in preceding articles, the single arrow-head marked upon a line will indicate the character of the stress. By measuring *8-9* with the same scale used to lay off *1-2*, and noticing the direction of the arrow-head marked upon it, it is found to represent a tensile stress of 27,490 pounds.

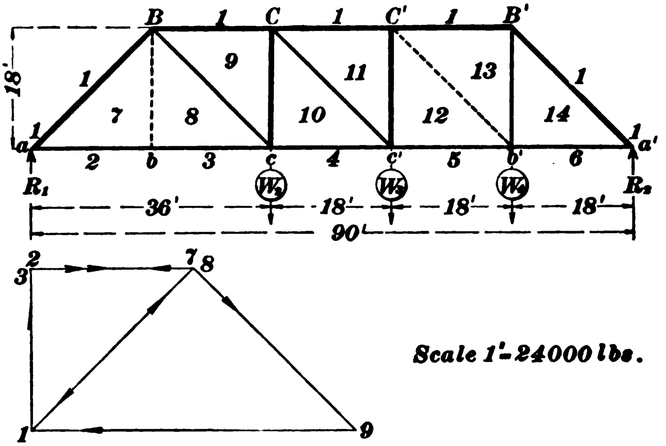


FIG. 269.

In Fig. 270 is represented the truss with joints *c'* and *b'* loaded, and the joints *b* and *c* unloaded; also the stress diagram for this condition, giving the maximum stresses in *C c'* and *C c*.

As no external forces act upon the truss between R_1 and W_3, the entire external space between these forces is distinguished by the same numeral. Likewise, as no force is acting in the member *B b*, the entire space between the members *B a* and *B c* is designated by the same numeral. For this condition of load, $R_1 = 16{,}200 \times \frac{18 + 36}{90} = 9{,}720$

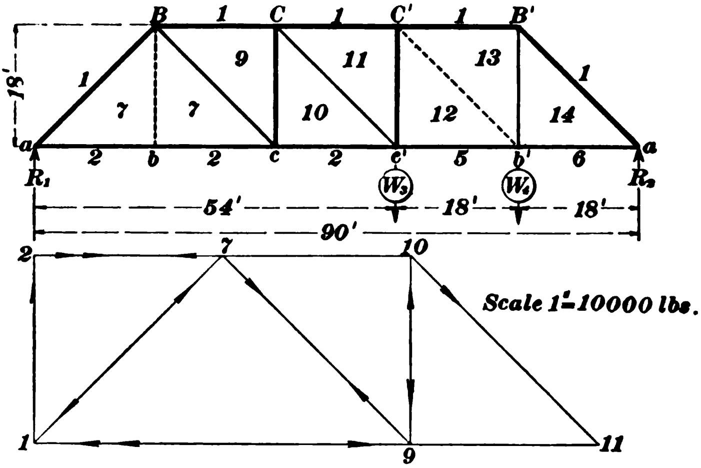


FIG. 270.

pounds; it is represented by the line *1-2*, and the polygon *1-2-7-1* is completed for joint *a*. As there is no stress in *B b*, *1-7-9-1* is the force polygon for joint *B*; *9-7-2-10-9* is the polygon for joint *c*; and *1-9-10-11-1* the polygon for joint *C*. The stress in *C c* is represented by *9-10* = + 9,720 pounds, and that in *C c'* is represented by *10-11* = 13,750 pounds; both stresses are maximum.

In Fig. 271, the truss is represented with only joint *b'* loaded. This condition gives the maximum positive shear in the panel *c' b'*; that is, it gives the maximum stress that could be produced by the left reaction in a counter having the position *C' b'*. This stress is found in order to determine whether a counter is required in this panel.

For this load, $R_1 = 16{,}200 \times \frac{18}{90} = \frac{16{,}200}{5} = 3{,}240$ pounds.

Commencing with *1-2*, laid off equal to R_1, the polygon *1-2-7-1* is completed for joint *a*. For joint *B*, *1-7-9-1* is the polygon; *9-7-2-10-9* is the polygon for joint *c*; *1-9-10-11-1*, for joint *C*; *11-10-2-12-11*, for joint *c′*, and *1-11-12-13-1*, the polygon for joint *C′*. The line *12-13* = − 4,580 pounds represents the maximum stress that could be caused in a member *C′ b′* by the left reaction. The stress in *C′ c′* from this condition can not be as great as that found for *C c* in the diagram of Fig. 270; and as *C′ c′* bears the same relation to the right reaction as *C c* does to the left, it will have the same maximum stress. But, as previously stated, when the two ends of the truss are alike, only those stresses caused by the left reaction are obtained; as the maximum stress in *C′ c′* is caused by the right reaction, it is not obtained.

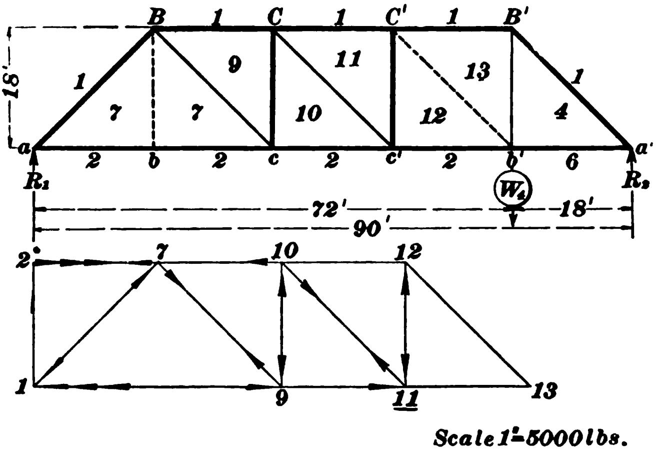


FIG. 271.

The maximum live load stresses have now all been found The maximum live load web stresses are:

Stress in $B\ c = -\ 27{,}490$ lb.

$C\ c = +\ 9{,}720$ lb.

$C\ c' = -\ 13{,}750$ lb.

$C'\ b' = -\ 4{,}580$ lb.

It is evident that Figs. 269, 270, and 271 could have been combined in one figure. How to do this will be explained later.

1296. It will be noticed that in the stress diagram of Fig. 271, $11\text{-}12 = 9\text{-}10 = 1\text{-}2 = R_1$. Since in this condition of load there are no external forces (loads) applied between a and b', the vertical shear in all panels between these points is uniform, equal to R_1, and positive. The student will notice particularly the fact that the shear in all panels at the left of b' equals R_1. The same would be true with any number of loads at the right of any given point, provided there were no loads at the left, i. e., between the given point and R_1. This shear is positive; that is, if at any point the truss be conceived to be cut by a vertical plane, the direction of the force on the left of the plane will be upwards, and of that on the right will be downwards. This will be referred to again.

THE DEAD LOAD.

1297. In order to satisfactorily decide the amount of dead load to be assumed for a structure, a certain amount of previous experience is very essential, if not really indispensable. As the stresses must be determined, the material proportioned, and an estimate of the weight of the structure be made before the amount of dead load is known, it is evident that the assumption of the dead load must be based largely on judgment and past experience. It is very desirable that the assumed dead load may represent, as nearly as possible, the actual weight of the structure. If it is assumed too light, the structure will not be strong enough; if too heavy, material will be wasted. In either case, if the error is considerable, it will be necessary to correct the dead load and repeat the operations.

It is a not uncommon practice among bridge engineers to consider 400 pounds per lineal foot as the minimum allowable dead load for bridges; and for very light structures of short span, in which the estimated dead load is very light, to never use less than this amount. This practice will here be followed.

1298. Owing to the various forms of floor, differing requirements of specifications, and varying conditions affecting the dead load, no general and satisfactory formula has yet been obtained for it. Indeed, it is extremely doubtful whether the many and varying conditions upon which the amount of dead load depends can be so formulated as to give satisfactory results. It is desirable, however, that the student be guided by a general formula in assuming the dead loads. The following empirical formulas, although but roughly approximate, are useful as guides in assuming the dead loads of ordinary truss bridges.

For a capacity of 100 pounds per square foot,

$$w = s + 5b + \frac{(b-6)l}{8} + \frac{l^2}{300}. \qquad \textbf{(90.)}$$

For a capacity of 80 pounds per square foot,

$$w = s + 5b + \frac{bl}{16} + \frac{l^2}{300}. \qquad \textbf{(91.)}$$

In both formulas, w is the dead load per lineal foot exclusive of the floor and joists, s is a quantity which will here be taken equal to the amount of live load per square foot, b is the clear width of roadway, and l is the length of span. In formula **90,** $s = 100$; in formula **91,** $s = 80$. For any other live load capacity s', find w by formula **90,** remembering that $s = 100$; call the required dead load w'. Then,

$$w' = w - \left[1 + \frac{(b-12)l}{320}\right](100 - s'). \qquad \textbf{(92.)}$$

These formulas have been derived from actual practice; when the results obtained by them are added to the estimated weight of floor, the resulting dead loads will approximate reasonably near to the actual loads for highway bridges of ordinary dimension and the usual type of construction. After some experience the formulas can be adapted to special forms of construction by modifying the values of s.

The student must bear in mind that these formulas are given for convenience and the purposes of instruction only; they are not to be relied upon in practice further than to afford reasonable checks.

1299. Having ascertained the kind of floor to be used, the weight of floor and joists must be separately calculated and added to the results obtained from the preceding formulas. For ordinary plank floors, supported on timber joists, the student may obtain the weight from the following table, which has been calculated for the three varieties of timber most commonly used:

TABLE 29.

Weight, in Pounds per Lineal Foot, of Timber Bridge Floor, Consisting of 3" Floor Plank, two 4" × 6" Wheel Guards, and the Required Stringers.

Length of Panel in Feet.	Yellow Pine.		White Oak.		White Pine.	
	For 12-Ft. Roadway.	For Each 2 Feet Additional Width, Add	For 12-Ft. Roadway.	For Each 2 Feet Additional Width, Add	For 12-Ft. Roadway.	For Each 2 Feet Additional Width, Add
12	245.6	36.4	262.0	38.8	152.5	22.5
13	245.6	36.4	276.0	40.8	152.5	22.5
14	258.8	38.3	292.8	43.2	161.3	23.8
15	274.5	40.5	292.8	43.2	161.3	23.8
16	274.5	40.5	309.6	45.6	171.5	25.2
17	274.5	40.5	309.6	45.6	171.5	25.2
18	290.3	42.8	309.6	45.6	181.7	26.7
19	290.3	42.8	329.2	48.4		
20	290.3	42.8	348.8	51.2		
22	327.0	48.0				

By substituting the proper value in formula **90,** the weight per lineal foot of the structure under consideration, exclusive of floor, is $100 + 5 \times 18 + \frac{(18 - 6)\,90}{8} + \frac{90^2}{300} = 352$ pounds.

The floor will be assumed to consist of long leaf yellow pine. From Table 29, this is found to weigh, for 18 feet panel length, $290.3 + 3 \times 42.8 = 418.7$ pounds per lineal foot. The total weight per lineal foot of the structure will, therefore, be $352 + 418.7 = 770.7$ pounds. The dead load will be taken at 770 pounds per lineal foot. As this load is considered to be uniformly distributed over the entire structure, the panel dead load will be $\frac{770 \times 18}{2} = 6{,}930$ pounds.

THE DEAD LOAD STRESSES.

1300. The dead load is considered to be concentrated at the joints of the truss. A common practice is to assume each panel dead load to be divided between the upper and lower chord joints of each panel. In an ordinary through bridge with plank floor, it is approximately correct to consider two-thirds of each panel load to be concentrated at the lower, and one-third at the upper chord joint.

The same results are arrived at in a somewhat simpler manner by the following

Rule.—*Obtain the dead load stresses by considering the entire dead load to be concentrated at the lower chord joints, denoting compression by the + sign, and tension by the − sign; correct the stress thus obtained for each vertical member, by adding to it algebraically a compressive stress equal to one-third of the panel load.*

This practice will here be followed; it has the advantage of giving a stress diagram similar in every respect to that for the live load chord stresses, as the conditions assumed in each case are the same. For, as the weight of the structure is always present in the structure, it is evident that, in obtaining the dead load stresses, the truss must be considered to be always fully loaded with its dead load. Therefore, the stress obtained for any member from the stress diagram for the dead load may be checked by multiplying the corresponding stress, obtained from the stress diagram for the full live load, by the dead load per lineal foot, and dividing the product by the live load per lineal foot. Or, by reversing the process, the live load stresses for the chords and end posts may be obtained from the corresponding dead load stresses.

NOTE.—As a low-truss bridge carries no horizontal bracing between the upper chords, the error due to assuming the entire dead load to be concentrated at the joint of the lower chord is very small, and may be entirely neglected. Therefore, *for low-truss bridges the entire dead load will be assumed to be carried at the joints of the lower chord, and the dead load stresses in the vertical members will not be corrected for the error due to this assumption.*

1301. By formula **89**, the dead load reaction $R_1 = \frac{6,930 \times (5 - 1)}{2} = 13,860$ pounds. In Fig. 272 is represented

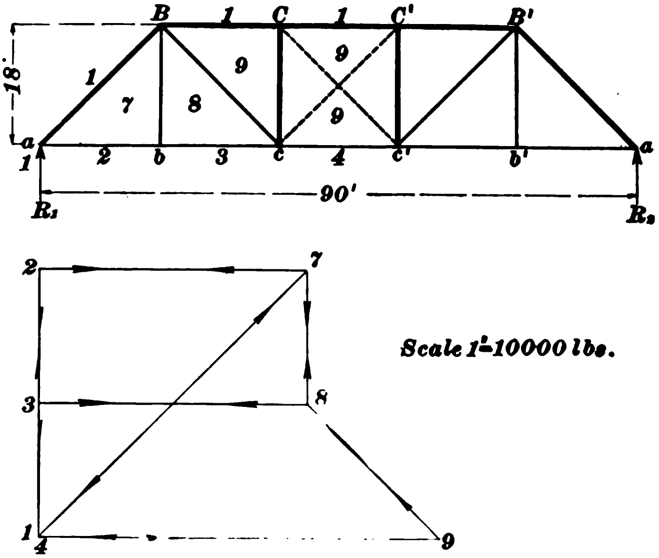


FIG. 272.

a diagram of the truss and that portion of the stress diagram for the dead load which refers to the left reaction; it is in every respect similar to the upper half of the stress diagram in Fig. 268. It will require no special explanation; the lines are drawn in the same order as in Fig. 268. The following stresses are obtained:

Stress in $a\ B = 1\text{-}7 = +\ 19{,}600$ lb.
Stress in $B\ B' = 1\text{-}9 = +\ 20{,}790$ lb.
Stress in $a\ b = 2\text{-}7 = -\ 13{,}860$ lb.
Stress in $b\ c = 3\text{-}8 = -\ 13{,}860$ lb.
Stress in $c\ c' = 4\text{-}9 = -\ 20{,}790$ lb.
Stress in $B\ b = 7\text{-}8 = -\ 6{,}930$ lb.
Stress in $B\ c = 8\text{-}9 = -\ 9{,}800$ lb.
Stress in $C\ c = 9\text{-}9 = 0{,}000$ lb.

The above stresses may be obtained or checked from those determined by the stress diagram of Fig. 268, by multiplying the latter stresses by the quantity $\frac{770}{1{,}800}$, which is the ratio of the dead load to the live load per foot of length. The stress for $B\ b$ is found to equal one panel load, or $-\ 6{,}939$ lb.; to this is to be added algebraically a compressive stress equal to one-third panel load, or $+\ 2{,}310$ lb. Therefore, the correct dead load stress in $B\ b$ is $-\ 6{,}930 + 2{,}310 = -\ 4{,}620$ lb. One-third panel load is also to be added to the stress obtained for $C c$; as the stress diagram gave no stress for this member, the correct dead load stress may be taken at one-third panel load, or $+\ 2{,}310$ lb.

COUNTER STRESSES.

1302. The effect of the dead load reaction has been entirely taken up by the loads at b and c, as explained for the full live load (Art. **1293**), and there is no dead load shear in the center panel $c\ c'$. It was also explained that, with a full uniform load, the effect of the shear is the same as though all panel loads at the right of the center were supported by the right reaction. As the dead load is a uniform load, it is evident that in the panel $c'\ b'$ there exists at all times a negative shear equal to the panel dead load at c', or 6,930 pounds, which is being transferred to the right reaction. This is greater than the positive shear of

3,240 pounds produced in the same panel by a panel live load at b'. (Art. **1295.**) It is thus found that no resultant positive shear can occur in this panel, and, therefore, no counter C' b' is required to resist it. The following principles are important:

(*a*) *The maximum live load positive shear in any panel of a truss is equal to the left reaction when all the joints at the right of the panel are loaded and all joints at the left of it are unloaded.*

(*b*) *The dead load negative shear in any panel at the right of the center is equal to the amount of dead load between the panel and the center of the span (including a half-panel load at the center if the truss has an even number of panels).*

(*c*) *No counter is required in any panel beyond the center of the truss in which the dead load negative shear exceeds the maximum live load positive shear; but in each panel beyond the center in which the live load positive shear can exceed the dead load negative shear, a counter is required.*

The latter principle may be better understood from the following considerations: If S is the dead load shear in any panel at the right of the center (negative with respect to the left reaction), and R_1 is the maximum live load positive shear in the same (see (*a*) above), then the resultant shear in the panel will be the algebraic sum of these two shears, or $R_1 + S$, the value of S being negative. If this sum is negative, that is, if R_1 is numerically less than S, the resultant shear will be of the same character as S, and the stress induced by it in any member will be of the same kind as that induced by the dead load. In this case, then, the diagonal member in the panel is designed to withstand either compression or tension alone, according as it may be a brace or a tie. But if R_1 is numerically greater than S, then the resultant shear $R_1 + S$ will be positive, that is, of a character the same as R_1 and opposite to S. In this case, consequently, the member must be designed to withstand stresses of both kinds (one induced by S when the truss is unloaded or fully loaded, and one of opposite kind induced by the resultant of $R_1 + S$ when the truss is partly loaded), or else a new member (counter) must be introduced to withstand the resultant shear $R_1 + S$ from a partial load.

The student should clearly understand the principles stated above. The subject of counter stresses, though simple, is usually found rather troublesome until it is thoroughly comprehended. For clearness the principle will be repeated.

(*d*) *A counter stress will obtain and a counter will be required in any panel at the right of the center of a truss when, with no live loads at the left of the panel, the left reaction, due to live loads at the right, is greater than the amount of dead load between the panel in question and the center of the truss (including a half-panel dead load at the center if the truss contains an even number of panels).*

WIND STRESSES.

1303. Bridge specifications differ somewhat in regard to the amount of wind load necessary to be provided for, the most common requirements varying from 100 to 150 pounds per lineal foot for the unloaded chord, and from 250 to 300 pounds per lineal foot for the loaded chord. Although a total wind load of 450 pounds per lineal foot is rather large for a highway bridge in the ordinary location, it will here be used. Throughout the following articles, and in the questions referring to them, a lateral wind force of 150 pounds per lineal foot will be assumed for the unloaded chord, and 300 pounds per lineal foot for the loaded chord. Of the latter, 150 pounds is to be treated as live load, and 150 pounds as dead load. The wind load upon the unloaded chord will be treated as dead load.

1304. Dead Wind Stresses in Lower Lateral System.—In the example, then, the total panel load of wind load for the lower chord is $300 \times 18 = 5{,}400$ pounds (Art. **1289**); but of this, one-half, or 2,700 pounds, is to be treated as live load, and the same amount is to be treated as dead load. For the wind dead load $R_1 = \frac{5-1}{2} \times 2{,}700$ lb. $= 5{,}400$ lb.

The "depth of truss" for the lateral systems is the width of the structure from center to center of chords. In the present case, as the chords are one foot in width, the width of the structure, center to center of chords, is $18 + \frac{1}{2} \times 2 =$

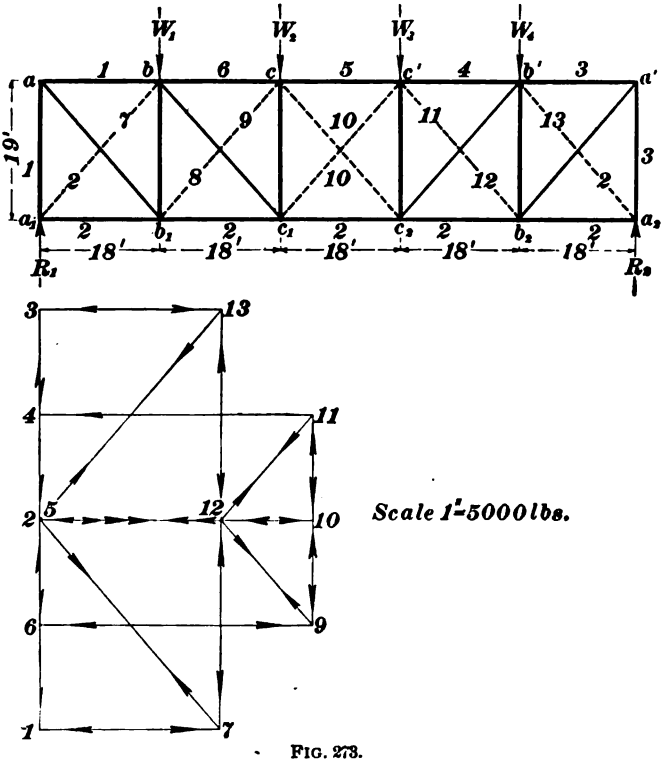


FIG. 273.

19 ft. A diagram of the lower lateral system, together with a stress diagram for the wind dead load, is represented in Fig. 273.

The force of the wind is assumed to be exerted against the windward joints *b*, *c*, *c'*, and *b'*. As the diagonals are designed to resist tension only, it is evident that but one diagonal in each panel acts with the wind against this side of the truss, the opposite diagonals acting only when the wind force is exerted from the opposite direction, or against the joints b_1, c_1, c_2, and b_2. As in the main truss, the diagonals in the center panel are not caused to act by the dead load. The members which do not act under the assumed conditions are, in the figure, represented by dotted lines. It is evident that the reactions R_1 and R_2 can not cause stress in the members a_1 b_1 and b_2 a_2, being exerted in directions perpendicular to those members. The members a a_1 and a' a_2 simply transfer the reactions from the joints a_1 and a_2 to the joints a and a'. Indeed, the left reaction may be resisted in equal amounts by the anchorage at a and at a_1; or it may be resisted entirely by the anchorage at a. The same is true of the right reaction. When shoe struts are used between the anchorages, it is customary to assume that the total reaction at each end of the bridge from the wind load is divided in equal parts between the two anchorages. For the purposes of the stress diagram, however, it will make no difference whether the reactions be assumed to be applied at the leeward or at the windward anchorages, or divided between the two; the total amount of each reaction and its line of action will remain the same.

In the diagram of the truss the reactions are represented as though applied at a_1 and a_2; it is evident that under such condition the compression in a a_1 and in a' a_2 is equal to R_1 and R_2, respectively. The stress diagram is drawn as though the reactions were applied at a and a'.

In the truss of a lateral system, the positions of the external forces correspond to those of a deck truss; the stress diagram for the lateral system will, therefore, also serve as an example of a stress diagram for a deck bridge.

In order that the load line may be upon the left side of the stress diagram, the external forces are laid off in order, beginning with R_1 and passing to the *left* around the truss; that is, the external forces *1-2*, *2-3*, *3-4*, *4-5*, *5-6*, and *6-1* are laid off in order, passing around the truss in a direction opposite to the movement of the hands of a watch. The lower half of the diagram represents the stress due to the left reaction, and the upper half represents those due to the right reaction. If the external forces were taken in the opposite order, passing to the right around the truss, the system of notation would require the stress diagram to be upon the left of the load line.

The student has become so familiar with the construction of the stress diagram that it will be unnecessary to explain the operation in detail.

The following stresses are obtained for the members in the left half of the lower lateral truss:

Stress in $a\ b$ = *1-7* = + 5,120 lb.
Stress in $b\ c$ = *6-9* = + 7,670 lb.
Stress in $b_1\ c_1$ = *2-8* = − 5,120 lb.
Stress in $c_1\ c_2$ = *2-10* = 7,670 lb.
Stress in $a\ b_1$ = *2-7* = − 7,440 lb.
Stress in $b\ b_1$ = *7-8* = + 5,400 lb.
Stress in $b\ c_1$ = *8-9* = − 3,720 lb.
Stress in $c\ c_1$ = *9-10* = + 2,700 lb.

1305. Live Wind Stresses in Lower Lateral System.—For the live wind load, the panel load is the same as for the dead wind load. For the members $a\ b_1$ and $b\ b_1$ (Fig. 273), the maximum stresses occur when the joint *b* and all joints at the right are loaded; that is, when the truss is fully loaded, or under the same condition as that for which the dead wind load stresses were found. Therefore, the live wind load stresses for $a\ b_1$ and $b\ b_1$ are the same as the dead wind load stresses for the same members. The same is true of the live wind load stresses for the chords.

1306. An essential difference between the deck truss and the through truss will now be noticed. If the truss were a through truss supporting its loads at the lower chord joints, this condition of load (i. e., with all joints loaded) would give the maximum stress for $a\ b_1$, but *not* for $b\ b_1$, as in the present case; in a through truss, the member $b\ b_1$

would obtain its maximum stress with all joints *except* b_1 loaded.

Each compression web member in a deck bridge and each tension web member in a through bridge receives its maximum stress when all joints at the right (including the joint at which the member connects) are loaded, no joint at the left of it being loaded. This condition of load also produces the maximum stress (of the opposite character) in the member which connects with the above member in the unloaded chord. (*See Art.* **1295.**)

For the maximum stresses in the members $b\,c_1$ and $c\,c_1$, the joint b must be unloaded, and the joints c, c' and b' remain loaded. For this condition of load, $R_1 = 2{,}700 \times \frac{54 + 36 + 18}{90} = 3{,}240$ lb.

1307. A diagram of the truss and that portion of the stress diagram necessary for determining the stresses in $b\,c_1$ and $c\,c_1$ are represented in Fig. 274. From the latter, the stresses in $b\,c_1$ and $c\,c_1$ are found to be 4,460 pounds and 3,240 pounds, respectively. These stresses are maximum for these members. The stress diagram will be readily understood.

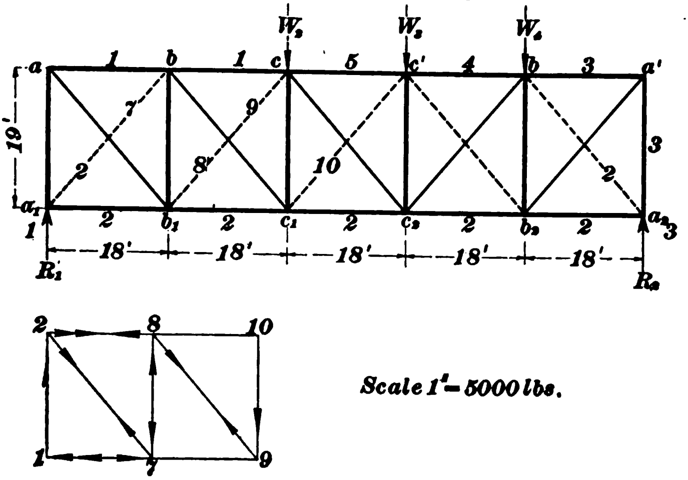


FIG. 274.

For the maximum stress in $c\,c_2$, the joints b and c must be unloaded and the joints c' and b' be loaded. For this condition of load, $R_1 = 2{,}700 \times \frac{36 + 18}{90} = 1{,}620$ lb. Fig. 275 shows the diagrams for this condition; the stress diagram will be readily understood. The stress in $c\,c_2$, which is maximum with this load, is found to be 2,230 pounds.

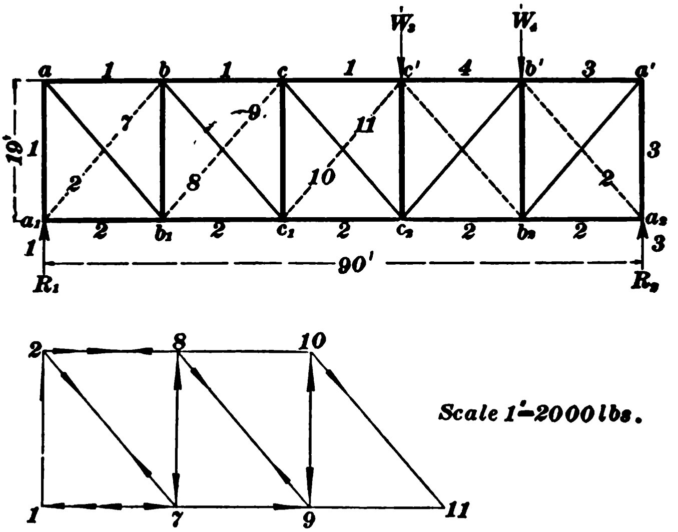


FIG. 275.

For all members at the right of the center, the stresses due to the right reaction will be identical with those which have been found for the members at the left of the center. Therefore, the maximum stress for $c'\,c_2$ will be greater than that given by this condition of load. Also, for those members which receive stress when the force of the wind is exerted from the opposite direction, the stress in each is of the same respective amount and character as found for the corresponding member above. As these members are the opposite diagonals in the panels, they will act as counters for the direction at present assumed for the wind. When a diagonal acts as a counter, the stress in it is less than when it acts as a main tie, and, therefore, no counter stresses need be found for the lateral system beyond the center panel. The maximum live wind stress for each member, as found above, must be added to the dead load wind stress, previously found, for the same. For the chords and for the members in the end panel, the live wind stresses are the same as the corresponding dead wind stresses, and the latter may be simply doubled. It is evident that Figs. 274 and 275 could have been combined.

1308. Wind Stresses in the Upper Lateral System.—The upper lateral system will be treated as a truss of three panels supported at the portals, the reactions being applied at the latter. The panel load is the same as the panel dead load for the lower lateral system, and $R_1 = 2{,}700 \times \frac{3-1}{2} = 2{,}700$ lb. Fig. 276 is a diagram of the upper lateral system with a complete stress diagram for the wind stresses in the same.

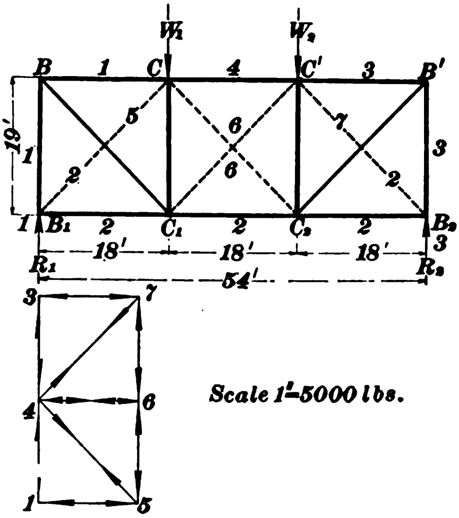


FIG. 276.

The stresses in $B\,C_1$ and $C\,C_1$ are found to be 3,720 pounds and 2,700 pounds, respectively. No stress is found for the diagonals in the center panel. The stress in each of the members $B\,C$, $C\,C'$, and $C_1\,C_2$ is equal to 2,560 pounds.

NOTE.—The student, having progressed thus far in the analysis of stresses, will probably have become able to readily recognize the character of each stress; therefore, the arrow-heads, which have been employed to distinguish the characters of the stresses, will be omitted in subsequent stress diagrams. Should the student be in doubt, however, as to the character of any stress, he should always resort to the system of marking the arrow-heads upon the lines of the stress diagram, as explained in former articles.

STRESSES IN A LATTICED PORTAL.

1309. The portal bracing extends across each end of the bridge between the upper portion of each pair of end posts; it serves to hold the trusses in a vertical position, and resists the lateral force of the wind against the side of the truss. Through the medium of the upper lateral system, the portal bracing, and the end posts, the wind pressure against the upper chords is transferred to, and finally resisted by, the anchorage at the bottoms of the end posts. In the present case, it has been found that the amount of wind pressure transmitted to each portal by the upper lateral system equals 2,700 pounds.

To this must be added the panel load of wind pressure against the ends of the chords and upper portions of the end posts, which is assumed to be applied directly at the portal, in equal portions on the windward and leeward sides. This is shown in Fig. 277 (*a*), in which are represented the upper lateral system, the portal bracing, and the end posts, together with the assumed positions of the wind forces. In Fig. 277 (*b*) are represented in outline the portal bracing and the end posts, together with the directions and positions of the external wind forces acting upon the same.

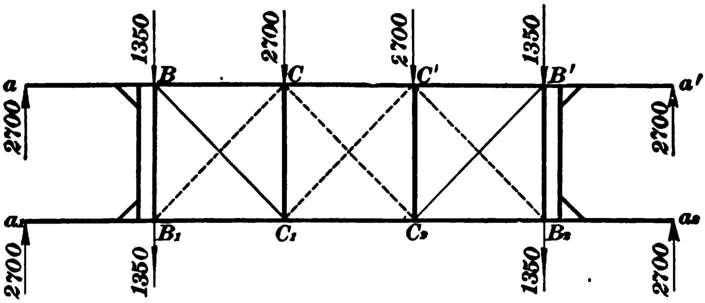


FIG. 277 (*a*).

The forces are considered to be acting *in the plane of the end posts.* The dimensions given for the portal bracing are the effective dimensions; they are really approximate, but they are sufficiently near to the actual dimensions for the purposes of computing the stresses. The length of the end post from center to center of pins, is 25′ 5⅝″; but, as the upper flange of the portal connects on top of the upper chord, for the purpose of computing the portal stresses the length of the end post will be considered to be 26 feet. The lateral wind pressure is assumed to be resisted in equal amounts by the anchorage at the foot of each end post, which assumption is probably very nearly correct.

Therefore, in the determination of the wind stresses in the portal bracing the following assumptions will be made, when not otherwise specially stated:

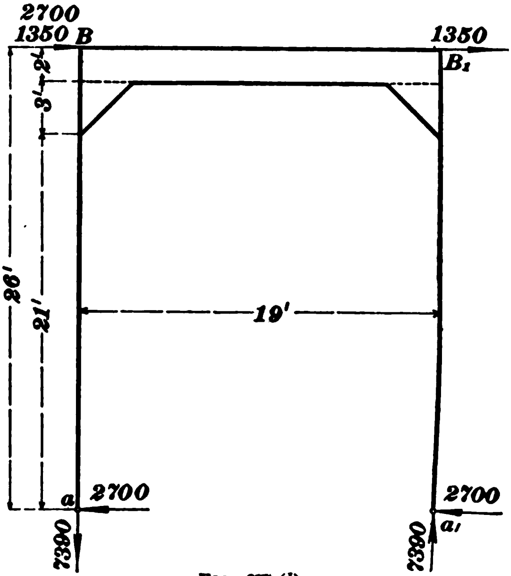


FIG. 277 (*b*).

(*a*) *When the calculated length of the end post contains a fraction, the fraction is to be taken equal to one foot.*

(*b*) *A panel load of wind pressure* (*or more, in certain cases, as will be hereafter noticed*) *will be assumed to be applied directly against the portal, one-half upon the windward and one-half upon the leeward side.*

(*c*) *The total wind pressure at the portal* (*including that applied directly at the portal and that delivered against it by* *the lateral system) will be assumed to be resisted in equal parts by the horizontal reaction at the bottom of the end posts; i. e., the horizontal reaction at the foot of each end post will be assumed to equal one-half the total wind pressure against the portal.*

1310. The force of the wind against the portal causes a downward pressure at a_1, the foot of the leeward end post, which is resisted by a corresponding upward reaction at the same point; it also causes an uplifting tendency at a, the foot of the windward end post, which is resisted by a corresponding downward, or negative, reaction at the same point. These reactions are not vertical except when the end posts are vertical; they always have the same directions as the end posts. The vertical components of these reactions are the vertical supports, or external forces, at the bottom of the end posts, but their horizontal components are internal forces or stresses in the chords which connect at these points. In order to distinguish them more clearly from the laterally horizontal reactions, these reactions, though really inclined, will be called vertical reactions; they are necessarily represented as vertical in the figure. The amounts of these reactions and the wind stresses in all portions of the portal bracing may be computed by resorting to the principles of moments. The vertical pressure at the foot of either end post is found by taking moments about the foot of the other end post. The total wind pressure against the portal is $2,700 + 1,350 + 1,350 = 5,400$ lb. The moment of this force about the foot of either end post must be balanced by the moment of the vertical reaction at the foot of the opposite end post about the same point; or, calling the wind pressure P and the reaction R, $26\,P - 19\,R = 0$, the moment of P being positive and that of R being negative. Therefore, $19\,R = 26\,P$, and as $P = 5,400$ lb., $R = \frac{26 \times 5,400}{19} = 7,389.47$ lb. (It will be near enough to call this reaction 7,390 pounds; in the following computations the results will be given to the nearest pound.) All the external forces acting upon the portal bracing and end posts have now been determined. The magnitude and position of each is shown in Fig. 277 (*b*).

1311. In order to determine the bending moment at any point in the portal it is only necessary to find the resultant moment of all the forces at the left about the point. Each bending moment thus found must be resisted by the moment of the internal force in the portal about the same point. Therefore, at any point, the moment of the internal forces must equal the bending moment at the same point; and, if the latter be divided by the lever arm of the resisting moment, the quotient will be the magnitude of the resisting force, or stress.

Fig. 278 is a diagram of the portal bracing with some dimensions exaggerated and the lattice bars omitted. Upon it are written, to the nearest ten pounds, the stresses in each flange at intervals of one foot along its length; i. e., at the points where the lattice bars connect. Treating the portal as a built beam, the bending moments are assumed to be entirely resisted by the flanges; the lattice bars resist the shear. It is impossible to correctly compute the stresses in the latter, on account of the different paths along which the applied forces may be transmitted.

In order to determine the stress at the point a, moments are taken about a'. The resultant moment of the external forces at the left taken horizontally and vertically about a', distinguishing those moments which tend to cause rotation to the right as positive, and those which tend to cause rotation to the left as negative, is $2,700 \times 22 + 4,050 \times 4 - 7,390 \times 1.5 = 64,515$ ft.-lb. This moment is positive; it is resisted by the moment of the internal force, or stress, at a, and if it be divided by the lever arm of this resistance, the quotient will be the amount of the stress. Therefore $\frac{64,515}{4} = 16,129$ lb. is the stress in the upper flange at a. It is evident that a positive bending moment at a', that is, a bending moment which tends to cause that portion of the portal at the left of a' to rotate to the right about that point, will cause compressive stress in the upper flange.

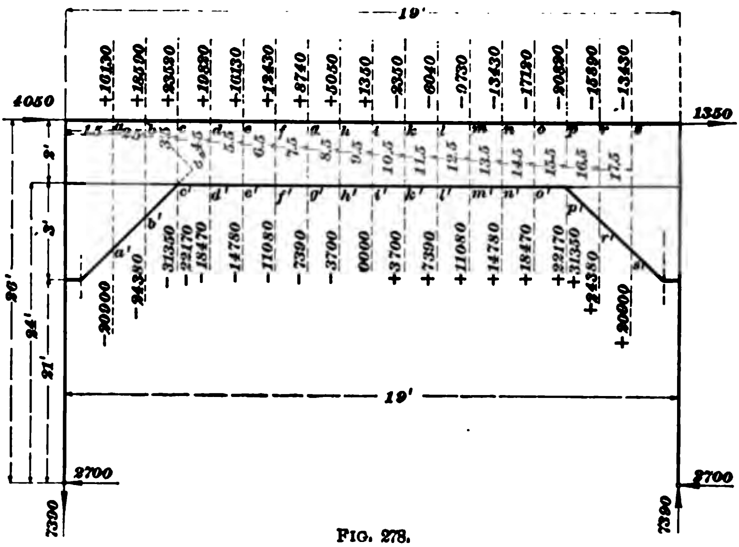


FIG. 278.

Therefore, this stress of 16,129 pounds at *a* is compression, it has the same sign as the bending moment.

NOTE.—It must be discerned that a + sign employed to denote positive bending moment is used in a sense entirely different from the meaning it has when it is placed before a stress to denote compression. In the present case, a positive bending moment will be found to produce compression in the upper flange, and a negative moment will produce tension in the same; in the lower flange, the opposite will be found to be the case.

Similarly, by taking moments about *b'*, the stress at *b* is found to be $\frac{2,700 \times 23 + 4,050 \times 3 - 7,390 \times 2.5}{3} =$ + 18,592 lb.

By taking moments about *i'*, the stress at *i* is found to be $\frac{2,700 \times 24 + 4,050 \times 2 - 7,390 \times 9.5}{2} = + 1,348$ lb. If the correct value 7,389.47 pounds were used for the vertical reaction, instead of 7,390 pounds, the correct stress would be found to be + 1,350 lb.

By taking moments about *k'*, the bending moment at this point is found to be $2,700 \times 24 + 4,050 \times 2 - 7,390 \times 10.5 =$

− 4,695 ft.-lb. This moment is negative, that is, it tends to cause that portion of the portal at the left of *k'* to rotate to the left about that point. It is evident that this tendency produces tensile stress in the upper flange at *k*, equal to $\frac{4,695}{2} = 2,348$ lb.

It will be found that the bending moment changes uniformly from maximum positive at the extreme left to maximum negative at the extreme right. The stress in the upper flange changes in the same manner except where the depth of the portal varies. The compressive stress at any point at the left of the center, however, will be found to be just 2,700 pounds greater than the tensile stress at the corresponding point at the right. This is because the "push" of the external forces (4,050 pounds) against the left end of this flange is 2,700 pounds greater than the corresponding "pull" of the external force (1,350 pounds) upon the right end. This difference of the external forces at the ends of the flanges becomes a resultant compression throughout the flange, increasing each compressive stress and decreasing each tensile stress by a uniform amount. The compressive stress in the left end of the flange is resisted by the lattice bars in the left half of the portal, an equal amount by each bar (except where the depth of portal varies). Near the center the compression in the upper flange becomes entirely taken up by the lattice bars, and the stress in the flange changes to tension, which increases towards the right in the same manner as the compression decreases; at the center of the upper flange, the compression is $\frac{2,700}{2} = 1,350$ lb. The amount of change in the stress at each point along the flange equals the horizontal component of the stress in the lattice bar which attaches at that point.

1312. The stress at any point along the lower flange is found by taking moments about the point in the upper flange vertically above it. No unequal external forces are applied against the ends of this flange. The stress at each point is found to be $\frac{2,700}{2} = 1,350$ pounds numerically less than, and of the opposite character to, the stress at the corresponding point in the upper flange; being zero at the center, compression at the right of the center, and tension at the left. The bending moment at each point has the same sign as the corresponding bending moment for the upper flange; but it is evident that a positive bending moment at any point (tending to cause that portion of the portal at the left of the point to rotate to the right about the point), while producing compression in the upper flange, would produce stress of the opposite character, or tension, in the lower flange. Thus, the stress at h', found by taking moments about h, equals $\frac{2,700 \times 26 - 7,390 \times 8.5}{2} = 3,693$ lb.; the bending moment producing this stress is positive, but the stress is tension and is given the $-$ sign. The bending moment producing the stress at i', found by taking moments about i (using the correct value of the vertical reaction), is $2,700 \times 26 - 7,389.47 \times 9.5 = 0$; the stress at i is, therefore, zero. Taking moments about k, the bending moment is $2,700 \times 26 - 7,390 \times 10.5 = -7,395$ ft.-lb. As previously explained, a negative bending moment causes tension in the upper flanges; but it causes stress of the opposite character, or compression, in the lower flange; therefore, the stress at k' is $\frac{7,395}{2} = +3,698$ lb. The compressive stress in the lower flange is found to increase from the center towards the right in precisely the same manner that the tensile stress increases towards the left.

1313. In the flanges of the brackets, at the left of c' and at the right of p', the conditions are slightly changed, owing to the fact that the changes in the direction of the flange change the lines of action of the internal forces acting along the flange. It must be kept in mind that the lever arm of a moment must always be taken perpendicular to the line of action of the force. For the stress in the lower flange at b', moments are taken about b; the bending moment at the latter point is $2,700 \times 26 - 7,390 \times 2.5 = 51,725$ ft.-lb. In order to obtain the stress at b', this bending moment must be divided by the perpendicular distance from the line of action of the stress to the center of rotation b, which is the distance $b\,b'$. This distance is approximately 2.121 feet, and the stress at b is $\frac{51,725}{2.121} = -24,387$ lb. The perpendicular distance $b\,b'$ may be either calculated or measured; instead of finding this distance, however, it is usually more expeditious to proceed as follows:

Find the horizontal component of the stress, that is, find the stress as though the flange were horizontal, by dividing the bending moment by the vertical lever arm; as $\frac{51,725}{3} = 17,242$ lb. Then, lay off the horizontal component thus found upon a horizontal line, and project it vertically (i. e., parallel to the lever arm used) upon a line drawn parallel to the line of action of the desired stress; that is, parallel to the flange. The operation is shown in Fig. 279. By measuring the latter line to the scale used, the stress in the flange at b' is found to be + 24,380 lb.

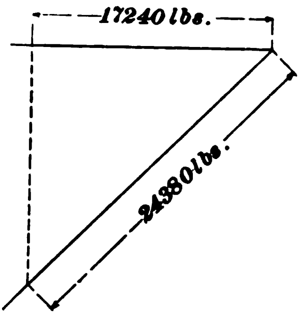


FIG. 279.

By applying the known principles of moments and proceeding as explained above, no special difficulty will be encountered in obtaining the stress at each point along the flanges of a latticed portal.

1314. The increment of the flange stress at any point is the horizontal component of the combined stress in the two lattice bars which connect at that point. If it be assumed that the entire increment of the flange stress is borne by the lattice bar which is in tension, then the difference in the flange stress at any two points becomes the horizontal component of the stress in the (tension) lattice bar which connects at the point having the greater flange stress, if tension, or the smaller flange stress, if compression.

To obtain the stress in the lattice bar, lay off upon a horizontal line the horizontal component of the stress, found by taking the difference of the stresses at two adjacent points, and project the same vertically upon line drawn parallel to the lattice bar.

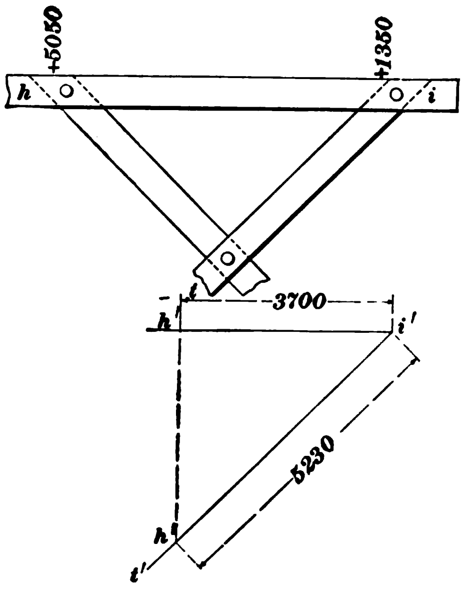


FIG. 280.

For example, the difference between the stresses at the points h and i is 3,700 pounds. (See Fig. 280.) This is laid off upon the horizontal line $h'\,i'$ by any convenient scale, and the point h' is projected vertically upon the line $t'\,i'$, drawn through i' parallel to the lattice bar $t\,i$. The line $h''\,i' = 5{,}230$ lb. represents the stress in the lattice bar $t\,i$.

EXAMPLE FOR PRACTICE.

Compute, to the nearest ten pounds, the stress in each flange of the portal at each point for which the stress is given in Fig. 278.

FLOOR BEAM STRESSES.

1315. The floor beams are the members which support the floor load and transfer it to the joints of the loaded chord. In most bridges the floor beams are small plate girders. Therefore, the following explanation of the method of obtaining the stresses in a floor beam will also apply to any plate girder carrying a uniform load. In a bridge having two trusses, the total live load upon a floor beam is twice the panel load supported by one truss; in the example the amount of live load supported by a floor beam is 16,200 × 2 = 32,400 lb.

The weight of the floor plank and joists, as previously obtained from Table 29, Art. **1299,** is 418.7 pounds per lineal foot. Calling this 419 pounds, the weight of floor supported by one beam is 419 × 18 = 7,542 lb.

The weight of the beam itself may be estimated by the following formula:

$$P = \frac{W\,l}{20\,d}, \qquad \textbf{(93.)}$$

in which P is the weight of the beam, W is the total live load supported by it, l is the distance in feet between supports, and d is the total depth of the beam in inches. This formula applies to ordinary iron floor beams built of plates and angles in the usual form; the results given by it are but roughly approximate, but are sufficiently accurate for the purposes of computing the stresses. The total live load upon the beam has been found to be 32,400 lb. = W. The distance between supports is 19 ft. = l. The value of d is taken at 24.5 inches; the depth of the beam will again be referred to further along. By the formula, the approximate weight of the beam is $\frac{32{,}400 \times 19}{20 \times 24.5} = 1{,}256$ lb.

1316. The total amount of live and dead load supported by a floor beam is, therefore, 32,400 + 7,542 + 1,256 = 41,198 lb., or say 41,200 pounds. This load is assumed to be uniformly distributed over that portion of the roadway supported by one beam. As the beam carries the entire load between its supports, the condition is that of a simple beam. The maximum bending moment, which occurs at the center, may readily be found by constructing the moment diagram, but, as it is a very simple case, it will be more expedient to compute it from a formula.

By applying the principles of moments to a beam carrying a uniform load of w pounds per foot, the maximum bending moment is found to be $M = \frac{w l^2}{8}$; or, since $w l =$ total load $= W$, we may write,

$$M = \frac{W l}{8}. \qquad \textbf{(94.)}$$

1317. For a uniformly loaded beam, the bending moment M_o at any point O, situated at a distance x from the center, is found by the formula

$$M_o = \frac{W l}{8} - \frac{W x^2}{2 l}; \qquad \textbf{(95.)}$$

or, if the weight w per foot be used,

$$M_o = \frac{w}{2}\left(\frac{l}{2} - x\right)\left(\frac{l}{2} + x\right). \qquad \textbf{(96.)}$$

The values of the characters used in the preceding formulas will be clearly understood by reference to Fig. 281; O is the point for which the bending moment is given by formulas **95** and **96.**

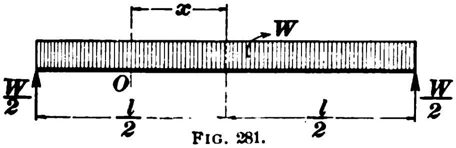


FIG. 281.

1318. Formula **94** is commonly used to obtain the maximum bending moment for the floor beams of highway bridges. As the flanges of such beams are usually made of uniform section throughout their length, the maximum moment is the only moment necessary to be found.

Formulas **94, 95,** and **96,** however, do not correctly apply to the floor beams of bridges, as the condition of a load distributed uniformly over the length of the beam does not obtain in a floor beam, even with the roadway uniformly loaded. The floor beam is not uniformly loaded over its entire length between supports, for it is supported at the centers of the chords, and can be loaded only over that central portion of its length corresponding to the clear width of roadway. In the present case, the distance between supports is 19 feet, while the clear width of roadway is 18 feet. The error arising from applying formula **94** to a floor beam is small, but it is an error on the dangerous side, always giving a bending moment slightly too small.

The following is a much better formula for the maximum or center bending moment M of a floor beam:

$$M = \frac{W(b+2c)}{8}, \qquad \textbf{(97.)}$$

in which b is the clear width of the roadway, and c is the width of one chord, while W represents the same value as before.

The bending moment M_o at any point O, situated at a distance x from the center of a floor beam is given, by the formula

$$M_o = \frac{W(b+2c)}{8} - \frac{Wx^2}{2b}, \qquad \textbf{(98.)}$$

in which all characters have the same significance as in formulas **95** to **97**, and will be clearly understood by reference to Fig. 282.

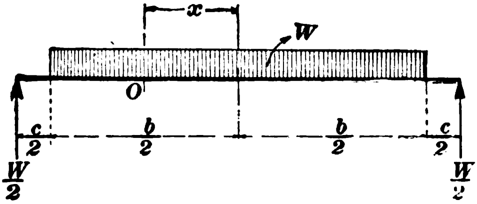


FIG. 282.

1319. Following the usual practice for short beams, however, the flanges of the floor beams in the example will be made of uniform section throughout their entire length; and, therefore, for the purpose of determining the required section, it will be unnecessary to determine any bending moment other than the maximum or center moment, which is given by formula **97.** In the example c, the width of each upper chord is 1 foot, and, by applying the formula, the maximum bending moment is found to be $\frac{41,200 \times (18+2)}{8} =$ 103,000 ft.-lb., or $103,000 \times 12 = 1,236,000$ in.-lb.

The lever arm of the resisting moment, which must equal the bending moment, is generally considered to be the distance between the centers of gravity of the flanges. *For ordinary floor beams, it is good practice to make the total depth of the beam equal to about one-tenth of the span.* This depth may be increased or diminished, according as the load is heavy or light, or it may be modified to meet the requirements of other controlling circumstances; the practice is by no means to be rigidly followed. In the present case, one-tenth the span is $\frac{19 \times 12}{10} = 22.8$ in. A web-plate 24 inches deep will be used. As the depth of the beam, out to out of flange angles, can usually be made from one-quarter to one-half inch deeper than the depth of the web-plate, the depth of the beam in the present case will be $24\frac{1}{2}$ inches.

In the ordinary sizes of angle bars used for floor beams, the distance from the center of gravity of the bar to the back of the longer or horizontal flange usually approximates $\frac{3}{4}$ of an inch, and may generally be taken at that amount. In the present case, then, $24.5 - (.75 \times 2) = 23$ inches may be taken as the distance between the center of gravity of the flanges. This is known as the **effective depth** of the beam.

The maximum bending moment divided by the effective depth of the beam (i. e., the lever arm of the resisting moment) will give the maximum resisting force required, or flange stress. As the effective depth is always expressed in inches, the bending moment must be expressed in inch-pounds. In the example, then, the maximum flange stress is $\frac{1,236,000}{23} = 53,740$ lb.

1320. It is evident that formulas **94** and **97** may be readily made to represent maximum flange stress instead of maximum bending moment by simply introducing a quantity representing the effective depth of the beam into the denominator of each. Let d represent the effective depth of the beam, and let S_m represent the maximum flange stress. Then, from formula **94**,

$$S_m = \frac{Wl}{8d}. \qquad \textbf{(99.)}$$

Similarly, from formula **97**,

$$S_m = \frac{W(b+2c)}{8d}. \qquad \textbf{(100.)}$$

1321. The maximum shear in the beam occurs at the supports, and is equal to the reaction (half of the load on the beam). At any other point the shear equals the reaction minus the load between the point and the support. In general, *the shear at any point of a uniformly loaded beam equals the load between that point and the center of the beam.* If w = load per foot, shear at a distance of x feet from center = $w\,x$.

In the example, each reaction equals $\frac{41{,}200}{2} = 20{,}600$ lb. This is also the maximum shear in the beam, uniform for about 6 inches towards the center from each reaction.

EXAMPLES FOR PRACTICE.

NOTE.—Find each result to the nearest hundred pounds.

1. For the above floor beam, (*a*) calculate the maximum bending moments, in foot-pounds, by formula **94**, and (*b*) reduce it to inch-pounds.
Ans. (*a*) 97,850 ft.-lb.; (*b*) 1,174,200 in.-lb.

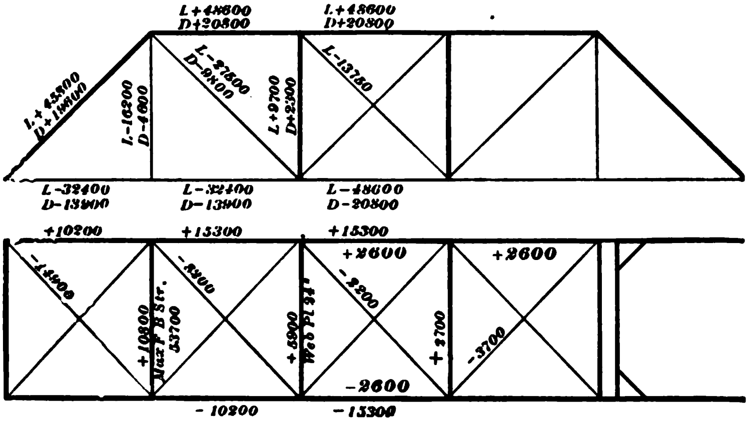
2. By formula **95**, find the bending moment, in inch-pounds, (*a*) at a distance of 4 feet, and (*b*) at a distance of 8 feet, from the center of the same beam.
Ans. (*a*) 966,000 in.-lb.; (*b*) 341,500 in.-lb.

3. Find, by formula **98**, the bending moments, in inch-pounds, for the same points in the floor beam.
Ans. (*a*) 1,016,300 in.-lb.; (*b*) 357,100 in.-lb.

4. Construct a moment diagram for the beam (i. e., for a simple beam 19 feet long between supports, carrying a load of 41,200 pounds, uniformly distributed over the central 18 feet of its length, leaving 6 inches between each support and the corresponding end of the load). Determine the bending moment in inch-pounds at the center of the beam, and at points 4 feet and 8 feet, respectively, from the center, ascertaining which of the above results are correct.

STRESS SHEETS.

1322. The maximum stresses in all the main members of the structure chosen as an example have now been found and each operation has been fully explained. In Fig. 283 these maximum stresses are shown written along the respective members in the left half of a diagram of the truss



Length of Span, C. to C. End Pins .... *= 90'-0"*
Height of Trusses, C. to C. Chord Pins .... *= 18'-0"*
No. of Panels = 5. Length of Each Panel .... *= 18'-0"*
Clear Width of Roadway .... *= 18'-0"*
Live Load per Sq. Ft. .... *= 100 lb.*
Live Load per Lin. Ft. .... *= 1800 lb.*
Dead Load per Lin. Ft. .... *= 770 lb.*

FIG. 283.

and in a diagram of the lateral systems. In the latter the left half represents one-half of the lower lateral system, while one half of the upper lateral system, together with the portal bracing, is represented in the right half. The stresses are written along the respective members in each. On each member in the diagram of the truss the live load stresses and dead load stresses are written separately, but in the lateral diagrams the live and dead wind stresses for each member are combined. The maximum floor-beam stress is written in the diagram of the lower lateral system. As the amount of stress in the floor beam depends upon the depth of the beam, it is necessary to designate the depth of the web plate.

The portal stresses are not given. Each stress is written correctly to the nearest hundred pounds only. The data for the bridge, giving the general dimensions and assumed loads, are written below the diagrams.

Such diagrams, with complete data, are usually constructed upon cap size sheets of paper, or larger when necessary, and are sometimes known as **strain sheets;** the name **stress sheets** applies more properly, however, and will here be used. All stress sheets required to be made throughout this Course should, whenever possible, be constructed upon sheets of paper of cap size.

The stress sheet shown in Fig. 283 is incomplete; the material for each member, when proportioned, is also to be written along it.

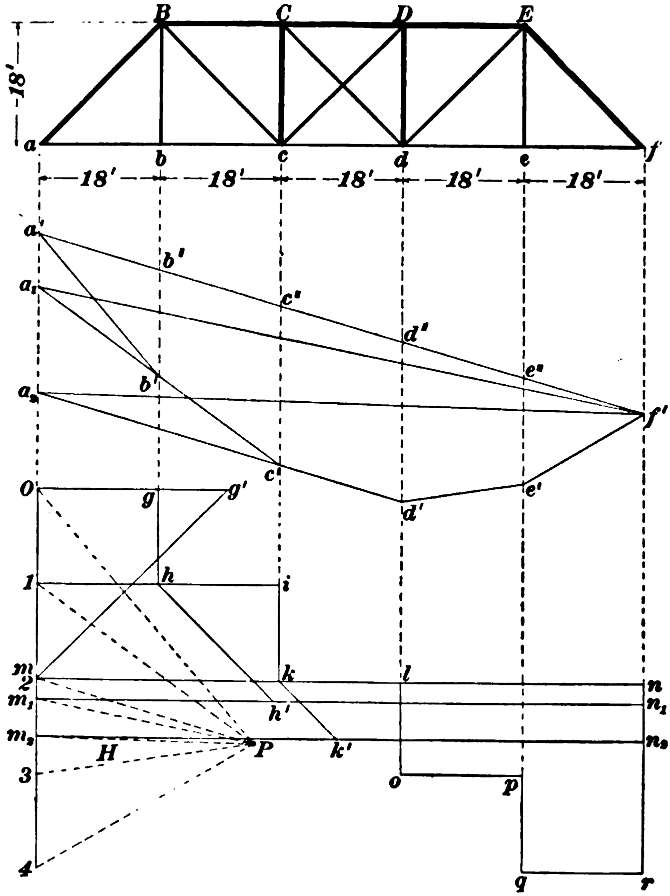
1323. The method of the graphical analysis of stresses, explained in the preceding pages, i. e., the method of determining the stresses by means of the stress diagram, or combined force polygons, constructed for the forces which act upon the several joints of the truss, is commonly known as the **Clerk Maxwell method,** being named after Prof. Clerk Maxwell, one of the early investigators in graphical statics, who largely developed and extended the method.

It is by no means the only method that can be employed for this purpose; but it is probably the best graphical method. After becoming familiar with the operations, the student will be able to somewhat shorten them. For the present, however, and until he has become thoroughly familiar with all the processes, it will be better for him to systematically follow the method as given.

METHOD BY THE MOMENT AND SHEAR DIAGRAMS.

1324. A method of obtaining the stresses by means of the moment and shear diagrams will now be briefly noticed. The live load stresses will be obtained for the members of the truss of the structure which was chosen to illustrate the previous method. The construction for the live load stresses, shown in Fig. 284, will sufficiently illustrate the method. A diagram of the truss, as *a f*, is first constructed; through the lower chord joints, *a*, *b*, *c*, *d*, *c*, and *f* lines are drawn verti-

Scale for Truss: 1 in. = 30 ft.



Scale for Loads: 1 in. = 32,000 lb.

FIG. 284.

cally downwards. Upon the vertical through *a*, the panel loads are laid off downwards in order, forming the load line. Thus, *0-1* = 16,000 pounds represents the panel load at *b*; *1-2*, the panel load at *c*, etc. The scale used for this purpose is not necessarily the same as that used for the diagram of the truss. Choose a pole *P*, and draw the rays of the force diagram *P0*, *P1*, *P2*, *P3*, and *P4*. The position of the pole should be so chosen that the pole distance *H* will be some convenient multiple (or factor) of the height of truss. In the example the height of truss is 18 feet, and the pole distance is made equal to 36,000 pounds. Construct the equilibrium polygon, or moment diagram, by drawing lines parallel to the rays of the force diagram, in order between the verticals through the joints of the truss, and finally the closing line $f' a'$, drawn from f', the terminus of the last line, to the starting point a'. The line *Pm*, drawn from *P* parallel to the closing line $f' a'$ of the moment diagram divides the load line into reactions $m\,4 = R_2$ and $m\,0 = R_1$.

1325. Stresses Due to Bending Moment.—The intercept in the moment diagram, on a vertical through any joint, measured to the scale used to construct the diagram of the truss, when multiplied by the pole distance and divided by the height of the truss, will give the chord stress in each chord between those two diagonals between which the vertical member at that joint is situated. The intercepts $b'' b'$ and $c'' c'$ are found to equal 16.2 ft. and 24.3 ft., respectively. Calling the height of truss *h*,

$$c'' c' \times \frac{H}{h} = 24.3 \times \frac{36,000}{18} = 48,600 \text{ lb.}$$

is the chord stress in the upper and lower chords between the diagonal members *Bc* and *Ed*. As the diagonals *Cd* and *Dc* do not act when the truss is fully loaded, the maximum stress in the upper chord is uniform from *B* to *E*. The intercept $b'' b'$ was found to equal 16.2 ft., and

$$b'' b' \times \frac{H}{h} = 16.2 \times \frac{36,000}{18} = 32,400 \text{ lb.}$$

is the chord stress in the lower chord members *a b* and *b c*, i. e., between the diagonal members *a B* and *B c*.

1326. Stresses Due to Shear.—As the line *P m*, Fig. 284, drawn parallel to the closing line of the moment diagram, divides the load line 0-4 into the reactions $m\,4$ and $m\,0$, a horizontal line, drawn through m, as the line $m\,n$, becomes the shear axis, and $0\,g\,h\ldots q\,r\,n$, drawn according to the instructions given in Art. **1228,** becomes the shear line for this load. As this condition of load gives the maximum shear in the panel $a\,b$, and, therefore, the maximum stress in $a\,B$, a line drawn parallel to $a\,B$ between the line $0\,g$, produced if necessary, and the shear axis, as the line $m\,g'$, will represent the maximum stress in that member. Measured to the scale used in laying off the load line, the line $m\,g'$ is found to equal 45,800 pounds.

For the maximum shear in the panel $b\,c$, and the maximum stress in the member $B\,c$, the panel load at b, represented by 0-1, is considered to be removed; 1-4 now becomes the load line and 1-4-P-1, the force diagram. The moment diagram remains the same as before, except that, as the line $P\,0$ no longer belongs to the force diagram, there is no longer any line in the moment diagram parallel to it, but the line $c'\,b'$ continues parallel to $P\,1$ till it intersects the vertical through a at a_1. The line $f'\,a_1$ becomes the new closing line of the moment diagram; $P\,m_1$ is drawn parallel to $f'\,a_1$, and $m_1\,n_1$ becomes the shear axis for this load, for which $1\,i\,k\,l\,o\,p\,q\,r$ is the shear line. The left reaction for this lead is represented by $1\,m_1 = 19{,}440$ pounds, which line also represents the shear at any point between a and c. A line drawn between the line $i\,1$ and the shear axis $m_1\,n_1$ parallel to $B\,c$, as the line $h\,h'$, will represent the maximum stress in $B\,c$. Measuring with the scale used for the load line, $h\,h'$ is found to equal 27,500 pounds.

For the maximum shear in the panel $c\,d$, and maximum stress in the member $C\,d$, the load at c, represented by 1-2, is also considered to be removed. For this condition of load, 2-4-P-2 becomes the force diagram; $a_2\,c'\,d'\,e'\,f'\,a_2$, the moment diagram; $m_2\,n_2$, the shear axis, and $2\,l\,o\,p\,q\,r$, the shear line. On the load line $m_2\,2 = 9{,}720$ pounds, represents the left reaction; it is equal to the shear at any point between a and d and to the maximum compression in $C\,c$. A line $k\,k'$, drawn parallel to $C\,d$ between the line 2-1 and the shear axis m_2 n_2, represents the maximum stress in the member $C d$. To the scale used for the load line, $m_2 \, 2 =$ 9,720 pounds and $k k' =$ 13,700 pounds. The stress in $B b$ is simply tension equal to one panel load = 16,200 pounds.

The maximum live load stresses have thus been found for all members in the left half of the truss; those for the corresponding members in the right half are identical with those found. The dead load stresses are found in substantially the same manner, the truss being considered fully loaded. The method will be readily understood from the above explanation of the process of finding the live load stresses.

This method, as applied to special cases, can be much shortened, but it can not be expeditiously applied to all forms of trusses.

EXAMPLES FOR PRACTICE.

1. For the same bridge construct the moment and shear diagrams for the dead load, determining the dead load stresses.

2. Construct the moment and shear diagrams determining the live wind stresses in the lower lateral system.

3. Construct the moment and shear diagrams determining the dead wind stresses in the same.

4. Construct the moment and shear diagrams determining the wind stresses in the upper lateral system.

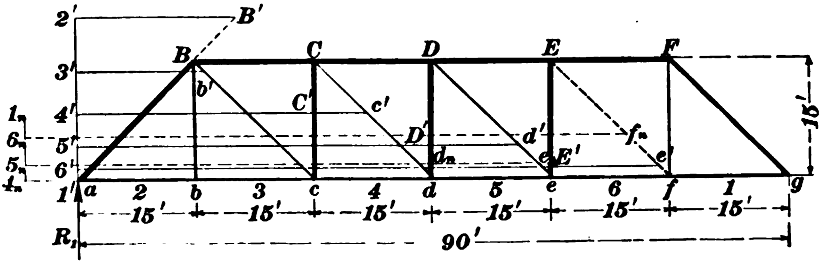
A SHORT METHOD FOR WEB STRESSES.

1327. The maximum live load web stresses for a truss having parallel chords may be obtained by a much shorter method than has been previously explained. Such a method will now be explained by an example.

For the truss of a through bridge of 90 feet span, the same data will be used that were given in Art. **1292,** except that the truss will be divided into 6 panels and the height of the truss will be 15 feet. The panel length is, therefore, $\frac{90}{6} = 15$ feet, and for one truss the live panel load is $\frac{1,800 \times 15}{2} = 13,500$ pounds. The dead load will be taken at 770 pounds per lineal foot, as obtained in Art. **1299**; for one truss, therefore, the panel dead load is $\frac{770 \times 15}{2} =$ 5,775 pounds.

A short method of determining the maximum live load stresses is as follows (see Fig. 285).

Construct a diagram of the truss to a rather large scale. From *a*, the joint at the left reaction, lay off upwards upon a vertical line the left reaction from a full load, as *1′-2′*. This constitutes the load line, upon which from *2′* lay off downwards, consecutively, those portions of the several panel loads supported by the left reaction, taking the loads in order and passing to the right across the truss. $R_1 =$ 13,500 $(\frac{5}{6} + \frac{4}{6} + \frac{3}{6} + \frac{2}{6} + \frac{1}{6}) = 11,250 + 9,000 + 6,750 + 4,500 +$ 2,250 = 33,750 lb. Then *1′-2′* = 33,750 lb. is laid off upwards, and *2′-3′* = 11,250 lb., *3′-4′* = 9,000 lb., *4′-5′* = 6,750 lb., *5′-6′* = 4,500 lb., and *6′-1′* = 2,250 lb. are laid off downwards in order to the starting point *1′* or *a*. Also, from *1′* lay off upwards upon the load line, or, as in the figure, from a point horizontally opposite *1′*, lay off upwards upon a vertical line $4_{\prime\prime}$-$5_{\prime\prime}$, $5_{\prime\prime}$-$6_{\prime\prime}$, and $6_{\prime\prime}$-$1_{\prime\prime}$, equal, respectively, to 2,888 lb., 5,775 lb., and 5,775 lb., the amounts of dead load at *d*, *e*, and *f*, respectively, that are supported by the right reaction. The right reaction due to *live* load is entirely neglected.



Scale for Loads: 1 in. = 40,000 lb.

FIG. 285.

Upon the load line, *1′-2′* represents the left reaction due to live load with the truss fully loaded, which condition gives the maximum stress in the member *a B*. Project the point *2'* upon *a B*, produced if necessary, by the horizontal line *B' 2'*; then, *a B'* will represent the stress in *a B*. The maximum stress in *B c* obtains with the load at *b* removed, all other joints being loaded. For this condition *1'-3'* represents the reaction, and by projecting the point *3'* upon *B c* (also produced, if necessary) by the horizontal line *b' 3'*, the line *b' c* will represent the stress in *B c*. The maximum stresses in *C c* and *C d* obtain with the load at *c* also removed, all joints at the right remaining loaded. The reaction for this condition is represented by *1'-4'*, and by drawing through *4'* the horizontal line *c' 4'*, the line *C' c* will represent the stress in *C c* and *c' d* will represent the stress in *C d*. The maximum stresses in *D d* and *D e* obtain with all loads at the left of the panel *d e* (numeral *5*) removed, all joints at the right of it remaining loaded. For this condition *1'-5'* represents the reaction, and by drawing through *5'* the horizontal line *d' 5'*, the lines *D' d* and *d' e* will represent what would be the stresses in *D d* and *D e*, respectively, if no negative dead load shear existed in the panel *d e*. The negative shear in this panel is represented by 4_n-5_n and by drawing through 5_n the horizontal line e_n-5_n, the lines $D'\ d_n$ and $d'\ e'_n$, will represent the resultant live load stresses in *D d* and *D e*, respectively. With the load at *e* also removed, all joints at the right remaining loaded, *1'-6'* represents the reaction. This condition will give the maximum stress for the member *E f*, if any stress can be found for that member; by drawing through *6'* a horizontal line *6' e'*, the line *e' f* will represent what would be the live load stress in *E f*, if no negative shear existed in the panel *e f*. But an amount of negative dead load shear represented by 4_n-6_n is always present in this panel; a horizontal line $6_n\ f_n$ drawn through 6_n will lie above the line *e' 6'*, showing that the negative dead load shear more than counteracts the positive live load shear in this panel. Therefore, no stress can be obtained for a tie in the position *E f*. The maximum stress in the vertical member

E e is produced by the right reaction, and is the same as has been found for *C c*.

It will be noticed that the dead load numeral 6_n is situated *above* the live load numeral $6'$, and that *no stress is produced* by positive shear in the panel *e f* under which the corresponding numeral *6* is situated; also, that the dead-load numeral 5_n is situated *below* the live load numeral $5'$, and that *stress is produced* by positive shear in the panel *d e* under which the corresponding numeral *5* is situated. This will always be the case, and it will always correctly indicate whether counter stress will be found in any panel of a truss having parallel chords.

1328. The method of laying off the load line and determining in what panel the counter stress ceases may be described in a general manner as follows:

(*a*) *The left reaction due to the live load is laid off upwards upon a vertical line erected at the left support; this constitutes the load line, the right reaction due to the live load being entirely neglected.*

From the top of the load line the respective portions of the live panel loads which are supported by the left reaction are laid off in order downwards, taking the loads from the left to the right across the truss; the last load will just reach to the bottom of the load line. From this point, or from a point on a vertical line horizontally opposite to it, the dead loads, supported by the right reaction, are laid off upwards, beginning at the center of the truss and passing in order towards the right until the numeral used to designate the dead load shall fall at a point upon the load line above the corresponding numeral employed to designate a live load.

When this occurs, no positive live load shear will be found in that panel of the truss under which the corresponding numeral is situated, nor in any panel to the right of it. But positive live load shear and counter stress will be found in each panel whose numeral used to designate the live load is situated at a point on the load line above the corresponding numeral used to designate the dead load.

1329. In determining the web stresses, proceed as follows:

(*b*) *From the point upon the load line indicating the upper limit of the reaction for each condition of load, a horizontal line is drawn intersecting those web members which obtain their maximum stress with that condition. If the members are at the right of the center of the truss, a horizontal line is also drawn to intersect them from the corresponding dead load numeral.*

On a line representing a member of the truss for which the maximum stress obtains in any of the several conditions, the portion below the horizontal line drawn from the live load numeral to intersect it (and above the line drawn from the corresponding dead load numeral, if the member is not at the left of the center) will represent the stress in the member.

EXAMPLES FOR PRACTICE.

1. By the method just explained, find the maximum live load web stress for the truss shown in Fig. 285, using the data given.

Ans. For $a\,B$ + 47,730 lb. For $C\,d$ − 19,090 lb.
For $B\,c$ − 31,820 lb. For $D\,d$ + 3,860 lb.
For $C\,c$ + 13,500 lb. For $D\,e$ − 5,460 lb.

2. By the same method obtain the maximum live load web stresses for the five panel truss of the previous articles.

THE HOWE TRUSS.

1330. The earliest form of simple truss of any considerable length of span that was built for bridge purposes in America is shown in Fig. 286. It was devised by William Howe, in 1840, and is known as the **Howe truss.** It is a very excellent form of truss, and is still much used in this country in localities where timber is cheap. For trusses

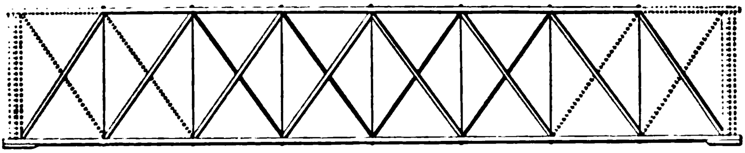


FIG. 286.

constructed entirely of metal, however, it is not as economical as the Pratt truss.

In the Howe truss all vertical members (except end posts) are tension members, and all diagonal members in the web system are compression members.

As originally constructed, the Howe truss had short panels with two diagonal braces in each panel, and vertical end posts; the floor was supported directly upon the lower chord, which, therefore, acted as a beam as well as a tension member. All parts of the truss were constructed of wood except the intermediate vertical members, which were iron rods. It is evident that the members represented in the figure by dotted lines can carry no stress; they serve no other purpose than to stiffen the truss.

1331. In modern examples of the Howe truss, those members which do not bear stress are omitted, and the lower chord is usually constructed of metal. Such a truss for a through bridge is represented in Fig. 287. For the sake of comparison, the same data are assumed for this truss that were assumed for the Pratt truss, treated in the preceding pages; namely, length of span, 90 ft.; panel length, 18 ft.; height of truss, 18 ft.; clear roadway, 18 ft.; live load, 1,800 lb., and dead load, 770 lb. per lineal foot.

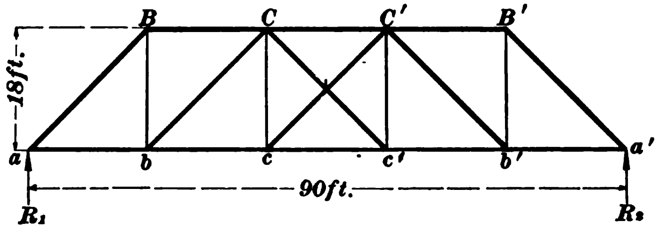


FIG. 287.

The Howe truss, of Fig. 287, is very similar to the Pratt truss, shown in the previous figures; the essential difference is that in the former all vertical members are ties and all diagonal members are struts, while in the latter the opposite is the case. It will be noticed that in the Howe truss the hip vertical is not merely an independent suspender, as in the Pratt truss, but is an important part of the entire web system; the amount of shear resisted by it is the same as that resisted by the end brace.

The method of constructing the stress diagram is substantially the same for both trusses, and after what has been explained concerning the Pratt truss, no difficulty will be encountered in obtaining the stresses for the members of the Howe truss. It must be noticed that, as in this truss the duties of diagonal and vertical members are the reverse of what they are in the Pratt truss, *the maximum stress in any vertical web member of a Howe truss, and in the diagonal meeting it at the upper chord, occur when the joint at the foot of the vertical member and all the joints at the right are loaded, the others being unloaded.*

(In the solution of Question 595, the student has drawn the stress diagram for a four-panel Howe truss carrying a full uniform load.)

EXAMPLES FOR PRACTICE.

1. Construct the stress diagram obtaining the dead load stress in each member in the left half of the truss shown in Fig. 287, using the data given above and correcting the stress in each vertical member.

Ans. For $a\ B$ + 19,600 lb.
For $B\ C$ + 13,860 lb.
For $C\ C'$ + 20,790 lb.
For $a\ \ b$ − 13,860 lb.
For $b\ \ c$ − 20,790 lb.
For $c\ c'$ − 20,790 lb.
For $B\ b$ − 11,550 lb.
For $b\ C$ + 9,800 lb.
For $C\ c$ − 4,620 lb.

2. Compute the live load stresses in the chords and end posts from the corresponding stresses obtained in the solution of the preceding example.

Ans. For $a\ B$ + 45,820 lb.
For $B\ C$ + 32,400 lb.
For $C\ C'$ + 48,600 lb.
For $a\ b$ − 32,400 lb.
For $b\ c$ − 48,600 lb.
For $c\ c'$ − 48,600 lb.

3. Construct the diagrams for the maximum live load web stresses.

Ans. For $B\ b$ − 32,400 lb.
For $b\ C$ + 27,490 lb.
For $C\ c$ − 19,440 lb.
For $c\ C'$ + 13,750 lb.

4. Solve the above example by means of the moment and shear diagrams.

THE WARREN GIRDER.

1332. The form of truss shown in Fig. 288 is known as a **Warren girder.** Constructed as a riveted girder, it is a very excellent truss for spans of moderate length. The distinguishing feature of this truss is that all web members are inclined at a uniform angle and no counter members are used, the counter stresses being reversed stresses in the main web members near the center.

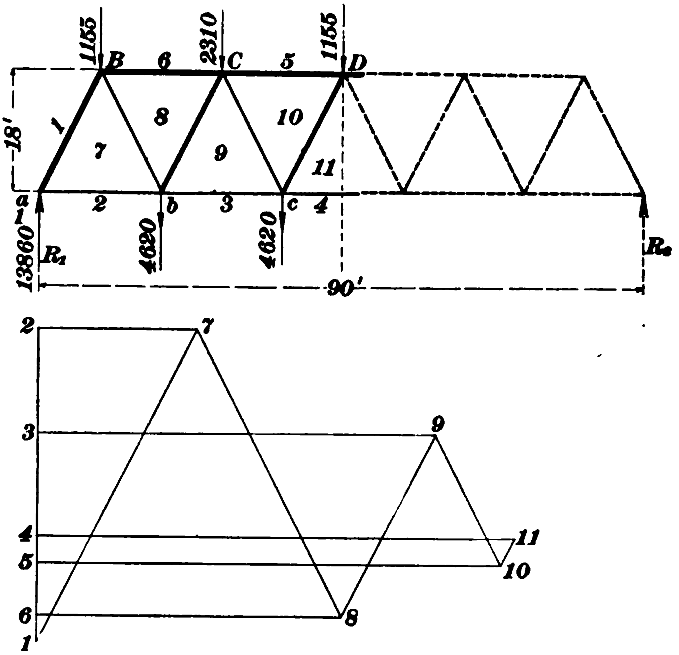


FIG. 288.

Although not giving the best possible proportions for a truss of this type, for the sake of comparison, the same data will again be assumed for the truss of Fig. 288 that were assumed for the trusses of the preceding articles, namely: length of span, 90 ft.; panel length, 18 ft.; height of truss, 18 ft.; clear width of roadway, 18 ft.; live load, 1,800 lb., and dead load, 770 lb. per lineal foot. As it is a through bridge, the joints of the lower chord carry the load; the joints of the upper chord are situated midway, horizontally, between the joints of the lower chord.

The stress diagrams are constructed according to the general principles used and explained in preceding articles. The student may also refer to Art. **1148,** where a Warren girder was given as an example.

1333. The assumption that one-third the dead load is carried at the upper chord joints is for through bridges approximately correct. If, in the construction of the stress diagram, it be assumed that each entire panel of dead load is carried at a lower chord joint, then the one-third of each panel load belonging to the upper chord is carried in equal parts by the two adjacent upper chord joints. Therefore, the amount of error in this assumption relating to each of the two web members which meet at each lower chord joint is $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ panel load. As the stress in each web member bears the same relation to the corresponding vertical shear that the length of the member bears to the vertical height of the truss, the stress in each web member could be readily corrected in much the same manner as explained for the vertical members in a Pratt truss. The error of the assumption also slightly affects the chord stresses, however, and these could not be so readily corrected. But the error is very small, and is usually entirely neglected.

1334. The correct dead load stresses may be obtained by constructing a stress diagram for one-half the truss, as shown in Fig. 288. This stress diagram is drawn for those stresses which relate to the left reaction, the right reaction, together with all loads and stresses relating to it, being entirely neglected. Only one-half of the one-third panel dead load at D belongs to this stress diagram, as only that portion of this load is supported by the left reaction. This diagram will be readily understood without further explanation.

In determining the maximum live load web stresses due to the left reaction, a maximum compression will be obtained for $D c$ and a maximum tension for $D c'$. It is evident that under corresponding conditions the right reaction will produce exactly the reverse of this, i. e., tension in *D c* and compression in *D c'*. Therefore, both maximum stresses must be written for each of these members which are thus found to undergo reversals of stress under varying conditions of load.

EXAMPLES FOR PRACTICE.

1. Construct the stress diagram for the dead load stresses in the members in the left half of the truss shown in Fig. 288, using the data given above and assuming the entire dead load to be concentrated at the joints of the lower chord.

Ans. For *a B* + 15,490 lb.
For *B C* + 13,860 lb.
For *C D* + 20,790 lb.
For *a b* − 6,930 lb.
For *b c* − 17,330 lb.
For *c c'* − 20,790 lb.
For *B b* − 15,500 lb.
For *b C* + 7,750 lb.
For *C c* − 7,750 lb.
For *c D* − 0,000 lb.

2. Compute the live load stresses in the chords and end posts from the corresponding dead load stresses.

Ans. For *a B* + 36,220 lb.
For *B C* + 32,400 lb.
For *C D* + 48,600 lb.
For *a b* − 16,200 lb.
For *b c* − 40,500 lb.
For *c c'* − 48,600 lb.

3. Construct the stress diagrams obtaining the maximum live-load web stresses.

Ans. For *B b* − 36,220 lb.
For *b C* + 21,730 lb.
For *C c* − 21,730 lb.
For $c\ D\left\{\begin{array}{l}+\ 10{,}870 \text{ lb.}\\ -\ 10{,}870 \text{ lb.}\end{array}\right.$
For $D\,c'\left\{\begin{array}{l}-\ 10{,}870 \text{ lb.}\\ +\ 10{,}870 \text{ lb.}\end{array}\right.$

4. Construct a stress diagram similar to that of Fig. 288, obtaining the correct dead load stresses, assuming two-thirds the dead load to be carried at the joints of the lower chord, and one-third to be concentrated at the joints of the upper chord.

Ans. For *a B* + 15,500 lb.
For *B C* + 13,280 lb.
For *C D* + 20,210 lb.
For *a b* − 6,930 lb.
For *b c* − 17,330 lb.
For *c c'* − 20,790 lb.
For *B b* − 14,200 lb.
For *b C* + 9,040 lb.
For *C c* − 6,460 lb.
For *c D* + 1,290 lb.

NOTE.—Compare the results obtained in the solution of Example 4 with those obtained in the solution of Example 1.

GENERAL REMARKS.

1335. In order to obtain reliable and accurate results in the graphical analysis of stresses, it is necessary that the diagram of the truss should be accurately drawn to as large a scale as convenient, and that the lines of the stress diagram should be drawn truly parallel to the corresponding members of the truss. Decimal scales should be used for stress diagrams; they should be large enough to be easily read to the nearest hundred pounds, except in cases where the use of so large a scale causes the stress diagram to be inconveniently large. Scales of 10, 20, 30, 40, 50, and 60 parts to the inch are the scales commonly used.

The best scales for graphical work are the ordinary paper (pasteboard) scales, which can be obtained 18 inches long; these scales also possess the advantage of being cheap. Besides the necessary scales, a complete outfit for graphical work consists of an ordinary drawing board, a **T** square, a pair of rather large 45° and 60° triangles, a rolling parallel rule, a hard pencil, a rubber, and a fine flat file to sharpen pencil. The parallel rule, though very convenient, is not indispensable. The paper commonly used is the ordinary cheap detail paper.

It is an absolute waste of time to attempt to obtain the stresses in a bridge or similar structure more accurately than to the nearest hundred pounds.

The student should guard against acquiring the habit of attempting to be unnecessarily exact. In this work, accuracy, using the term in a large sense, is absolutely indispensable, but painful and unnecessary exactness should be avoided. Results containing no error greater than one per cent. will be considered correct.

After the student becomes thoroughly familiar with the methods of determining stresses which have been explained in the preceding pages, he will be able to shorten the process considerably in nearly every case.

ANALYSIS OF STRESSES.

(CONTINUED.)

TRUSSES WITH INCLINED CHORDS.

1336. All the stress diagrams in the preceding articles refer to trusses having horizontal chords. But such trusses do not afford the greatest degree of economy for long spans.

An inclination of 45 degrees, or midway between horizontal and vertical, is the most economical for the inclined web members of a truss carrying vertical loads. While this statement applies to all the inclined web members, it, of course, applies to a greater degree to those members which bear greatest stress. Considerations of economy also require the height of a truss at the center to bear a certain ratio to the length of span; this ratio varies with different conditions, but is usually between one-fifth and one-eighth. In trusses of considerable length of span this center height becomes so great that the diagonal members can not be given economical inclinations without making the panels longer than is permissible for wood stringers or desirable for the lateral systems. In such cases, therefore, special expedients are resorted to in order to obtain, as nearly as possible, economical inclinations for the diagonal members, especially for those near the ends of the truss, in which the stresses are very great.

1337. In bridges of long span, some degree of economy of material may be obtained by properly inclining one of the chords. A somewhat greater economy is usually obtained by "curving" the chord, i. e., by making the inclination of the chord between joints near the ends of the truss greater than between those near the middle, giving an outline for the chord to some extent approximating a curve. The

For notice of copyright, see page immediately following the title page.

chord remains straight between joints. When in different portions of the chord the degree of inclination is thus varied, it is called **a curved chord.** When the degree of inclination of the chord is constant from the center to either end, it is called **an inclined chord.** Curved chords give not only better economy of material, but also a more graceful outline to the truss than can be obtained by chords having the same inclination throughout. In through bridges it is almost invariably the upper chord that is inclined or curved, although the end panels of the lower chord are sometimes inclined. In a curved or inclined chord the middle portion is usually horizontal; if the truss has an odd number of panels, the chord is always horizontal in the center panel.

The method of constructing the stress diagram for a bridge having curved or inclined chords is substantially the same as for a bridge having parallel chords, though the *form* of the diagram is somewhat different. An example of a through bridge having a curved upper chord will now be given.

NOTE.—All stress diagrams noticed in the following pages refer to *through* bridges, which are the bridges most commonly met with. When the principles are thoroughly understood with reference to through bridges, no difficulty will be experienced in applying them to deck bridges.

CURVED CHORDS.

1338. In Fig. 289 is represented a truss of 8 equal panels, having a span of 140 feet. The general design of the truss involves the principle of the Pratt truss; that is, in the web system the vertical members are compression members, and the diagonal members are tension members. It differs from the ordinary Pratt truss in that the vertical members are of different lengths, thus varying the height of the truss, as shown in the figure. The clear width of the roadway supported by the two trusses is 16 feet. The live load is assumed as 100 pounds per square foot of roadway.

1339. The Dead Load Stresses.—By formula **90,** Art. **1298,** the dead load per lineal foot, exclusive of the floor, is found to be $100 + 5 \times 16 + \frac{(16 - 6) \times 140}{8} + \frac{140^2}{300} =$ 420 lb. Assuming the floor to be white oak, from Table 29, Art. **1299,** the weight per lineal foot of floor for panel lengths of 18 feet is found to be $309.6 + 2 \times 45.6 = 400.8$ lb.

The total dead load per lineal foot of structure is $420 + 400.8 = 820.8$ lb., say 820 lb. As the length of each panel is $\frac{140}{8} = 17.5$ ft., the panel dead load is $\frac{820.8 \times 17.5}{2} = 7{,}182$ lb., or say 7,200 lb. For this load, by formula **89,** Art. **1293,** $R_1 = R_2 = \frac{7{,}200 \times (8 - 1)}{2} = 25{,}200$ lb.

In Fig. 289 is also shown a complete stress diagram for the dead load. By the aid of the notation it will be easily understood. The character of each stress will be readily distinguished. It will be noticed that the stresses in the web members are small, especially the stresses in the vertical posts. This is due to the fact that in each panel where the chord is inclined, the vertical component of the chord stress takes up the greater portion of the shear; thus the vertical component of *1-12*, the stress in *B C*, is nearly equal to *1-4*, or the shear in the panel *c d*, which is carried by the members *C D* and *C d* to the point *C*, and *12-13*, the amount of shear in this panel remaining to be carried by the post *C c*, is very small. It is especially noticeable that the vertical component of *1-14*, the stress in *C D*, is greater than *1-5*, or the shear in the panel *d e*; and, therefore, *14-15*, the stress in *D d*, is tension instead of compression. It is evident that the stress in *D d* will also be tension when the truss is *fully* loaded with the live load, as the conditions are then the same as are assumed for the dead load.

From the upper half of the stress diagram the following dead load stresses are obtained for the members in the left half of the truss:

Stress in *a b* = *2-10* = – 24,500 lb.
Stress in *b c* = *3-11* = – 24,500 lb.
Stress in *c d* = *4-13* = – 34,360 lb.

Stress in *d e* = *5-15* = − 39,380 lb.
Stress in *a B* = *1-10* = + 35,150 lb.
Stress in *B C* = *1-12* = + 35,250 lb.

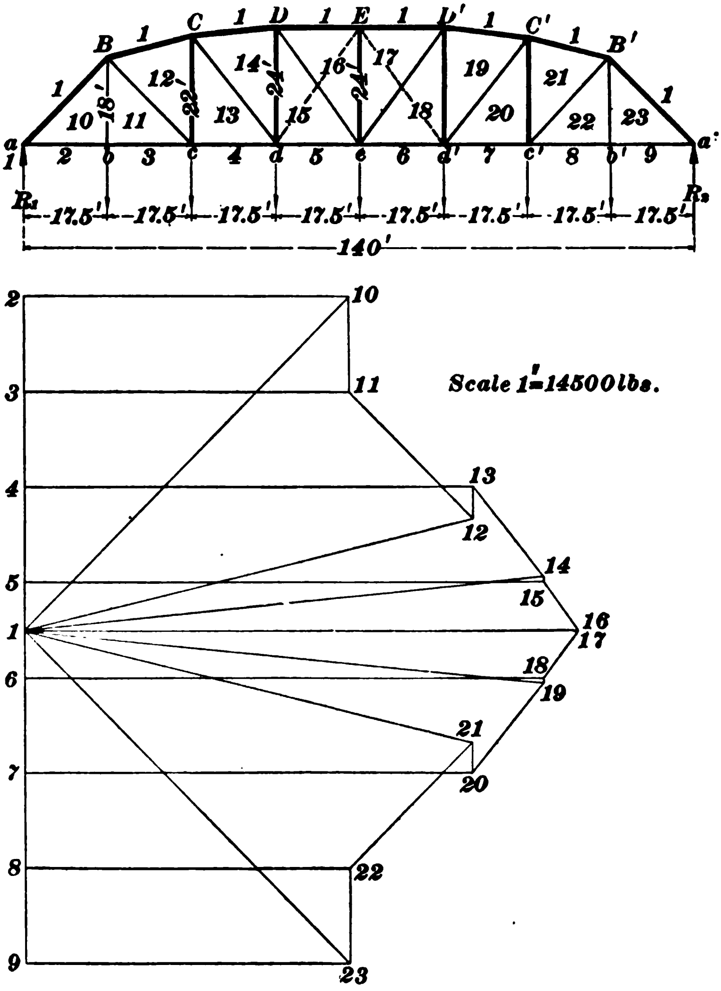


FIG. 289.

Stress in *C D* = *1-14* = + 39,630 lb.
Stress in *D E* = *1-16* = + 42,000 lb.
Stress in *B b* = *10-11* = − 7,200 lb.

Stress in $B\ c = 11\text{-}12 = -\ 14{,}150$ lb.
Stress in $C\ c = 12\text{-}13 = +\ 2{,}950$ lb.
Stress in $C\ d = 13\text{-}14 = -\ 8{,}050$ lb.
Stress in $D\ d = 14\text{-}15 = -\ 900$ lb.
Stress in $D\ e = 15\text{-}16 = -\ 4{,}450$ lb.
Stress in $E\ e = 16\text{-}17 = 0{,}000$ lb.

The above stresses for the vertical members are each to be corrected by adding algebraically one-third the panel load considered as a compressive stress, according to the rule given in Art. **1300.** In the present case, the correction to be added is $\frac{7{,}200}{3} = 2{,}400$ lb. Therefore, the corrected stresses in the vertical members are as follows:

Stress in $B\,b = -\ 7{,}200 + 2{,}400 = -\ 4{,}800$ lb.
Stress in $C\,c = +\ 2{,}950 + 2{,}400 = +\ 5{,}350$ lb.
Stress in $D\,d = -\ 900 + 2{,}400 = +\ 1{,}500$ lb.
Stress in $E\,e = 000 + 2{,}400 = +\ 2{,}400$ lb.

1340. The Live Load Stresses.—As previously stated, the live load assumed for the trusses is a uniform load of 100 pounds per square foot of roadway. The live load per lineal foot is, therefore, $100 \times 16 = 1{,}600$ lb., and the live panel load is $\frac{1{,}600 \times 17.5}{2} = 14{,}000$ lb. The live load stresses in the chords and end posts may be obtained by constructing a stress diagram in every respect similar to that explained for the dead load stresses, but they will be most easily obtained by observing that, since the condition of loading for the maximum live load chord stresses is the same as for the dead load stresses, the only difference being in the amount of the respective panel loads, *the stress in the two cases must be proportional to the corresponding loads* (either per panel or per lineal foot). Let S_d be the dead load stress in a member, and D the dead load per lineal foot (for which may be substituted the panel load); S_l the live load stress in the same member, when the whole truss is loaded, and L the live load per lineal foot (for which the panel load may be substituted). Then, $S_l : S_d ::$

$L : D$, and $S_l = S_d \times \frac{L}{D}$; that is, *the maximum live load chord stresses are found by multiplying the corresponding dead load stresses by the ratio of the live load to the dead load.*

In the example, the panel live load is 14,000 lb., and the panel dead load is 7,200 lb. Therefore, $\frac{L}{D} = \frac{14,000}{7,200} = \frac{35}{18}$.

NOTE.—It may not be out of place to state here that the calculating instrument commonly known as a slide rule, though not a necessity, will be found very convenient in the analyses of stresses and various other computations relating to bridge design.

Applying the preceding rule to our example, we get:

Stress in $a\ B = +\ 35,150 \times \frac{35}{18} = +\ 68,350$ lb.
Stress in $B\ C = +\ 35,250 \times$ " $= +\ 68,500$ lb.
Stress in $C\ D = +\ 39,630 \times$ " $= +\ 77,100$ lb.
Stress in $D E = +\ 42,000 \times$ " $= +\ 81,700$ lb.
Stress in $a\ b = -\ 24,500 \times$ " $= -\ 47,600$ lb.
Stress in $b\ c = -\ 24,500 \times$ " $= -\ 47,600$ lb.
Stress in $c\ d = -\ 34,360 \times$ " $= -\ 66,800$ lb.
Stress in $d\ e = -\ 39,380 \times$ " $= -\ 76,600$ lb.
Stress in $D d = -\ 900 \times$ " $= -\ 1,750$ lb.

It is evident that the live load stress in the hip vertical $B b$ is tension equal to the panel load at b supported by it, or $-$ 14,000 lb.

1341. The maximum live load stresses in the members of the web system remain to be found. As these maximum stresses obtain for the different members under varying conditions of load, the operations for finding them, as explained in Art. **1295,** involve the construction of several stress diagrams, one for each condition of load. A method of obtaining the maximum live load web stresses by means of a single figure, in which the necessary portions of the several stress diagrams are combined, together with a special method of shortening the operations, will now be explained.

In Fig. 290 is a diagram of the truss in which are shown those web members only which transfer their load to the left reaction. It will always be found expedient to designate the external forces and, so far as possible, the members of the truss by numerals corresponding to those that were used for the same purposes when obtaining the dead load stresses. Below the diagram of the truss are the combined stress diagrams for the varying live load, which are constructed as follows:

Upon the load line are laid off in order the *left* reaction from a full live load and that portion of each separate panel load that is supported by it; taking the external forces in order, passing around the truss to the left, but omitting all portions of the live load relating to the *right* reaction, as explained in Arts. **1327** to **1329.** The portion of the panel load (14,000 lb.) at each joint, which is supported by the left reaction, is as follows:

Load at *b* or *2-3* = $\frac{7}{8}$ × 14,000 = 12,250 lb.
Load at *c* or *3-4* = $\frac{6}{8}$ × 14,000 = 10,500 lb.
Load at *d* or *4-5* = $\frac{5}{8}$ × 14,000 = 8,750 lb.
Load at *e* or *5-6* = $\frac{4}{8}$ × 14,000 = 7,000 lb.
Load at *d'* or *6-7* = $\frac{3}{8}$ × 14,000 = 5,250 lb.
Load at *c'* or *7-8* = $\frac{2}{8}$ × 14,000 = 3,500 lb.
Load at *b'* or *8-1* = $\frac{1}{8}$ × 14,000 = 1,750 lb.

Total reaction R_1 from full load = 49,000 lb.

Upon the load line, *1-2* is laid off upwards equal to R_1, then *2-3*, *3-4*, *4-5*, etc., each equal to that portion of each respective panel load supported by the left reaction, are laid off in order downwards to the starting point. Also, from *1* are laid off *upwards* the amounts 5_n-6_n and 6_n-7_n, equal, respectively, to the half panel load of dead load at *e* (*5-6*) and to the panel load of dead load at *d'* (*6-7*) which are supported by the *right* reaction. This is conveniently obtained from the load line in the stress diagram for the dead load, Fig. 289. In Fig. 290 the dead load supported by the right reaction is laid off *upwards* along the load line in precisely the same manner that it extends *downwards* upon the load line in Fig. 289, except that it is laid off to the scale used for the live load, and that only such portion of the dead load need be thus laid off as will bring some numeral for the dead load, as 7_n, above the corresponding numeral for the live load, as *7*. It will be expedient to draw through *1* the lines *1*-12_0, *1*-14_0, and *1*-16_0, of indefinite length, and

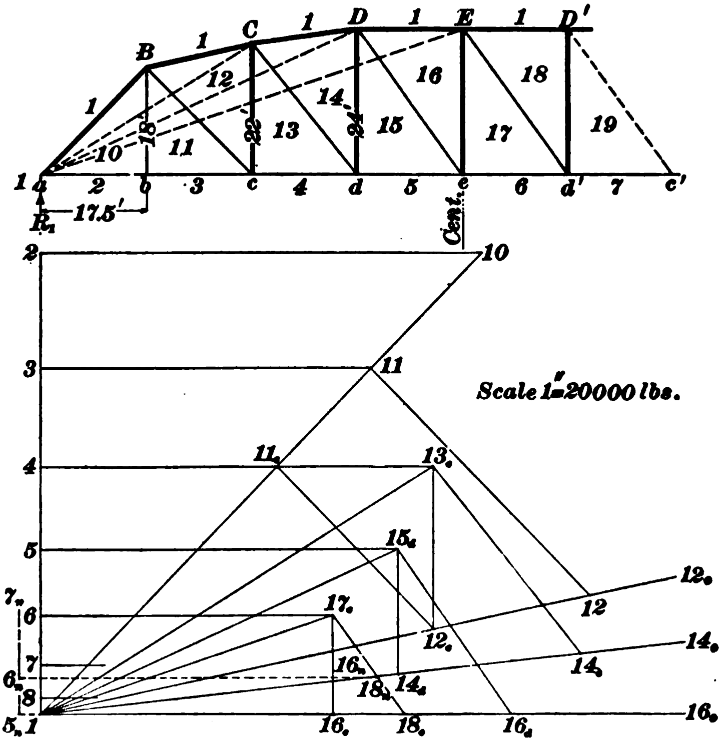


FIG. 290.

parallel, respectively, to the upper chord members *B C*, *C D*, and *D E*.

On the load line, *1-2* represents the reaction from a full load, which condition gives the maximum stresses in the chords and end post; *1-2-10-1* is the force polygon for joint *a*, giving the maximum stresses in *a B* and *a b*. As these stresses have been obtained above from the dead load stresses, this force polygon serves merely to check the accuracy of the work.

The stress in *B c* obtains its maximum value with joint *b* unloaded, all other joints remaining loaded. Since in this condition no external force is applied at joint *b*, the external force R_1 applied at *a* is designated by *1-3*; also, as no internal force is acting in *B b*, the entire space between *a B* and *B c* is designated by the one numeral *11*.

(*a*) *When two or more original spaces are combined as one space, on account of the intervening force or forces having been removed, consider the entire space thus combined to be designated by the numeral of the original space furthest at the right.*

On the load line, *1-3* is the reaction from this load and *1-3-11-1* is the force polygon for joint *a*; for joint *B* *1-11-12-1* is the force polygon, giving the maximum stress in *B c*.

For the maximum stresses in *C d* and *C c*, the joints *b* and *c* must carry no load, and all other joints must be loaded. For this load, *1-4* represents the reaction and $1\text{-}4\text{-}11_c\text{-}1$ is the force polygon for joint *a*. For joint *B*, $1\text{-}11_c\text{-}12_c\text{-}1$ is the force polygon; $12_c\text{-}11_c\text{-}4\text{-}13_c\text{-}12_c$ is the polygon for joint *c*; and $1\text{-}12_c\text{-}13_c\text{-}14_c\text{-}1$ is the polygon for joint *C*, in which $12_c\text{-}13_c$ and $13_c\text{-}14_c$ represent the maximum stresses in *C c* and *C d*, respectively. The operation of finding the maximum stresses in these members may be somewhat shortened by a special method based upon the following principle:

(*b*) *If, with no loads at b and c, all portions of the truss at the left of the member C d were considered to be removed and replaced by the member a C, represented by a dotted line, and the chord a d, the stress in C d and in each member of the truss at the right of C d will remain unchanged.*

This is a general principle, and may be stated as follows:

(*c*) *If, while the entire loaded portion of any truss remains unchanged, all that portion of the truss, situated between the reaction and the nearest load, be considered removed and replaced by two members which extend, respectively, from the reaction directly to the nearest upper and* *lower chord joints in the unchanged portion of the truss, then the stress in each member in the latter portion will remain the same as though the entire truss were unchanged.*

In the diagram of the truss, as thus modified, the entire space at the left of *C d* will be represented by the numeral *13*; that is, by the numeral which originally represented the first space at the left of that member. On the load line, *1-4* represents the reaction, and $1\text{-}4\text{-}13_c\text{-}1$ is the force polygon for joint *a*, the lines $4\text{-}13_c$ and $13_c\text{-}1$ being drawn parallel to the new members *a d* and *a C*, respectively. The force polygon $1\text{-}13_c\text{-}14_c\text{-}1$ may now be drawn for joint *C*, giving the line $13_c\text{-}14_c$ to represent the maximum stress in *C d*. But as, in drawing this polygon, *C c* was assumed to have been removed, the polygon does not give the stress for this member. To obtain this stress, consider the members of the truss at the left of *C d* to be restored to their original positions; then, by passing in the *opposite* direction around the joint *C*, the polygon $1\text{-}14_c\text{-}13_c\text{-}12_c\text{-}1$ may now be drawn, in which $12_c\text{-}13_c$ represents the maximum stress in *C c*.

For the maximum stress in *D e* and *D d*, the joints *b*, *c*, and *d* must carry no load, and all other joints, i. e., all joints to the right of the panel *d e*, must be loaded. All members of the truss at the left of *D e* are considered to be removed, and the members *a D* and *a e* substituted; the entire space at the left of *D e* is represented by *15*, the numeral furthest to the right in this space.

The reaction from this load is represented on the load line by *1-5*. For joint *a* of the truss as modified, $1\text{-}5\text{-}15_d\text{-}1$ is the force polygon; the force polygon for joint *D* is $1\text{-}15_d\text{-}16_d\text{-}1$, giving $15_d\text{-}16_d$ as the maximum stress in *D e*. The members of the truss at the left of *D e* are now considered to be restored to their original positions, and in this condition the reversed polygon for joint *D* is $1\text{-}16_d\text{-}15_d\text{-}14_d\text{-}1$, in which $14_d\text{-}15_d$ represents the maximum stress in *D d*

For the maximum stresses in *E d'* and *E e*, all joints to the left of the panel *e d'* must be unloaded, and all joints at the right of it must be loaded. To obtain the stress produced in *E d'* by this load, the members *a E* and *a d'* are considered to be substituted for all portions of the truss at the left of *E d'*. The entire space at the left of *E d'* is now represented by the numeral *17*, and, since no external forces are applied between *a* and *d'*, the left reaction is represented by *1-6*. On the load line *1-6* represents the reaction for this load; for the truss as modified, $1\text{-}6\text{-}17_e\text{-}1$ is the force polygon for joint ***a***. Considering the members of the truss at the left of *E d'* to be restored to their original positions, the force polygon for joint *E*, found by taking the forces in the reverse order around the point, is $1\text{-}18_e\text{-}17_e\text{-}16_e\text{-}1$.

The lines $16_e\text{-}17_e$ and $17_e\text{-}18_e$ would represent the maximum live load stresses in *E e* and *E d'*, respectively, were it not that the panel *e d'* is situated at the *right* of the center of the truss, and, therefore, the *negative dead load shear* in this panel counteracts a portion of what would otherwise be the stresses in these members produced by the positive live load shear. One-half the panel of dead load at *e* is supported by the right reaction; in traveling towards the reaction this half-panel load produces negative shear in the panel *e d'*. In the stress diagram this half panel of negative shear is represented by $5_n\text{-}6_n$. Therefore, through 6_n draw the horizontal line $6_n\text{-}18_n$; then, $16_n\text{-}17_e$ and $17_e\text{-}18_n$ will represent the resultant maximum live load stresses in *E e* and *E d'*, respectively.

For the maximum live load stress in *D' c'*, the joint *d'*, as well as all joints to the left of it, is unloaded, while all joints to the right of it remain loaded. For this load, the left reaction is represented on the load line by *1-7*, which also represents the positive shear in the panel *d' c'*, as well as in all panels at the left of it. But the negative shear in the panel *d' c'* is equal to one-half the panel load of dead load at *e* plus the panel load at *d'*, represented on the load line by $5_n\text{-}6_n$ and $6_n\text{-}7_n$, respectively, the total amount being represented by $5_n\text{-}7_n$. As $5_n\text{-}7_n$ is greater than *1-7*, no positive shear can occur in this panel, and hence no stress can be obtained for a counter *D' c'*. In the figure the dead load numeral 7_n is situated at a point upon the load line above the live load numeral *7*; therefore, no positive live load shear is found in the panel *7*, or *d' c'*. But the dead load numeral 6_n is below the live load numeral *6*, and positive live load shear is found to exist in the panel *6*, or *e d'*.

The above method should be employed for determining the counter stresses in all panels in which both chords are horizontal, as in the panel *e d'*. It is not to be relied upon when this is not the case.

By measuring the lines of the stress diagram to the scale used in laying off the load line, the following maximum stresses are obtained:

Stress in $a\ b$ = 2-10 = − 47,640 lb.
Stress in $a\ B$ = 1-10 = + 68,350 lb.
Stress in $B\ c$ = 11-12 = − 32,620 lb.
Stress in $C\ c$ = 12_c-13_c = + 16,700 lb.
Stress in $C\ d$ = 13_c-14_c = − 25,160 lb.
Stress in $D\ d$ = 14_d-15_d = + 13,130 lb.
Stress in $D\ e$ = 15_d-16_d = − 21,660 lb.
Stress in $E\ e$ = 16_n-17_e = + 6,900 lb.
Stress in $E\ d'$ = 17_e-18_n = − 8,540 lb.

1342. The Lateral Stresses.—The dimensions of the chords are necessarily greater in bridges of long span than in bridges of short span. For short spans, the width of a chord may, for the purpose of calculation, be taken equal to 1 foot. *For spans longer than 120 feet, consider the width of the chord in inches to be* $\frac{1}{10}$ *the length of the span in feet.* If l is the length of the span in feet, then, width of chord $= \frac{l}{10}$ inches $= \frac{l}{120}$ feet.

The width, center to center of chords, equals the clear width of roadway plus the width of one chord.

Let $C =$ width center to center of chords, $b =$ clear width of roadway, $l =$ length of span. Then,

$$C = b + \frac{l}{120}. \qquad \textbf{(101.)}$$

1343. For lateral systems of bridges having curved or inclined chords, the panel concentrations of wind load are found in the same manner as for horizontal chords, since the amount of wind force *per lineal foot of structure* is the load usually (though not always) specified. But in drawing the diagram of the truss of the lateral system extending along the curved or inclined chord, from which to construct the stress diagram, the length of each respective panel should be made equal to the correct length of the panel *measured along the inclined chord.*

Thus, in the truss shown in Figs. 289 and 290 the panel length, measured horizontally, is 17.5 feet; but for the upper lateral system the lengths of the panels *B C*, *C D*, and *D E* are, respectively, 17.95, 17.61 and 17.5 feet. For this purpose it is sufficiently accurate to measure their distances on the diagram of the truss.

1344. In a system of lateral bracing of the Pratt truss type the live load stress for the lateral strut at the center, as given by the stress diagram, will sometimes be less than a panel load; in such cases the live load stress should be taken equal to one panel live load, as it is evident that with the adjacent panels loaded, the stress in this strut will equal that amount. This statement applies also to the vertical post at the center of a Pratt truss deck bridge, in which *the maximum live load stress is never less than a panel load.*

EXAMPLES FOR PRACTICE.

1. Using the data given in this article, construct the stress diagram shown in Fig. 289, obtaining to the nearest hundred pounds the dead load stresses which have been given for the same.

2. Construct the stress diagram, obtaining the live load stresses in the chords and end posts.

3. Check the results obtained in the solution of the preceding example by multiplying the corresponding dead load stresses by the ratio $\frac{L}{D}$.

4. Construct the combined diagrams, obtaining to the nearest hundred pounds the maximum live load web stresses given above.

FORMULAS FOR CHORD STRESSES.

1345. The stress in any horizontal portion of either chord in a truss of the Pratt or Howe type, when fully loaded with a uniform load, may be obtained or verified by applying the following formula:

$$C_h = \frac{P p m m'}{2 h} = \frac{P p}{2 h} \times m m', \qquad \textbf{(102.)}$$

in which P is the panel load, p is the panel length, h is the height of the truss at a vertical member, m is the number of panels at the left, and m' the number of panels at the right of the vertical member, and C_h is the stress in that portion of either chord situated between the same two diagonal members between which is situated the vertical member at which h is taken. The formula, of course, is to be applied *to one-half of the truss only;* the stresses in the other half are the same, respectively, as those found for the first half.

This is a very convenient formula, and its use is illustrated in the following

EXAMPLE.—Find the dead and live load chord stresses for a Pratt truss of 8 panels, in which panel length = 20 ft.; height of truss = 24 ft.; panel live load = 11,200 lb., and panel dead load = 7,400 lb. Let the portions of the lower chord be $a b$, $b c$, $c d$, $d e$, $e d'$, $d' c'$, $c' b'$, $b' a'$.

SOLUTION.—For dead load, we have,

$$\frac{P p}{2 h} = \frac{7{,}400 \times 20}{2 \times 24} = 3{,}083, \text{ nearly.}$$

The values of $m m'$ for the different panels are:

For $b c$, $m m' = 7 \times 1 = 7$;
For $c d$, $m m' = 6 \times 2 = 12 = 7 + 5$;
For $d e$, $m m' = 5 \times 3 = 15 = 12 + 3$.

Then, we have,

$$\begin{array}{lrl}
 & 7 \times 3{,}083 = & -21{,}581 \text{ lb.} = \text{stress in } b c. \\
\text{Add} & 5 \times 3{,}083 = & \underline{15{,}415} \\
 & & -36{,}996 \text{ lb.} = \text{stress in } c d. \\
\text{Add} & 3 \times 3{,}083 = & \underline{9{,}249} \\
 & & -46{,}245 \text{ lb.} = \text{stress in } d e.
\end{array}$$

It will be noticed that, since, for instance, 12 = 7 + 5, in order to multiply 3,083 by 12, we have to multiply by 5 only, and add the product to 7 × 3,083, already found. The same with other products.

For live load,

$$\frac{Pp}{2h} = \frac{11,200 \times 20}{2 \times 24} = 4,667, \text{ nearly.}$$

Then, as before,

$$\begin{array}{lrl} & 7 \times 4,667 = & -\ 32,669 \text{ lb.} = \text{stress in } ab \text{ and } bc. \\ \text{Add} & 5 \times 4,667 = & \underline{23,335} \\ & & -\ 56,004 \text{ lb.} = \text{stress in } cd. \\ \text{Add} & 3 \times 4,667 = & \underline{14,001} \\ & & -\ 70,005 \text{ lb.} = \text{stress in } de. \end{array}$$

Stresses in other half are the same. For upper chord, *the stress in every portion of it is the same in amount as, but of opposite kind to, the stress in the portion of the lower chord included between the same main diagonals.* Thus, stress in BC (first portion of upper chord) = *minus* stress in cd = + 56,004.

1346. If the portion of either chord to which the formula is applied is *inclined,* then C_h will represent the *horizontal component* of the chord stress, and the latter may be obtained by applying the following formula:

$$C = \frac{C_h k}{p} = C_h \times \frac{k}{p}, \qquad \mathbf{(103.)}$$

in which k is the panel length measured along the inclined chord (i. e., the length of the inclined member in question), C is the stress in the same, and p is the panel length, as before.

Chord stresses may readily be obtained by applying the preceding formulas **102** and **103.** But the dead load chord stresses and web stresses are both obtained by constructing the stress diagram; and if the live load chord stresses are obtained by multiplying the corresponding dead load stresses by the ratio $\frac{L}{D}$, the entire operation may be checked by applying the formulas to verify the results.

EXAMPLE.—Verify the dead load stress obtained graphically for the lower chord member de of the truss shown in Fig. 289, using the data given therefor in Art. **1338.**

SOLUTION.—In this case, $P = 7,200$, $p = 17.5$, and $h = 24$; as Dd is the vertical member between the diagonals Cd and De, between which de is situated, $m = 3$ and $m' = 5$.

$$\text{Therefore, } C_h = \frac{7,200 \times 17.5 \times 3 \times 5}{2 \times 24} = 39,375 \text{ lb.}$$

Example.—Verify the dead load stress obtained graphically for the upper chord member $C\,D$ of the same truss.

Solution.—As $C\,D$ and $d\,e$ lie between the same two diagonals, the amount of stress found above for $d\,e$ is also the horizontal component of the stress in $C\,D$. The length of $C\,D = \sqrt{17.5^2 + 2^2} = 17.614$ ft. $= k$, and $p = 17.5$, as before. Therefore, $C = \frac{39{,}375 \times 17{,}614}{17.5} = 39{,}631$ lb.

EXAMPLES FOR PRACTICE.

1. Verify all the dead load chord stresses for the truss shown in Fig. 289, Art. **1338**, using the data given therefor in that article.

2. Verify all the live load chord stresses for the same.

INCLINED CHORDS.

1347. As previously stated, trusses with inclined chords are usually less economical, and have not as graceful outlines

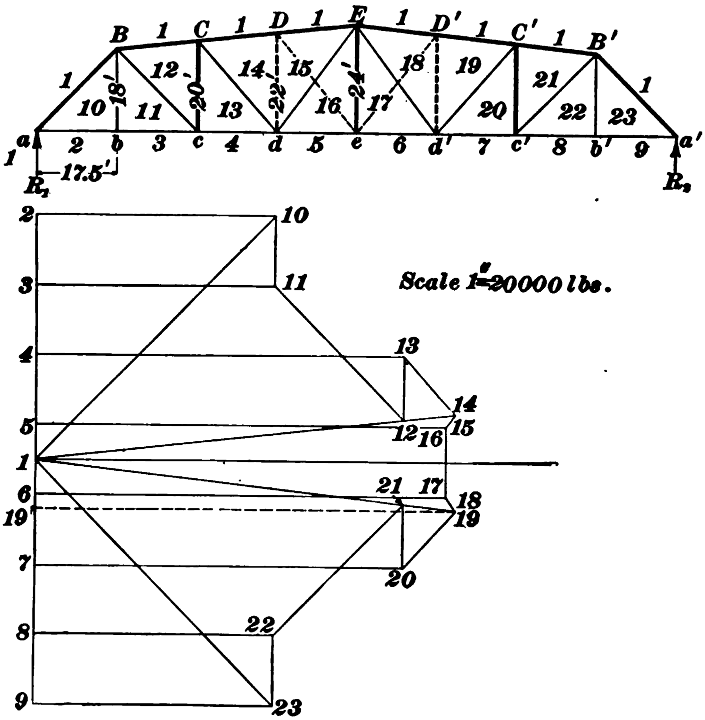


Fig. 291.

as trusses with curved chords. Nevertheless, bridge trusses having inclined chords are not uncommon.

When the inclination of the chord is considerable, the counter stresses, and even the dead load web stresses near the center, usually become quite irregular, and sometimes rather complicated. In trusses of the Pratt type, however, they do not become ambiguous. An example illustrating this feature will now be given.

1348. The length of span, number of panels, and the amount of both live and dead loads will be assumed to be the same as in the example for curved chords, explained in Art. **1338.**

The general form of the truss will also be the same, except that while the heights of the truss at the hips and at the center will be, respectively, 18 and 24 feet, as in the former example, the upper chord will be straight between those points, as represented in Fig. 291.

The Dead Load Stresses.—The stress diagram for the dead load, assuming the panel loads to be concentrated at the joints of the lower chord, is also shown in Fig. 291. Some peculiarities in this diagram will be noticed. The force polygons for joints *a*, *b*, *B*, *c*, and *C* are each constructed in the usual manner, and will be readily understood. For joint *d*, the lines *14-13* and *13-4*, parallel, respectively, to the members *d C* and *d c*, are retraced, the pencil is passed down the load line the amount *4-5*, equal to the panel load at *d*, and from *5* it must be returned to the starting point *14* by lines drawn parallel, respectively, to the two remaining forces which act upon this joint, of which one is stress in the chord member *d e*, and the other must be stress in either the compression member *D d* or in the tension member *d E*. For, by drawing through the point *5* a line of indefinite length parallel to *d e*, as the line *5-16*, it can be readily discerned that the polygon could be closed by a single line drawn from some point on the line *5-16* parallel to either of these members, showing that it is necessary for but one of them to act; it can also be seen that, as the line *5-16* lies below the starting point *14*, the general direction of the closing line must be upwards, showing that the corresponding force or stress acts in an upward direction upon joint *d*. It is evident that a stress in either *D d* or *d E*, acting in an upward direction upon this joint, must necessarily be tension, and must act in a correspondingly downward direction upon the joint at the opposite end of the member. It is evident that this downward action can not be resisted at *D* without causing bending moment in the upper chord between *C* and *E*, which is not permissible; it is also evident that this downward force, if transferred by the member *d E* to joint *E*, can be resisted at that point by the vertical component of the stress in the upper chord. As it is then found that the forces acting upon joint *d* can be held in equilibrium only through tensile stress acting in the diagonal *d E*, therefore this member is known to act in this condition of load (i. e., when the truss is loaded with the dead load only); consequently, a line must be drawn parallel to it in the polygon for joint *d*. The line *16-15*, in this polygon, can be so drawn that the point *15* will coincide with the starting point *14*; the line *15-14*, which represents the stress in *D d*, has, therefore, no length (i. e., there is no line *15-14*), and the stress in *D d* is zero. As but one diagonal in a panel can act in the same condition of load, the diagonal *D e* does not act when the truss is loaded with the dead load only, as will also be determined by the force polygon for joint *e*. Considering the forces which act upon this joint, the line *16-5* is retraced, the pencil is passed downwards upon the load line the amount *5-6*, equal to the panel load at *e*, and it is found that from *6* the pencil can be returned to the starting point *16* by the lines *6-17* and *17-16*, drawn parallel, respectively, to the members *e d'* and *e E*, giving no stress for the members *D e* and *e D'*

In this polygon the line *6-17*, representing the stress in *e d'*, is found to equal *16-5*, which represents the stress in *d e*, as should be the case at the center of a symmetrical truss having an even number of panels. Also the line *17-16*, which represents the stress in *E e*, is found to equal *5-6*, or the panel load at *e*, which evidently should be the case when there is no stress in the members *D e* and *e D'*. Considering next the forces which act upon joint *E*, the lines *1-15*, *15-16*, and *16-17* are retraced, and it is found that from *17* the pencil can be returned to the starting point *1* by lines *17-18* and *18-1*, drawn parallel, respectively, to *E d'* and *E D'*; this gives *17-18*, the stress in *E d'*, which is equal to *15-16*, the stress in *E d*, and also gives *18-1*, the stress in *E D'*, which is equal to *1-15*, the stress in *E D*, as should be the case.

The polygons for the joints in the right half of the truss are the same as for those in the left half; they will require no special explanation. In the diagram of the truss those members which do not act when the truss is fully loaded are represented by dotted lines.

By measuring the lines of the stress diagram, the following stresses are obtained:

Stress in *a B* = *1-10* = + 35,150 lb.
Stress in *B C* = *1-12* = + 38,040 lb.
Stress in *C D* = *1-14* = + 43,230 lb.
Stress in *D E* = *1-15* = + 43,230 lb.
Stress in *a b* = *2-10* = – 24,500 lb.
Stress in *b c* = *3-11* = – 24,500 lb.
Stress in *c d* = *4-13* = – 37,800 lb.
Stress in *d e* = *5-16* = – 42,000 lb.
Stress in *B b* = *10-11* = – 7,200 lb.
Stress in *B c* = *11-12* = – 9,080 lb.
Stress in *C c* = *12-13* = + 6,480 lb.
Stress in *C d* = *13-14* = – 7,830 lb.
Stress in *D d* = *14-15* = + 0,000 lb.
Stress in *d E* = *15-16* = – 1,620 lb.
Stress in *E e* = *16-17* = – 7,200 lb.

The above stresses for the vertical members must be corrected by adding to each, algebraically, one-third of a panel load, or 2,400 lb. Therefore,

Stress in $B\,b = -7{,}200 + 2{,}400 = -4{,}800$ lb.
Stress in $C\,c = +6{,}480 + 2{,}400 = +8{,}880$ lb.
Stress in $D\,d = 0{,}000 + 2{,}400 = +2{,}400$ lb.
Stress in $E\,e = -7{,}200 + 2{,}400 = -4{,}800$ lb.

1349. The Live Load Stresses.—The maximum live load stresses in the chords and end posts may be obtained either by multiplying the corresponding dead load stresses by the ratio $\frac{L}{D}$, as previously explained (Art. **1340**), or by constructing a stress diagram similar to that in Fig. 291. Neither method will require special explanation. For the same condition of load and in the same operation the stresses in $d\,E$ and $E\,e$ must be obtained. The stress thus obtained for $E\,e$ will be the maximum live load *tension* in that member, while the stress thus obtained for the tension member $d\,E$ may or may not be the maximum tensile stress in that member; it may or may not be exceeded by the stress due to a partial load. The latter stress will be found for the corresponding web member $E\,d'$; the greater of the two stresses must be taken as the maximum live load stress for each member $d\,E$ and $E\,d'$.

The live load stresses in the chords, end posts, and in the two web members mentioned above, are as follows:

Stress in $a\,B = +35{,}150 \times \frac{35}{18} = +68{,}350$ lb.
Stress in $B\,C = +38{,}140 \times$ " $= +73{,}960$ lb.
Stress in $C\,D = +43{,}230 \times$ " $= +84{,}050$ lb.
Stress in $D\,E = +43{,}230 \times$ " $= +84{,}050$ lb.
Stress in $a\,b = -24{,}500 \times$ " $= -47{,}640$ lb.
Stress in $b\,c = -24{,}500 \times$ " $= -47{,}640$ lb.
Stress in $c\,d = -37{,}800 \times$ " $= -73{,}500$ lb.
Stress in $d\,e = -42{,}000 \times$ " $= -81{,}670$ lb.
Stress in $d\,E = -1{,}620 \times$ " $= -3{,}150$ lb.
Stress in $E\,e = -7{,}200 \times$ " $= -14{,}000$ lb.

In applying formulas **102** and **103** for verifying the chord stresses, it must be noticed that with the truss fully loaded the vertical member $D d$ is situated between the diagonals $C d$ and $E d$, and, therefore, formula **102** applied at $D d$ will give the horizontal component of the stress in the chord $C E$ ($C D$ and $D E$). Also, that the vertical member $E e$ is situated between the diagonals $E d$ and $E d'$, and, therefore, formula **102** applied at $E e$ will give the stress in the chord $d e$ and $e d'$, or $d d'$.

1350. For the web stresses which obtain from partial loads, the combined stress diagrams are shown in Fig. 292. The respective portions of the panel loads which are supported by the left reaction are laid off upon the load line in precisely the same manner as was explained in Art. **1341** for the load line in Fig. 290, but the dead loads are not laid off upwards from the bottom of the load line, as in that example.

Beginning with the polygon $1\text{-}2\text{-}10\text{-}1$, the force polygons for the several joints and for the various conditions of loading are drawn in the same manner as explained or Fig. 290, as far as and including the reversed polygon $1\text{-}16_d\text{-}15_d\text{-}14_d\text{-}1$ for joint D. For joint E, the polygon $1\text{-}17_e\text{-}18_e\text{-}1$ and the reversed polygon $1\text{-}18_e\text{-}17_e\text{-}16_e\text{-}1$ are also drawn in the same manner, but from the stresses thus obtained *no deduction is made for the negative shear from dead load*, as was done in the previous case. No negative dead load shear is deducted from the positive live load shear in the panel $e d'$, which produces stress in $E d'$ and $E e$, because, by consulting the stress diagram for the dead load (Fig. 291), it is found that the entire panel load of dead load at e, by which the negative shear in this panel is produced, is carried by the member $E e$ to the upper chord joint E, and is there taken by the vertical component of the chord stress; consequently, the negative dead load shear in the panel $e d'$ does not in any way affect the diagonal members $E d'$ and $e D'$ in this panel.

The maximum positive shear in the panel $d' c'$, due to the live load, occurs when joints d', c', and b' are loaded, and all joints to the left are unloaded. The left reaction from this condition of loading is represented on the load line by *1-7*.

This being known, the next step is to ascertain to what extent the negative shear in the panel *d' c'* affects the diagonals in that panel, in order to determine whether any resultant positive shear can obtain in the panel. One-half the panel load of dead load at *c*, together with the entire panel load of dead load at *d'*, produce negative shear in this panel (*d' c'*); but, by again referring to the stress diagram for the dead load, it can be seen that not only the entire panel load

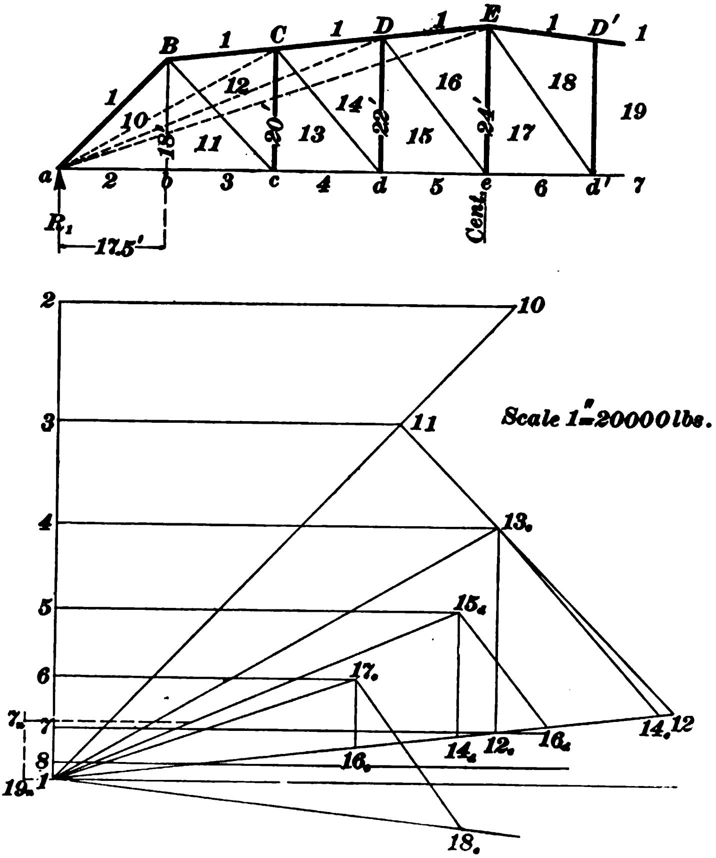


FIG. 291.

of dead load at *e*, but also a portion of the dead load at *d'*, is carried to joint *E*, and there taken by the vertical component of the chord stress. Therefore, the dead load at *e*, and as much of the dead load at *d'* as is carried by the diagonal *d' E* to the joint *E*, being supported wholly by the chord, do not affect the diagonals in the panel *d' c'*. But that portion of the dead load at *d'* which is carried by the diagonal *d' C'* (not shown in figure) is the only portion of the dead load which affects the diagonals in this panel; it may produce the amount of dead load negative shear resisted by *d' C'*, or it may counteract a corresponding amount of positive live load shear. This amount of negative shear which affects the diagonals in the panel *d' c'* is equal to the vertical component of the line *19-20* in the stress diagram for the dead load, Fig. 291. If in this stress diagram the point *19* be projected upon the load line by the horizontal line *19-19'*, then *1-19'* will represent the amount of negative shear in the panel *d' c'* carried by the chord *D' C'*, and *19'-7* will represent the amount of negative shear which affects the diagonals in that panel. By laying off upwards from *1* on the load line of the stress diagram for the live load (Fig. 292), the amount 19_n-7_n, equal to the amount *19'-7* obtained from the load line of the stress diagram for the dead load, making due allowance for the difference of scales, if any, a comparison is obtained between the positive live load shear and the negative dead load shear *affecting the diagonals* in the panel *d' c'*. The amount 19_n-7_n = 5,890 lb. is found to be greater than the live load reaction *1-7* = 5,250 lb., and, therefore, no positive live load shear can obtain in the panel *d' c'*; or, in other words, as the dead load numeral 7_n is situated at a point upon the load line above the live load numeral *7*, no positive live load shear can obtain in the panel under which the corresponding numeral is situated. Since, however, the point designated by 7_n is so near to the point designated by *7*, or, in other words, as 5,890 lb. is so little in excess of 5,250 lb., it will be best to place a light counter tie *D' c'* in this panel, although no stress is obtained for it.

By measuring the lines of the combined diagrams thus constructed, the following web stresses are obtained for the respective conditions of partial live load:

Stress in $a\,B = 1\text{-}10 = +68{,}350$ lb.
Stress in $B\,c = 11\text{-}12 = -41{,}000$ lb.
Stress in $C\,c = 12_c\text{-}13_c = +21{,}000$ lb.
Stress in $C\,d = 13_c\text{-}14_c = -25{,}370$ lb.
Stress in $D\,d = 14_d\text{-}15_d = +12{,}730$ lb.
Stress in $D\,e = 15_d\text{-}16_d = -14{,}910$ lb.
Stress in $E\,e = 16_e\text{-}17_e = +7{,}000$ lb.
Stress in $E\,d' = 17_e\text{-}18_e = -18{,}900$ lb.

By comparing the stress (18,900 lb.) in *E d'* as found above for a partial load, with the stress (3,150 lb.) as previously found for the corresponding web member *d E*, when the truss was considered to be fully loaded, the former stress (18,900 lb.) is found to be the maximum stress for each of these members.

As the dead load is always present upon the truss, the *corrected* dead load tension in *E e* (– 4,800 lb.) must be deducted from (added algebraically to) the live load compression (+ 7,000 lb.) obtained for it above, in order to obtain the net or resultant live load stress in that member. The correct maximum compression obtained for *E e* is, therefore, $+7{,}000 - 4{,}800 = 2{,}200$ lb.

1351. In trusses with inclined chords the stresses in the web members near the center of the truss, developed by a full load, are not always of the same nature as in the example which has just been illustrated. For instance, if in the truss of that example the inclination of the upper chord were less, a full (live or dead) load would produce stress in the diagonals *D e* and *e D'* instead of in *d E* and *E d'*. Or the inclination of the chord could be such that neither the dead load nor a full live load upon the truss would produce stress in any diagonal in the two center panels. In a truss with curved chords, the chord could have such inclinations that no web stresses would be produced by a full quiescent load.

A careful comparison of the stress diagrams, and of the stresses obtained therefrom, for these different forms of trusses having the same span, the same loads, and nearly the same form, the stress diagrams for two of which have been explained in detail, will be found very instructive. From such comparisons may be obtained practical and valuable ideas concerning the principles of economic design.

EXAMPLES FOR PRACTICE.

1. Construct the stress diagram for the truss having inclined chords, when carrying a full live load, obtaining the live load stresses in the chords and posts as previously given.

2. Construct the combined stress diagrams for the partial live loads, obtaining the maximum web stresses as previously given.

1352. The truss shown in Figs. 291 and 292 is a **single intersection truss with inclined chords**; it is commonly known as a **camel-back truss.** The latter name is also applied to the form of truss shown in Figs. 289 and 290; but this truss is more properly a **single intersection truss with curved chords.** These two forms of trusses are also known as the **Pratt truss with inclined chords** and the **Pratt truss with curved chords,** respectively. By the latter names these trusses are distinguished from all other forms of trusses, although, correctly speaking, neither truss has the exact form of a Pratt truss. But, by having vertical compression members and diagonal tension members extending horizontally through but one panel, both trusses embody the prominent features of the Pratt truss.

THE WHIPPLE TRUSS.

1353. As previously stated, an angle of 45 degrees with the horizontal chord is the most economical inclination for the diagonals of a truss. In a Pratt truss, therefore, considerations of economy require the panel length to be approximately equal to the height of the truss. As the height of the truss increases with the length of the span (Art. **1336**), it follows that in long spans the diagonal members of a Pratt truss can not be given an economical inclination without making the panels longer than would be desirable. In long (and consequently high) trusses, the diagonals may be given an economical inclination without requiring excessively long panels, by combining two systems of Pratt truss bracing, forming what is known as a **Whipple double intersection**, or **double quadrangular truss.**

A truss of this type is shown in the upper portion of Fig. 293. In this truss the panel length is equal to one-half the height; the diagonals have an inclination of 45 degrees. One system of web bracing is shown by dotted lines, and one system is shown by full lines. An analysis of the stresses may be obtained by separating the two systems. In (*a*), Fig. 293, is shown the system of bracing which in the upper figure is represented by full lines, while the system represented in the latter figure by dotted lines is shown at (*b*). In Figs. (*a*) and (*b*), those diagonals of the respective systems which do not bear stress when the system to which they belong is fully loaded are represented by dotted lines.

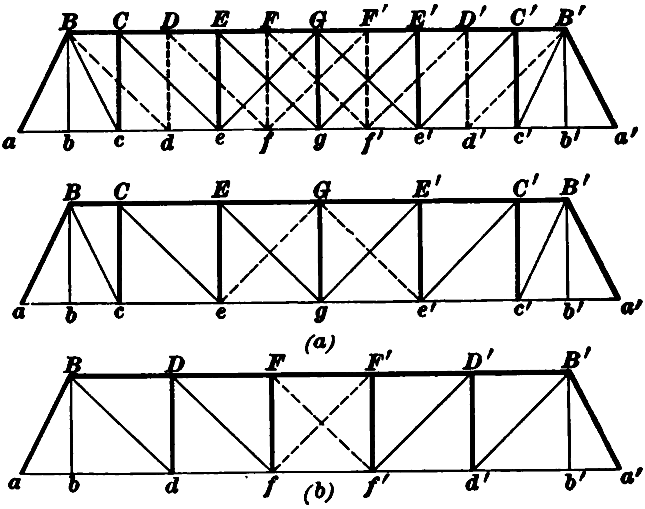


FIG. 293.

A slight ambiguity occurs in the web stresses, due to the fact that it is not known by which system the panel loads at *b* and *b'* are carried; but the assumption usually made, namely, that one-half of each load is carried by each system, must be very nearly correct. The error, if any, is small.

1354. The following data are assumed for the truss shown in Fig. 293: The length is 198 ft.; as it is divided into 12 equal panels, the length of each panel is $\frac{198}{12} = 16.5$ ft. The height of the truss is 33 ft. The clear width of roadway supported by the two trusses is 20 ft., and the live load for the trusses is a uniform load of 80 lb. per sq. ft. of roadway. The live load per lineal foot is, therefore, $80 \times 20 = 1{,}600$ lb., and for one truss the panel live load is $\frac{1{,}600 \times 16.5}{2} = 13{,}200$ lb.

1355. The Dead Load Stresses.—By formula **91**, Art. **1298,** the dead load per lineal foot, exclusive of the floor, is $80 + 100 + 248 + 131 = 559$ lb.

From Table 29, Art. **1299,** the weight per lineal foot of a yellow pine floor for a panel length of 17 feet is $274.5 + 4 \times 40.5 = 436.5$ lb. Therefore, the dead load per lineal foot is $559 + 437 = 996$ lb., or say 1,000 lb. For one truss the panel load is, therefore, $\frac{1{,}000 \times 16.5}{2} = 8{,}250$ lb.

Considering first the system of bracing shown in (*a*), Fig. 293, and remembering that but one-half of the panel loads at *b* and at *b'* are borne by this system, the value of either reaction is $8{,}250\left(\frac{11}{12} \times \frac{1}{2} + \frac{10}{12} + \frac{8}{12} + \frac{6}{12} + \frac{4}{12} + \frac{2}{12} + \frac{1}{12} \times \frac{1}{2}\right) = 8{,}250 \times \frac{36}{12} = 24{,}750$ lb. The full loads and reactions are laid off upon the load line, and the stress diagram is constructed in substantially the same manner as previously explained for the dead load. The extreme upper and lower portions of the load line represent the half panel loads

(4,125 lb.) at *b* and *b'*, respectively, while each intermediate division represents a full panel load (8,250 lb.), otherwise the construction is regular, and will require no special explanation. Bow's system of notation can be used.

1356. The **live load stresses** in the chords and end posts of this system may be obtained by constructing a stress diagram similar to that for the dead load; or they may be obtained by multiplying the corresponding dead load stresses by the ratio $\frac{L}{D} = \frac{1,600}{1,000} = 1.6$.

For the live load web stresses, the left reaction and those portions of the respective panel loads which are supported by it are laid off upon the load line in the manner explained in Art. **1341.** The right reaction due to live load is entirely neglected, but the portions of dead load producing negative shear in the panels at the right of the center are laid off upwards from the bottom of the load line, as was also explained in Art. **1341.** For this system the left reaction from live load and the portions of each panel load supported by it are as follows: $R_1 = 13,200(\frac{11}{12} \times \frac{1}{2} + \frac{10}{12} + \frac{8}{12} + \frac{6}{12} + \frac{4}{12} + \frac{2}{12} + \frac{1}{12} \times \frac{1}{2}) = 6,050 + 11,000 + 8,800 + 6,600 + 4,400 + 2,200 + 550 = 39,600$ lb. The reaction is laid off upwards and the proper portions of the respective loads, as thus found, are laid off in order downwards to the starting point. The combined stress diagrams for the partial loads are constructed substantially as explained in Art. **1341.**

The process of obtaining the stresses for the system of bracing shown in (*b*), Fig. 293, is very similar to the above. The principal difference is that the system of bracing shown in (*b*), Fig. 293, has an odd number of panels, while that shown in (*a*), Fig. 293, has an even number of panels. For dead load $R_1 = R_2 = 20,625$ lb. The portions of the live panel loads supported by the left reaction are, respectively, $6,050 + 9,900 + 7,700 + 5,500 + 3,300 + 550 = 33,000$ lb. No difficulty should be experienced in drawing the stress diagrams.

When the stresses are obtained for each system of bracing, the two systems should be combined, forming the truss shown in the upper portion of Fig. 293. The corresponding stresses for the chords and for all members common to both systems of bracing must, of course, be added.

1357. Formula **102**, Art. **1345**, may be applied at *B b* for the stress in *a c*, and at *G g* for the stress in *E E'*. It will give the stresses in the two panels at each end of the lower chord and in the four panels at the center of the upper chord; i. e., in that portion of each chord where the two systems act as one system.

EXAMPLES FOR PRACTICE.

1. For the system of bracing shown in (*a*), Fig. 293, having all loads and dimensions as given above, construct the stress diagram for the dead load stresses.

2. Correct the stresses as thus obtained for the vertical members.

NOTE.—Correct the stresses obtained in *B b* and *B' b'* for this system by one-third of *one-half* panel load for each. Do the same in the solution of Example 6, thus making the sum of the corrections for each member in both systems equal to one-third panel load.

3. For the same system of bracing, compute the live load stress in the chords and end posts from the corresponding dead load stresses.

4. For the same system of bracing, construct the combined diagrams for the maximum live load web stresses.

5. For the system of bracing shown in (*b*), Fig. 293, having loads and dimensions as above, construct the stress diagram for the dead load stresses.

6. Correct the stresses thus obtained for the vertical members.

7. For the same system of bracing, construct the stress diagram for the live load stresses in the chords and end posts.

8. Check the stresses obtained in the solution of the preceding example by computing them from the corresponding dead load stresses.

9. For the same system of bracing, construct the combined diagrams for the maximum live load web stresses.

10. Mark the stresses obtained in the solution of the preceding examples upon the respective members in the left half of a diagram of

the truss similar to Fig. 293 (without dotted lines), adding the respective stresses for those members which are common to both systems of bracing.

	Live Load Stresses.		Dead Load Stresses.
	For $a\,B$ + 81,160 lb.		+ 50,730 lb.
	For $B\,C$ + 79,200 lb.		+ 49,500 lb.
	For $C\,D$ + 99,000 lb.		+ 61,880 lb.
	For $D\,E$ + 112,200 lb.		+ 70,130 lb.
	For $E\,F$ + 118,800 lb.		+ 74,250 lb.
	For $F\,G$ + 118,800 lb.		+ 74,250 lb.
	For $a\,b$ − 36,300 lb.		− 22,690 lb.
	For $b\,c$ − 36,300 lb.		− 22,690 lb.
	For $c\,d$ − 52,800 lb.		− 33,000 lb.
	For $d\,e$ − 79,200 lb.		− 49,500 lb.
	For $e\,f$ − 99,000 lb.		− 61,880 lb.
	For $f\,g$ − 112,200 lb.		− 70,130 lb.
Ans.	For $B\,b$ − 13,200 lb.	Ans.	− 5,500 lb.
	For $B\,c$ − 37,510 lb.		− 23,060 lb.
	For $B\,d$ − 38,110 lb.		− 23,330 lb.
	For $C\,c$ + 22,550 lb.		+ 15,130 lb.
	For $C\,e$ − 31,890 lb.		− 17,500 lb.
	For $D\,d$ + 17,050 lb.		+ 11,000 lb.
	For $D\,f$ − 24,110 lb.		− 11,670 lb.
	For $E\,e$ + 13,750 lb.		+ 6,880 lb.
	For $E\,g$ − 19,440 lb.		− 5,830 lb.
	For $F\,f$ + 9,350 lb.		+ 2,750 lb.
	For $F\,f'$ − 13,220 lb.		− 0,000 lb.
	For $G\,g$ + 3,030 lb.		+ 2,750 lb.
	For $G\,e'$ − 4,280 lb.		− 0,000 lb.

11. By formula **102**, Art. **1345**, verify the stress in $a\,c$ ($a\,b$ and $b\,c$) and in $E\,E'$ ($E\,F$ and $F\,G$).

THE BALTIMORE TRUSS.

1358. A form of truss will now be noticed that affords economical inclinations for the diagonal members, and for long spans retains the economical features of the Whipple truss, without ambiguity of stress. The truss shown in Fig. 294 embodies the feature known as the **divided panel.** It will be noticed that it is simply a Pratt truss in which, by means of an additional short diagonal member, each original panel is divided into two panels of one-half the original panel length; the shorter panels thus obtained are called **sub-panels.** This form of truss is known as the **Baltimore truss.** The stresses will be analyzed for a truss of this type.

The data assumed will be the same as those assumed for the Whipple truss of the preceding pages; namely, span, 198 feet, number of panels, 12; panel length, 16.5 feet; height, 33 feet; clear roadway, 20 feet; live and dead loads per lineal foot, 1,600 and 1,000 pounds, respectively.

1359. The Dead Load Stresses.—A stress diagram for the dead load is also shown in Fig. 294. For one truss the panel load is $\frac{1,000 \times 16.5}{2} = 8,250$ lb., as before. By formula **89,** Art. **1293,** $R_1 = R_2 = \frac{8,250 \times 11}{2} = 45,375$ lb. The loads and reactions are laid off upon the load line in the usual manner; thus, *1-2-3-4-5-6-7-8-9-10-11-12-13-1* is the load line. It will be found convenient to divide each panel load into two equal parts.

For joint *a*, the polygon is *1-2-14-1*; for joint *b* it is *15-14-2-3-15*, and for joint B_1 it is *1-14-15-16-1*. Of the forces acting upon joint *C*, *1-16*, or the stress in *B C*, is apparently the only known force, leaving three unknown forces, *16-17*, *17-20*, and *20-1*, acting upon this joint. Likewise, for joint *c*, of the three forces acting in *C c*, *c* D_1, and *c d*, none have yet been obtained by the stress diagram. However, from an inspection of the diagram of the truss, it is evident that the hip vertical *C c* supports one-half of each of the panel loads at *b* and *d*, besides the entire panel load at *c*, or two panel loads in all. Knowledge of this fact determines the length of the line *16-17*, common to the polygons for joints *C* and *c*; and as this leaves but two unknown forces acting upon each joint, the polygon for both joints can be drawn. Again, it is evident that the stress in D_1 *c* equals that in B_1 *c*, and that the stress in *c d* equals that in *b c*; therefore, by retracing the lines *16-15*, *15-3*, and *3-4*, the lines *4-18* and *18-17* may be drawn, respectively, parallel to *c d* and D_1 *c*, but equal to *15-3* and *16-15*, respectively, leaving *17-16* as the closing line of the polygon for joint *c*.

A similar difficulty will be met with in the polygons for

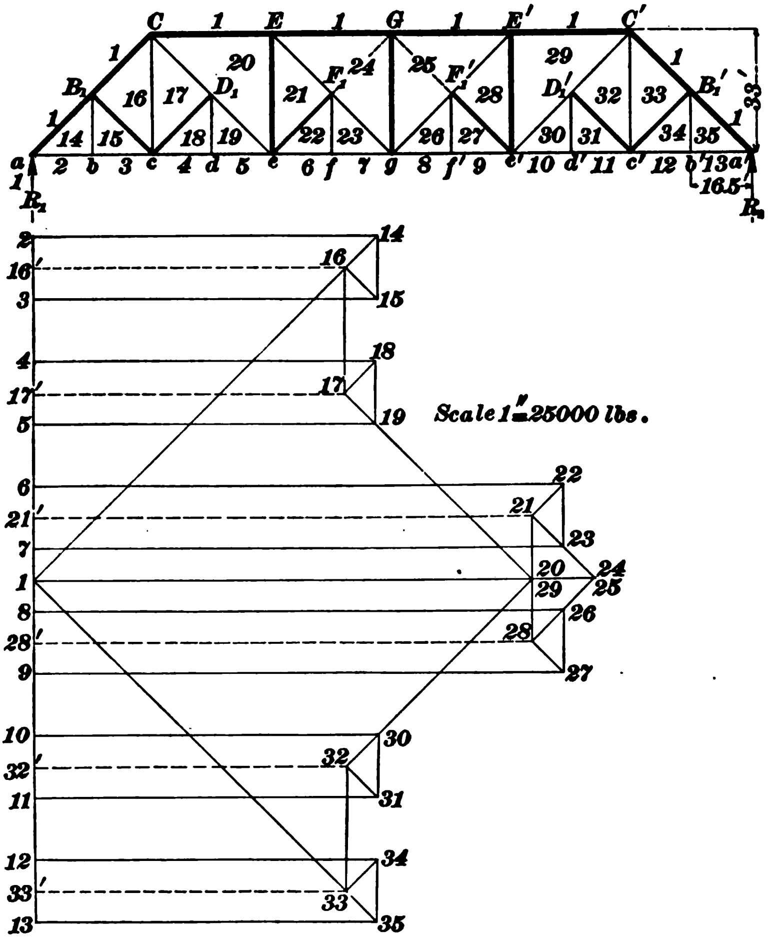


FIG. 294

joints *E* and *c*. But by noticing that the stress in F_1 *e* is equal to the stress in $D_1 c$, or, in other words, that the vertical component of the stress in $F_1\,e$ is equal to one-half the panel load *6-7* at f, the polygon for joint e may be drawn. The lines *20-19*, *19-5*, and *5-6* are retraced, and a line of indefinite length *6-22* is drawn parallel to $e\,f$; by drawing the horizontal line *21′-21* through the point *21′*, which, on the load line, divides the load *6-7* into two equal parts, and a line *20-21* through the starting point *20*, parallel to $E\,e$, the length of the line *21-20*, representing the stress in $E\,e$, will be determined, and the polygon *20-19-5-6-22-21-20* may be completed for joint e.

Thus, the slight difficulty resulting from the additional member necessary to the divided panel is readily overcome; no other difficulty will be encountered in the construction of the stress diagram for the dead load stresses. It is evident that no dead load stress will obtain in the diagonals $F_1\,G$ and $G\,F_1'$, and that the stress diagram will give no dead load stress for $G\,g$.

The stresses for the members $B_1\,b$, $D_1\,d$, and $F_1\,f$ are each to be corrected by $\frac{1}{3}$ panel load as usual. But the stresses for $E\,e$ and $G\,g$ are each to be corrected by $\frac{1}{3}$ of 2 panel loads, = $\frac{2}{3}$ panel load, and the stress for $C\,c$ is to be corrected by $\frac{1}{3}$ of $1\frac{1}{2}$ panel loads, = $\frac{1}{2}$ panel load; the reason for this is obvious.

1360. In trusses of this type the stresses in the *upper* chord may be verified by applying formula **102,** Art. **1345,** at the main vertical members, as at $E\,e$ and $G\,g$, taking m and m' equal to the number of *sub-panels* at the left and right, respectively. The stress C_s in any portion of the lower chord will have the following value:

$$C_s = C_h + \frac{Pp}{h}, \qquad \textbf{(104.)}$$

in which C_h, P, p, and h have the same values as in formula **102,** Art. **1345,** except that if the height of the truss varies, h will represent the height at the vertical post at the *left* (towards end of truss) of the double panel. C_h equals the corresponding stress in the upper chord.

EXAMPLES FOR PRACTICE.

1. For the truss shown in Fig. 294 and the dead load assumed as above, construct the stress diagram, determining the dead load stresses.

Ans.

For $a\ B_1$	+ 64,170 lb.	For $E\ e$	+ 8,250 lb.
For $B_1\ C$	+ 58,340 lb.	For $G\ g$	+ 0,000 lb.
For $C\ E$	+ 66,000 lb.	For $B_1 c$, $D_1 c$, $F_1 e$	+ 5,830 lb.
For $E\ G$	+ 74,250 lb.	For $C\ D_1$	— 35,000 lb
For $a b$, $b c$, $c d$, $d e$	— 45,380 lb.	For $D_1\ e$	— 29,170 lb
For $e f$ and $f g$	— 70,130 lb.	For $E F_1$	— 11,670 lb.
For $B_1 b$, $D_1 d$, $F_1 f$	— 8,250 lb.	For $F_1\ g$	— 5,830 lb.
For $C\ c$	— 16,500 lb.		

2. Correct the stresses in the vertical members.

Ans.

For $B_1\ b$, $D_1\ d$, and $F_1 f$	— 5,500 lb.
For $C\ c$	— 12,380 lb.
For $E\ e$	+ 13,750 lb.
For $G\ g$	+ 5,500 lb.

3. Check the stresses in the chords by formulas **102** and **104.**

1361. The Live Load Stresses.—For the chords and end posts, the live load stresses will be most easily

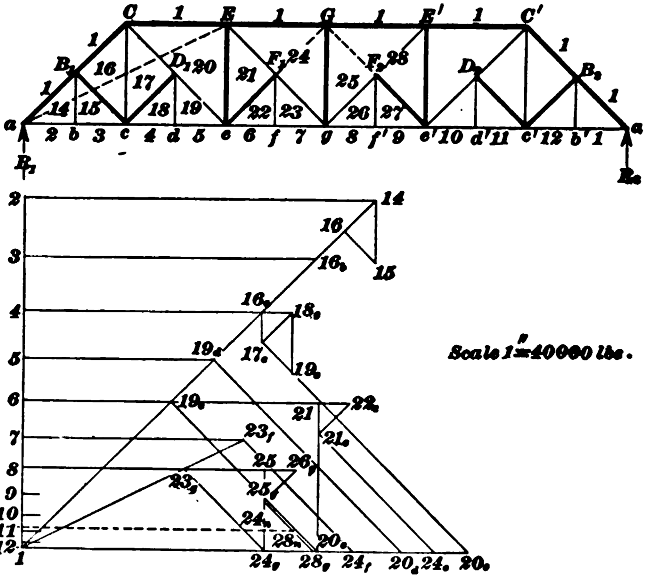


FIG. 295.

obtained by multiplying the corresponding dead load stresses by the ratio $\frac{L}{D}$, which in this case equals $\frac{1,600}{1,000} = 1.6$.

For the maximum live load web stress the combined diagrams may be constructed in much the same manner as previously explained. The left reaction and that portion of each panel load supported by it are laid off in order upon the load line, and from the bottom of the load line the amounts of dead load which cause negative shear in the panels at the right of the center are laid off upwards. The portions of the respective panel loads supported by the left reaction are found as usual by computing the reaction. Thus, $R_1 = 13,200\ (\frac{11}{12} + \frac{10}{12} + \frac{9}{12} + \frac{8}{12} + \frac{7}{12} + \frac{6}{12} + \frac{5}{12} + \frac{4}{12} + \frac{3}{12} + \frac{2}{12} + \frac{1}{12}) = 12,100 + 11,000 + 9,900 + 8,800 + 7,700 + 6,600 + 5,500 + 4,400 + 3,300 + 2,200 + 1,100 = 72,600$. On the load line of Fig. 295, *1-2* = 72,600 lb. is the reaction for a full load, and *1-2-14-1* is for this load the polygon for joint *a*, giving *14-1* as the maximum stress in $a\,B_1$. As one-half the load at *b* is carried by $B_1\,c$ to the joint *c*, this condition of load will also give the maximum stress in $B_1\,C$. For in this condition the stress in $B_1\,b$ equals the panel load at *b*, and by making *14-15* equal to a panel load, the polygon *1-14-15-16-1* is drawn for joint B_1, giving *16-1* as the stress in $B_1\,C$; while by removing the load from *b*, *1-3* becomes the reaction, and, there being no stress in either $B_1\,b$ or $B_1\,c$, $1\text{-}16_b$ represents the stress in the end post for this condition, uniform from *a* to *C*. The stress $1\text{-}16_b$ is less than the stress *1-16*. For the maximum shear in the panel *c d* and the maximum stress in $C\,D_1$, the loads at *b* and *c* must be removed. There being now no external force between *a* and *d*, *1-4* represents the reaction and $1\text{-}4\text{-}16_c\text{-}1$ is the force polygon for joint *a*. As no forces are now acting in $B_1\,b$ or $B_1\,c$, the polygon for joint B_1 is the straight line $1\text{-}16_c$. It is known that the member *C c* supports one-half the panel load at *d*, which by the member $D_1\,c$ is transferred to joint *c*; therefore, by making $16_c\text{-}17_c$ equal to one-half panel load, i. e., the stress in *C c*, the polygon $1\text{-}16_c\text{-}17_c\text{-}20_c\text{-}1$ can be drawn for joint *C*, giving $17_c\text{-}20_c$ as the maximum stress in $C\,D_1$.

If, instead of the strut $D_1 c$, a tie $D_1 E$ were used to effect the divided panel, this condition of load would also give the maximum stress in $D_1 e$. But such is not the case when the panel is divided by means of a strut which carries the load *towards the reaction*. For in this condition, i. e., with *d* loaded, 20_c-17_c-18_c-19_c-20_c is the polygon for joint D_1, giving 19_c-20_c as the stress for $D_1 e$. By removing the panel load from joint *d*, *1-5* becomes the reaction; as in this condition of load the members $B_1 b$, $B_1 c$, $C c$, $D_1 c$, and $D_1 d$ carry no stress, the polygons for joints B_1 and D_1 are simply straight lines, and the entire space between the end post *a C* and the diagonal *C e* is designated by the numeral *19*. The polygon for joint *a* is 1-5-19_d-1, and for joint *C* it is 1-19_d-20_d-1; the stress in $D_1 e$ represented for this condition by 19_d-20_d is found to be greater than as represented by 19_c-20_c.

The maximum stress in *E e* obtains with the condition giving the maximum stress in $E F_1$, that is, with the load at *e* also removed. For this condition, *1-6* is the reaction, *1-6-19-1* is the polygon for joint *a*, and 1-19_e-20_e-1 is the polygon for joint *C*. For joint *e*, 20_e-19_e and 19_e-6 are retraced, leaving the three members *e f*, $F_1 e$, and *E e* connecting at *e*, in which the stresses have not been determined. But, as has been explained for $D_1 c$, it is evident that the vertical component of the stress $F_1 e$ is equal to one-half the panel load at *f*. Therefore, from *6*, a line of indefinite length, as 6-22_e, is drawn parallel to *e f*, and from the starting point 20_e the line 20_e-21 is drawn upwards (or backwards in the polygon) parallel to *E e*; the intersection of these two lines is at *21*. By laying off one-half panel load downwards from this intersection, the point 21_e is located; and by drawing through 21_e a line 21_e-22_e parallel to $e F_1$ intersecting the line 6-22_e the polygon 20_e-19_e-6-22_e-21_e-20_e may be completed for joint *e*, giving 21_e-20_e as the maximum stress in *E e*. For joint *E*, 1-20_e-21_e-24_e-1 is the polygon, giving 21_e-24_e as the maximum stress in $E F_1$.

For the maximum stress in $F_1 g$, the panel load at *f* is also removed. Since for this condition no external force is applied between a and g, all that portion of the truss at the left of $E\,g$ may be considered to be removed, and the two members $a\;E$ and $a\;g$ substituted. The reaction for this condition is *1-7*; for joint a, *1-7-23-1* is the polygon, and, as the members $F_1\,e$ and $F_1\,f$ have been removed, the polygon $1\text{-}23_f\text{-}24_f\text{-}1$ gives $23_f\text{-}24_f$ as the stress in $E\;g$ for this condition, or the maximum stress in $F_1\,g$.

For the maximum stress in $G\;F_2$ and in $G\,g$ the load at g is also removed. The members $a\;E$ and $a\;g$ are still considered to be substituted for all portions of the truss at the left of $E\,g$. For this load *1-8* is the reaction; the polygon for joint a is $1\text{-}8\text{-}23_g\text{-}1$; for joint E it is $1\text{-}23_g\text{-}24_g\text{-}1$. For joint g the polygon $24_g\text{-}23_g\text{-}8\text{-}26_g\text{-}25_g\text{-}24_g$ is constructed in the same manner as explained for joint e; and for joint G, $1\text{-}24_g\text{-}25_g\text{-}28_g\text{-}1$ is the polygon. But the panel $g\;f'$ is at the right of the center, and the dead load shear in this panel must be deducted from $24_g\text{-}25_g$ and from $25_g\text{-}21_g$ in order to obtain the correct stresses in $G\,g$ and $G\;F_2$. If from the negative dead load numeral 8_n on the load line, corresponding to the numeral *8* below the panel $g\,f'$, the horizontal line $8_n\text{-}28_n$ be drawn, it will cut off the negative shear in the panel $g\,f'$, and the lines $24_n\text{-}25_g$ and $25_g\text{-}28_n$ will remain as the stresses in $G\,g$ and $G\;F_2$, respectively.

For the maximum tension in $F_2\;e'$, caused by positive shear, the load at f' is removed. For this condition *1-9* on the load line represents the reaction. As the live load numeral *9* falls below the corresponding dead load numeral 9_n, there can be no resultant positive shear in the corresponding panel $f'\;e'$, and, therefore, no tension in $F_2\;e'$.

EXAMPLES FOR PRACTICE.

1. From the corresponding dead load stresses compute the corresponding live load stresses in the cnords and end posts of the bridge just considered.

Ans.
For $a\;B_1$ + 102,670 lb.
For $B_1\;C$ + 93,330 lb.
For $C\;\;E$ + 105,600 lb.
For $E\;G$ + 118,800 lb.
For $a\,b$, $b\,c$, $c\,d$, and $d\,e$ − 72,600 lb.
For $e\,f$ and $f\,g$ − 102,200 lb.

2. By formula **102**, Art. **1345**, and formula **104**, Art. **1360**, check the stresses obtained for the chords in the solution of the preceding example.

3. For the same bridge construct the combined stress diagrams determining the maximum live load web stresses.

Ans.
For $a\ B_1$ + 102,670 lb.
For $B_1\ C$ + 93,330 lb.
For $C\ \ c$ − 26,400 lb.
For $C\ D_1$ − 60,670 lb.
For $D_1\ e$ − 56,000 lb.
For $E\ \ e$ + 24,200 lb.
For $E\ F_1$ − 34,220 lb.
For $F_1\ g$ − 32,670 lb.
For $G\ \ g$ + 5,780 lb.
For $G\ F_2$ − 8,170 lb.
For $B_1\,b$, $D_1\,d$, and
For $F_1\ \ f$ − 13,200 lb.
For $B_1\,c$, $D_1\,c$, and
For $F_1\ \ e$ + 9,330 lb.

THE PETIT TRUSS.

1362. A modified form of the Baltimore truss is shown in Fig. 296. The general form of the truss is the same as that of the truss shown in Figs. 294 and 295; but, except in the end panels, the additional diagonals employed to subdivide the panels are ties instead of struts. The feature of subdividing the panels by means of additional ties is called the **half hitch**, and the ties so used are sometimes called **sub-ties.** The half hitch does not give as good economy as is obtained when the subdivision is effected by means of a strut, and the diagonal web members thus connected are also susceptible of greater vibrations,

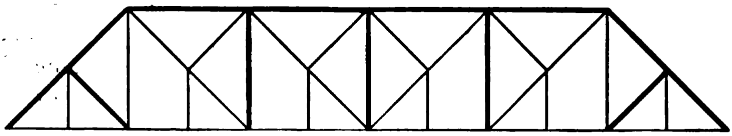


FIG. 296.

but it is very commonly used. As subdivided panels and curved chords are both well adapted to long spans, these two features are usually combined. When the panels are subdivided, either by means of struts or ties, in a truss having curved chords, the truss is usually known as the **Petit truss**; it is one of the most economical forms of a simple truss for long spans. The general form of the Petit truss commonly used for highway bridges, together with the stress diagram for the dead load stresses in the same, is shown in Fig. 297. The stress diagrams for this truss will serve as general illustrations of the graphical analysis for half-hitch trusses. It is evident that the panels could have been subdivided by means of struts instead of ties. In the two end panels this truss differs materially from those shown in Figs. 291 and 292; the hip is at *B* instead of at *C*, and *C c* is a compression member instead of a tension member. The portions of the truss between *a* and *c* and between *a'* and *c'* are similar to the corresponding portions of a Pratt truss, while between *c* and *c'* the panels are subdivided.

Except in regard to height, the data assumed for this truss will be the same as for the Whipple truss and the Baltimore truss in the preceding articles; namely, length of span, 198 feet, divided into 12 panels 16.5 feet long each; clear width of roadway, 20 feet; live and dead loads per lineal foot, 1,600 pounds and 1,000 pounds, respectively. The heights of the truss at the hip vertical and at the intermediate posts, in order towards the center, are 22, 28, 33, and 33 feet, respectively.

1363. The Dead Load Stresses.—For each truss the dead load per panel is 8,250 pounds, and $R_1 = R_2 = 45,375$ lb., as before. The loads and reactions are laid off in order upon the load line in the usual manner. No difficulty should be experienced in drawing the stress diagram for the dead load. The force polygons for joints *a*, *b*, *B*, *c*, and *C* are constructed as for an ordinary Pratt truss with curved chords, *1-16-17-18-1* being the polygon for joint *C*. For joint *d* the polygon is *17-4-5-19-17*; for joint D_1, it is *18-17-19-20-18;* for joint *e*, it is *20-19-5-6-21-20*; and for joint *E*, it is *1-18-20-21-22-1.* Further explanation is unnecessary; the entire stress diagram will be readily understood.

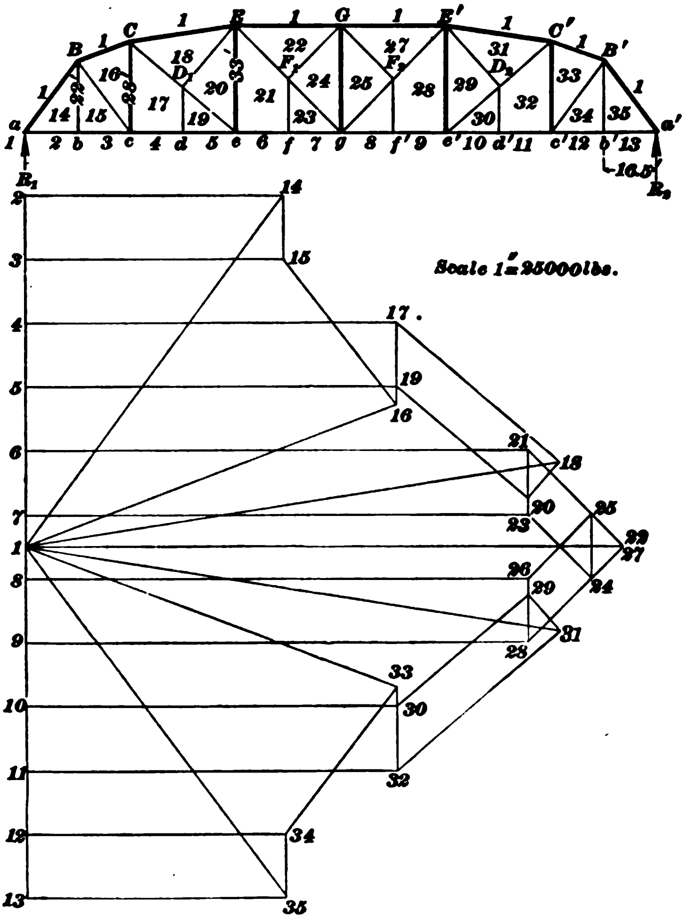


FIG. 297.

In correcting the stresses for the vertical members, the stress for *B b* is corrected by $\frac{1}{3}$ of 1 panel load, the stresses for *E e* and *G g* are each corrected by $\frac{1}{3}$ of 2 panel loads, or

⅔ panel load, while the stress for $C\,c$ is corrected by ⅓ of 1½ panel loads, or ½ panel load.

In half-hitch trusses the stress obtained for the lower chord may be verified by applying formula **102,** Art. **1345.** Where the upper chord is horizontal, its stress will have the value of C_s in formula **104,** Art. **1360.** Where the upper chord is not horizontal, C_s will represent the horizontal component of its stress; the stress may be found by substituting the value of C_s for C_h in formula **103,** Art. **1346.** Where the height of the truss varies, use for h in formula **104** the height at the vertical post at the *right* (towards the center of the truss) of the double panel.

EXAMPLES FOR PRACTICE.

1. For the truss shown in Fig. 297, the data being as above, construct the stress diagram determining the dead load stresses.

Ans.	For $a\,B$	+ 56,720 lb.	For $C\,c$,	+ 11,200 lb.
	For $B\,C$	+ 51,730 lb.	For $C\,D_1$	– 28,210 lb.
	For $C\,E$	+ 70,930 lb.	For $D_1\,e$	– 21,830 lb.
	For $E\,G$	+ 78,380 lb.	For $D_1\,E$	– 5,460 lb.
	For $a\,b$ and $b\,c$	– 34,030 lb.	For $E\,e$	+ 5,880 lb.
Ans.	For $c\,d$ and $d\,e$	– 48,620 lb.	For $E\,F_1$	– 17,500 lb.
	For $e\,f$ and $f\,g$	– 66,000 lb.	For $F_1\,g$	– 11,670 lb.
	For $B\,b$, $D_1\,d$. $F_1\,f$	– 8,250 lb.	For $F_1\,G$	– 5,830 lb.
	For $B\,c$	– 24,310 lb.	For $G\,g$	– 8,250 lb.

2. Correct the stresses for the vertical members.

Ans.	For $B\,b$, $D_1\,d$, and $F_1\,f$	– 5,500 lb.
	For $C\,c$	+ 15,330 lb.
	For $E\,e$	+ 11,380 lb.
	For $G\,g$	+ 13,750 lb.

1364. The Live Load Stresses.—For one truss the panel live load is 13,200 pounds. The maximum live load stresses in the chords and end posts may be found by multiplying the corresponding dead load stresses by the ratio $\frac{L}{D}$, as previously explained. The maximum live load web stresses may be obtained by constructing the combined stress diagrams as shown in Fig. 298. $R_1 = 13{,}200 \times (\frac{11}{12} + \frac{10}{12} + \frac{9}{12} + \frac{8}{12} + \frac{7}{12} + \frac{6}{12} + \frac{5}{12} + \frac{4}{12} + \frac{3}{12} + \frac{2}{12} + \frac{1}{12}) = (12{,}100 + 11{,}000 + 9{,}900 + 8{,}800 + 7{,}700 + 6{,}600 + 5{,}500 + 4{,}400 + 3{,}300 + 2{,}200 + 1{,}100) = 72{,}600$ lb. From *1*, *1-2* $= R_1$ is laid off upwards, and *2-3* = 12,100 lb., *3-4* = 11,000 lb., etc., are laid off downwards to the starting point. Also from *1*, 7_n-8_n, and 8_n-9_n, equal, respectively, to the amount of negative dead load shear produced at the right of the center by the dead loads at *g* and *f'*, all in the usual manner. No difficulty will be encountered in constructing the diagrams, but some irregularity will be noticed in the conditions of loading which produce maximum stresses in certain members.

With the truss fully loaded, *1-2* represents the reaction, and the polygon *1-2-14-1* for joint *a* gives the maximum stresses in *a b* and *a B*. For the maximum stress in *B c* the load at *b* is removed; *1-3* is the reaction for this condition and *1-3*-15_b-*1* is the polygon for joint *a*; as there is no stress in *B b*, *1*-15_b-16_b-*1* is the polygon for joint *B*; giving 15_b-16_b as the maximum stress in *B c*.

The maximum stresses in *C c*, *C* D_1, and D_1 *e* occur with no loads upon *b* and *c*, all other joints being loaded. For the stresses in *C* D_1 and D_1 *e*, consider members *a C* and *a d* to be substituted for those portions of the truss at the left of *C* and *d*. The reaction is *1-4*; *1-4*-17_c-*1* is the polygon for joint *a*, and *1*-17_c-18_c-*1* is the polygon for joint *C*, giving 17_c-18_c as the maximum stress in *C* D_1. On the load line, *4*-5_c is laid off downwards equal to the full panel load at *d*; 17_c-*4*-5_c-19_c-17_c is the polygon for joint *d*, and 18_c-17_c-19_c-20_c-18_c is the polygon for joint D_1, giving 19_c-20_c as the stress in D_1 *e* and 20_c-18_c as the stress in D_1 *E*. That the stress in D_1 *e* with joint *d* loaded is greater than with that joint unloaded may be readily shown. With no load at *d* *1-5* becomes the reaction, *1-5*-19_d-*1* is the polygon for joint *a*, and *1*-19_d-20_d-*1* the polygon for joint *C*, giving 19_d-20_d as the uniform stress from *C* to *e* for this condition; it is found to be less than 19_c-20_c. By considering the original portions of the truss to be restored to the irrespective positions, the *reversed* polygon *1*-18_c-17_c-16_c-*1* gives 16_c-17_c as the stress in *C c*.

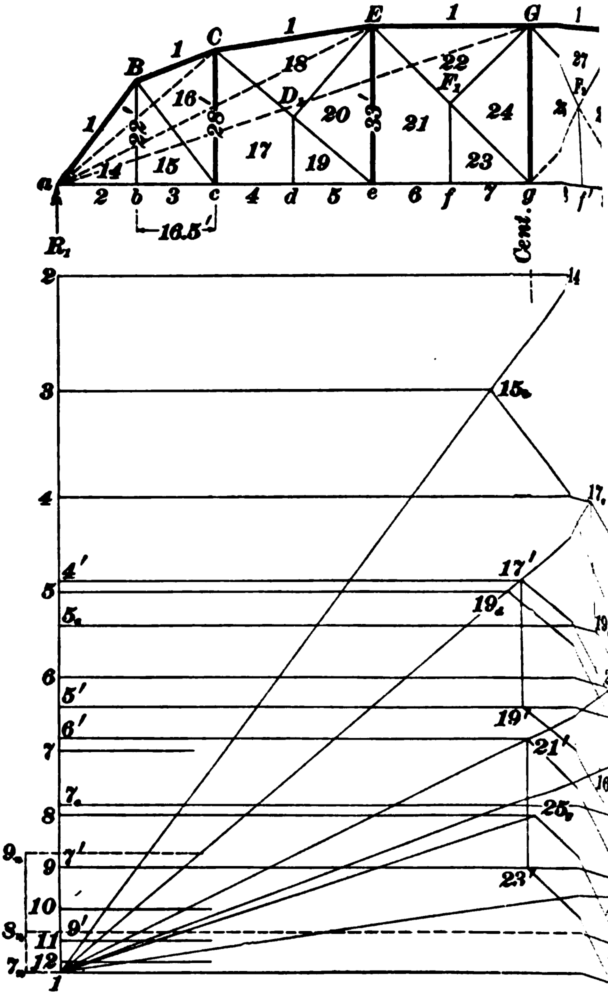
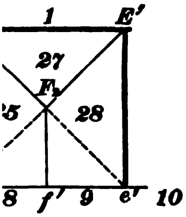
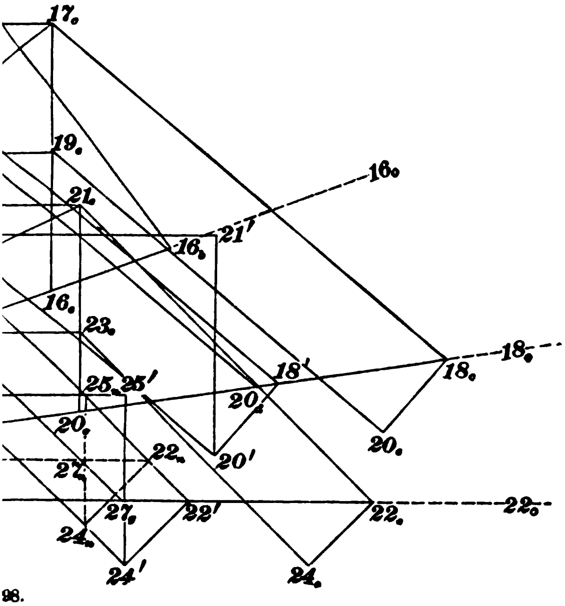

a
B
C
E
G
1
1
1
1
1
1
18
22
F₁
16
D₁
20
33
21
24
28
22
14
15
17
19
23
2
b
3
c
4
d
5
e
6
f
7
g
f′
16.5′
R₁
Cent.
2
14
3
15₀
4
17₀
4′
17′
5
19₄
5₀
6
5′
19′
6′
21′
7
7₀
25₀
8
9₀
7′
9
23′
10
S₀
9′
11
7₀
12
1

FIG. 21



714

Scale 1"=20000 lbs.



98.

For the maximum stresses in $E F_1$ and $F_1 g$, all joints at the right of the panel *e f* are loaded, no load being at the left of that panel. The members *a E* and *e f* are considered to be substituted for all portions of the truss at the left of *E* and *f*. For this load, *1-6* is the reaction, and, by proceeding as before, 21_e-22_e, 23_e-24_e, and 22_e-24_e are obtained as the maximum stresses in $E F_1$, $F_1 g$ and $F_1 G$, respectively. By considering the original members of the truss to be restored, and drawing the reverse polygon for joint *E*, 20_e-*21* is obtained as the stress in *E e* for this condition of load.

But this is not the maximum stress in this member; for it will be found that with all joints to the right of the panel *e f* loaded, an additional load placed upon joint *d* will increase the stress in *E e*. As *1-6* represents the reaction with all joints at the right of the panel *e f* loaded, and the others unloaded, to obtain the reaction with the additional load at *d*, lay off upon the load line above *6* the amount *4'-6*, equal to *4-5* (i. e., equal to the amount of the left reaction produced by the load at *d*); *1-4'* is the left reaction from all the loads upon the truss in this condition.

Considering *a C* and *a d* to be substituted for those portions of the truss at the left of *C* and *d*, the polygon *1-4'-17'-1'* is obtained for joint *a*, *1-17' 18'-1* is obtained for joint *C*, *17'-4'-5'-19'-17'* for joint *d*, *18'-17'-19'-20'-18'* for joint D_1, and *20'-19'-5'-21'-20'* for joint *e*. The stress *20'-21'* for *E e* is found to be greater than the stress 20_e-21_e; it is the maximum stress for *E e*. For the maximum stress in $G F_2$ all joints at the right of the panel *g f'* are loaded, no load being at the left of that panel. For this condition *1-8* is the reaction, and, with *a G* and *a f'* substituted for all portions of the truss at the left of *G* and *f'*, *1-8-*25_g*-1* is the polygon for joint *a*, and *1-*25_g-27_g*-1* the polygon for joint *G*. In the panel *g f'* there exists an amount of negative dead load shear represented by 7_n-8_n; therefore, by drawing through 8_n the horizontal line 8_n-22_n, the effect of the negative shear, or 27_n-27_g, is cut off, leaving 25_g-27_n as the correct stress in $G F_2$. If stress can occur in a member $F_2 e$, it will obtain its maximum limit with this condition of load. The negative dead load shear in the panel $f'\ e'$ equals 7_n-9_n; the live load reaction is *1-8*. The polygon for joint *a* is 1-8-25_0-1, and for joint *G* it is 1-25_0-27_0-1. To draw the polygon for joint f', retrace 25_0-8, pass downwards upon the load line 8-$9'$, the amount of the entire panel load at f', and without proceeding further it is found that the point $9'$, as well as the point *9*, falls below the point designated by the corresponding dead load numeral 9_n, and, therefore, no positive shear can obtain in the corresponding panel *9* or $f'\ e'$; consequently, no stress can be produced in a counter $F_2\ e'$.

The maximum stress in *G g* will obtain with joint *f* and all joints at the right of the panel $g f'$ loaded. The amount $6'$-$8 = 6$-7 is laid off upon the load line above *8*; 1-$6'$ is the reaction for this condition. With *a E* and *a f* substituted for those portions of the truss at the left of *E* and *f*, 1-$6'$-$21'$-1 is the polygon for joint *a*. For joint *E* the polygon is 1-$21'$-$22'$-1; for joint *f* it is $21'$-$6'$-$7'$-$23'$-$21'$; for joint F_1 it is $22'$-$21'$-$23'$-$24'$-$22'$, and for joint *g* it is $24'$-$23'$-$7'$-$25'$-$24'$.

The line $25'$-$24'$ would represent the live load stress in *G g*, if no negative dead load shear existed in the panel $g f'$; but as the stress in *G g* is produced by the stresses in $F_2\ G$ and $F_1\ G$, the same amount of negative dead load shear must be deducted from the stress $25'$-$24'$, obtained for *G g*, that was deducted from the maximum stress previously obtained for $G\ F_2$. This negative shear is represented by 7_n-8_n, and a convenient method of deducting it is to consider the stress in $E\ F_1$ to be represented by $21'$-22_n instead of $21'$-$22'$; then 22_n-$21'$-$23'$-24_n-22_n becomes the polygon for joint F_1 and 24_n-$23'$-$7'$-25_n-24_n the polygon for joint *g*, giving 25_n-24_n as the correct maximum stress in *G g*. The stress in each suspender *B b*, $D_1\ d$, and $F_1\ f$ is tension equal to the panel of live load supported by it.

It is evident that the maximum stress produced in $G\ F_1$ by the right reaction will be the same as the maximum stress in $G\ F_2$ due to the left reaction. Therefore, both members must be given the greater stress found above for either of these members.

EXAMPLES FOR PRACTICE.

1. From the data given for the truss shown in Figs. 297 and 298 and the dead load stresses previously obtained for the same, compute the live load stresses in the chords and end posts.

Ans.
For $a\ B +$ 90,750 lb.
For $B\ C +$ 82,770 lb.
For $C\ E +$ 113,480 lb.
For $E\ G +$ 125,400 lb.
For $a\ b$ and $b\ c -$ 54,450 lb
For $c\ d$ and $d\ e -$ 77,790 lb.
For $e\ f$ and $f\ g -$ 105,600 lb.

2. Construct the combined stress diagrams for the maximum live load web stresses for the same.

Ans.
For $a\ B$ + 90,750 lb.
For $a\ b$ − 54,450 lb.
For $B\ b$, $D_1\ d$, and $F_1\ f$ − 13,200 lb.
For $B\ c$ − 43,220 lb.
For $C\ c$ − 28,290 lb.
For $C\ D_1$ − 53,320 lb.
For $D_1\ e$ − 44,670 lb.
For $D_1\ E$ − 10,070 lb.
For $E\ e$ + 22,770 lb.
For $E\ F_1$ − 43,560 lb.
For $F_1\ g$ − 34,220 lb.
For $G\ g$ + 13,480 lb.
For $F_2\ G = F_1\ G$ − 17,500 lb.

3. Check the chord stress obtained in the solution of Example 1 above.

ERROR IN THE ASSUMPTION FOR MAXIMUM WEB STRESSES.

1365. Throughout the foregoing investigation of the stresses in trusses it has been assumed that each loaded joint receives its panel load wholly independent of the load upon any other joint; for uniform loads this is the usual assumption and practice. The assumption is not exactly correct, as may be seen from an inspection of Fig. 290. We have assumed that it is possible for joint c, for example, to be fully loaded with a whole panel load, while b was unloaded; but, in reality, in order that c may carry a full panel load, the panels $b\ c$ and $c\ d$ must be loaded, half of the load on each going to c; were $c\ d$ loaded and $b\ c$ unloaded, c would carry only *one-half* of a panel load. But, then, with $b\ c$ loaded, b carries half a panel load $(= \frac{1}{2} P$, calling P the panel load). The reaction R_1' will be what we have found it when b was assumed to be unloaded (R_1), plus the part of the $\frac{1}{2} P$ carried by b, which goes to the left support; that is, $R_1' = R_1 + \frac{7}{8} \times \frac{P}{2}$; and the shear in the panel *bc*, instead of being R_1, as assumed, will be R_1', which acts upwards, minus $\frac{P}{2}$, which acts downwards at *b*; that is, shear $= R_1' - \frac{P}{2} = R_1 + \frac{7}{8} \times \frac{P}{2} - \frac{P}{2} = R_1 - \frac{1}{8} \times \frac{P}{2}$. As this is smaller than R_1, the corresponding stresses will be smaller than those we have found, and the members also smaller. By neglecting the load at *b* the members are, therefore, made larger than they actually have to be. As the error is both small and on the side of safety, it is usually disregarded, as we have done.

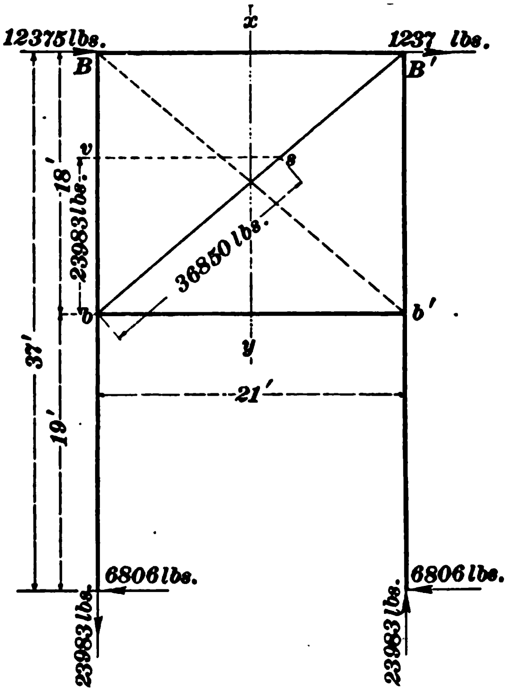


FIG. 299.

STRESSES IN A BRACED PORTAL.

1366. In Arts. **1309** to **1314,** the method of obtaining the stresses in a latticed portal by means of moments was explained. But in certain cases it is desirable to construct other forms of portals. The Whipple truss, shown in Fig. 293, requires a very deep portal, and for it the form of portal shown in Fig. 299 is more economical than a latticed portal. This form of portal is called a **braced portal.** It consists of two struts $B\,B'$ and $b\,b'$ connected by the two diagonal rods $B'\,b$ and $B\,b'$, called **sway rods;** $b\,b'$ is called a **sub-strut.** The widths of the end posts are not yet known, but the width center to center of chords may be approximately calculated by formula **101,** Art. **1342.** Here $b = 20$, and $C = 20 + \frac{198}{120} = 21.65$ ft. But for present purposes it will be convenient to take the distance center to center of end posts at 21 feet. As the panel length is 16.5 feet and the height of truss is 33 feet, the length of each end post is $\sqrt{33^2 + 16.5^2} = 37$ ft. (nearly).

The depth of the portal $B\,b$ (in plane of end posts) will be taken at 18 feet. By applying the principles of moments to this portal a very easy analysis is obtained. If the wind pressure against the upper chord be taken at 150 pounds per lineal foot, the panel wind load is $150 \times 16.5 = 2{,}475$ pounds. As there are ten panels in the upper lateral system, the amount of wind pressure carried by it against each portal is $\frac{10 - 1}{2} \times 2{,}475 = 11{,}138$ lb. To this must be added the panel of wind load directly at the portal, one-half of which, or 1,237 pounds, is assumed to be applied against the windward and the same amount against the leeward side. In the figure, the wind is assumed to be from the left; $11{,}138 + 1{,}237 = 12{,}375$ lb. is the pressure against the windward side, and 1,237 lb. the pressure against the leeward side of the portal. The horizontal and the vertical reactions at the foot of each end post are found as explained in Arts. **1309** and **1310.** The horizontal reaction at the foot of each post is $\frac{12{,}375 + 1{,}237}{2} = 6{,}806$ lb.

The amount of each so-called vertical reaction equals $(12,375 + 1,237) \times \frac{37}{21} = 23,983$ lb., positive at the foot of the leeward post, and negative at the foot of the windward post. The positions and directions of all external forces acting upon the portal are shown in Fig. 299. With the wind pressure from the left the member $B\ b'$ does not act, and need not be considered; the members $B\ B'$, $b\ B'$, and $b\ b'$ are, therefore, the only members of the portal considered in the following analysis. With the wind from the right, the opposite conditions would obtain, and the tie $B\ b'$ would act instead of the tie $b\ B'$.

1367. In Art. **1128,** it was stated that *the moment of a force about any point in its line of action is zero.* If now the portal be conceived to be cut by a vertical plane $x\ y$, this plane will pass through the acting members $B\ B'$, $b\ B'$, and $b\ b'$, each of which represents the line of action of an internal force. If we take moments of all the forces at the left of the plane about b, the intersection of $b\ B'$ and $b\ b'$, the bending moment thus obtained must be resisted entirely by the moment of the force in $B\ B'$ about that point, as the moments of the forces in $b\ B'$ and $b\ b'$ about b are zero. If then we divide the bending moment thus obtained by $B\ b$, the lever arm of the force $B\ B'$, the quotient will equal the internal force, or stress, in $B\ B'$. Likewise, if we take moments about B', the forces in $B\ B'$ and $b\ B'$ will have no rotative effect, and the bending moment thus obtained must be resisted entirely by the moment of the force in $b\ b'$; by dividing this bending moment by the lever arm $B'\ b'$, the quotient will equal the stress in $b\ b'$.

This principle is perfectly general, and may be stated as follows:

Wherever a truss can be conceived to be cut by a plane in such manner as to entirely divide it without cutting more than three members, the stress in any one of the members may be found by taking moments of all the forces upon either side *of the plane about the point of intersection of the other two members.*

By taking moments of the forces at the left of $x\,y$ about b, Fig. 299, the compression in $B\,B'$ is found to be $\frac{12{,}375 \times 18 + 6{,}806 \times 19 - 23{,}983 \times 0}{18} = 19{,}559$ lb. The same result may be obtained by taking moments of the forces at the right of $x\,y$ about the same point. In this case the negative sign of the result will simply signify that the resultant moment of the forces at the right of $x\,y$ tends to cause rotation to the left; it is plain that this will cause compression in $B\,B'$. The stress in $b\,b'$ is found by taking moments of the forces at the left of $x\,y$ about B'; thus, $\frac{6{,}806 \times 37 + 12{,}375 \times 0 - 23{,}983 \times 21}{18} = -13{,}990$ lb. Here, again, the negative sign of the result signifies that the resultant moment of the forces at the left of $x\,y$ tends to cause rotation to the left about B', which evidently produces compression in $b\,b'$. The stress in $b\,B'$ could be found by taking moments about the intersection of $B\,B'$ and $b\,b'$, if those members intersected, but they are parallel and do not intersect.

1368. It is evident that the vertical shear in the portal at the plane $x\,y$ is equal to either vertical reaction, negative on the left and positive on the right of the plane. As this shear is vertical, by laying it off upon a vertical line and projecting it upon a line drawn parallel to $b\,B'$, the projected length of the latter line will represent the tension in $b\,B'$. It will be convenient to lay off the vertical shear above b on $b\,B$ (produced if necessary). If $b\,v$ is laid off by any convenient scale equal to the vertical shear = 23,983 lb., then, by drawing the horizontal line $v\,s$, the line $b\,s$ thus cut off will represent the stress in $b\,B'$.

1369. It will be well to notice that the amount of wind pressure to be assumed, as applied directly at the portal, may, in certain cases, be more than one panel load. For instance, in the Baltimore truss, shown in Fig. 294, if lateral struts are placed only at *C*, *E*, *G*, *E'*, and *C'*, and each upper lateral tie extends longitudinally through two panels, as from *C* to *E*, the amount of wind pressure to be assumed as applied directly at *C* would be two panel loads. But if, with the lateral ties extending through two panels, the end post extends horizontally through but one panel, the amount of wind pressure to be assumed to be applied directly at the hip *B* would be one and one-half panel loads.

EXAMPLES FOR PRACTICE.

1. Find the stress in $b\,B'$, Fig. 299. Ans. – 36,850 lb.

2. Find the stress in $B\,B'$ by taking moments of the forces at the right of $x\,y$.

3. Find the stress in $b\,b'$ by taking moments of the forces at the right of $x\,y$.

FLOOR BEAMS WITH SIDEWALK CANTILEVERS.

1370. When, in addition to the roadway, a bridge carries sidewalks, the latter are usually supported outside of the truss upon projecting or cantilever ends of the floor beams, usually called **sidewalk brackets.** A floor beam having sidewalk brackets is shown in Fig. 300.

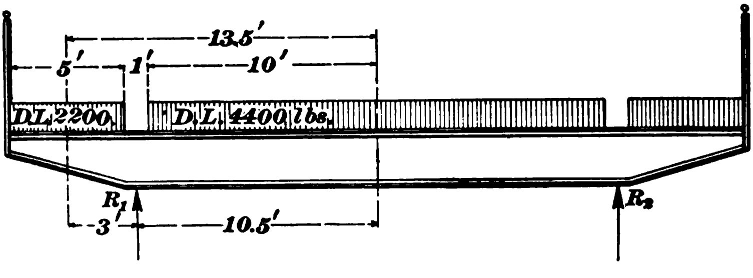


FIG. 300.

In estimating the dead load for a bridge having sidewalks, for the quantity *b*, in formulas **90** and **91,** Art. **1298,** use the clear width of roadway plus the width of *one* sidewalk. Also for *l* in formula **93,** Art. **1315,** use the distance between centers of supports plus the width of one sidewalk; but for W, in the same formula, use only the live load upon the beam *between supports.* In finding the maximum bending moment at the support and at the center of the beam, find first the bending moment at each point due to the entire dead load; then find the bending moment at the support caused by the live load upon the sidewalk cantilever, and the bending moment at the center caused by the live load upon the roadway *with no live load upon the sidewalks.* The live and dead load bending moments, as found for each point, should then be combined.

The live load bending moment at the center may be found as for a simple beam by formula **97**, Art. **1318.** The dead load moments may be found by constructing the moment diagram, or by computing the resultant moment of all the forces at the left.

EXAMPLE.—The floor beam of a bridge having 16-foot panels supports a 20-foot roadway and two 5-foot sidewalks. What is the weight of a white oak floor for the same (*a*) per lineal foot of bridge, and (*b*) per lineal foot of the loaded portions of the beam ?

SOLUTION.—(*a*) From Table **29**, Art. **1299**, for 16-foot panels the weight per lineal foot of a white oak floor 30 feet in width is $309.6 + 9 \times 45.6 = 720$ lb. Ans. (*b*) The total floor load upon the beam is $720 \times 16 = 11{,}520$ lb., and the weight of the same per loaded foot of beam is $\frac{11{,}520}{30} = 384$ lb. Ans.

EXAMPLE.—Assuming the live load to be 100 pounds per square foot, the width of each chord to be 1 foot, and the total depth of the beam to be 24.5 inches, what is (*a*) the total weight of the beam, and (*b*) the weight per lineal foot of beam ? (Assume the total weight of the beam to be distributed along its loaded portion.) (*c*) What is the total amount of dead load per lineal foot of beam upon the loaded portion ?

SOLUTION.—(*a*) The total live load upon the beam between supports equals $16 \times 20 \times 100 = 32{,}000$ lb. $= W$; and the length of beam between centers of supports plus the width of one sidewalk is $21 + 5 = 26 = l$. The total estimated weight of the beam is, therefore, $\frac{32{,}000 \times 26}{20 \times 24.5} = 1{,}698$ lb. Ans. (*b*) $\frac{1{,}698}{30} = 56.6$ lb. Ans. (See Ans. to (*b*) of preceding example.) (*c*) $384 + 56.6 = 440.6$. Ans.

EXAMPLE.—Calling the dead load 440 pounds per lineal foot of the loaded portion of the beam, what is the bending moment in inch-pounds due to the dead load (*a*) at the left support R_1, and (*b*) at the center of the beam?

SOLUTION.—See Fig. 300. (*a*) The total dead load upon the sidewalk cantilever is $5 \times 440 = 2,200$ lb. The distance between the center of gravity of this load and R_1 is $2.5 + .5 = 3$ ft. It will be remembered that the bending moment of a cantilever is negative. Therefore, the bending moment at R_1 is $-(2,200 \times 3) = -6,600$ ft.-lb. $= -6,600 \times 12 = -79,200$ in.-lb. Ans. (*b*) The amount of load supported by R_1 is $2,200 + 10 \times 440 = 6,600$ lb. The resultant bending moment at the center is $6,600 \times 10.5 - 2,200 \times 13.5 - 4,400 \times 5 = 69,300 - 29,700 - 22,000 = 17,600$ ft.-lb. $= 17,600 \times 12 = 211,200$ in.-lb. Ans.

EXAMPLE.—What is the bending moment in inch-pounds produced by the live load (*a*) at the left support R_1, and (*b*) at the center of the beam?

SOLUTION.—(*a*) The total live load upon the sidewalk cantilever is $5 \times 16 \times 100 = 8,000$ lb., and its moment about R_1 is $8,000 \times 3 = 24,000$ ft.-lb. $= 24,000 \times 12 = 288,000$ in.-lb. Ans. (*b*) By formula **97**, of Art. **1318**, the bending moment at the center produced by the live load upon the roadway, with no live load upon the sidewalks, is $\frac{32,000 \times (20 + 2)}{8} = 88,000$ ft.-lb. $= 88,000 \times 12 = 1,056,000$ in.-lb. Ans.

EXAMPLE.—What is the total bending moment (*a*) at R_1 and (*b*) at the center of the beam?

SOLUTION.—(*a*) The total bending moment at R_1 is $79,200 + 288,000 = 367,200$ in.-lb. Ans. (*b*) The total bending moment at the center of the beam is $211,200 + 1,056,000 = 1,267,200$ in.-lb. Ans.

EXAMPLES FOR PRACTICE.

1. A bridge having 15-foot panels carries one 16-foot roadway and two 4-foot sidewalks. What is the weight of a white oak floor (*a*) per lineal foot of bridge, and (*b*) per lineal foot of the loaded portion of the beam?

Ans. { (*a*) 552 lb.
(*b*) 345 lb.

2. Assuming the total depth of the beam to be 21 inches, the width of each chord to be 1 foot, and the live load upon the beam to be 80 pounds per square foot, what is (*a*) the total weight of the beam, and (*b*) the weight per lineal foot of the loaded portion?

Ans. { (*a*) 960 lb.
(*b*) 40 lb.

3. What is the bending moment in inch-pounds produced by the dead load (*a*) at the support, and (*b*) at the center of the beam?

Ans. { (*a*) 46,200 in.-lb.
(*b*) 120,120 in.-lb.

4. What is the bending moment in inch-pounds produced by the live load (*a*) at the support, and (*b*) at the center of the beam?

Ans. { (*a*) 144,000 in.-lb.
(*b*) 518,400 in.-lb.

5. What is the total bending moment in inch-pounds in the floor beam of the four preceding examples (*a*) at either support, and (*b*) at the center?

Ans. { (*a*) 190,200 in.-lb.
(*b*) 626,520 in.-lb.

CONCENTRATED WHEEL LOADS.

1371. The floor members of highway bridges constructed in cities and manufacturing districts are generally designed to carry not only the specified uniform load but also certain specified concentrated or wheel loads, such as an electric car, a steam road roller, etc. In such bridges, metal stringers are commonly used. The bending moment at any point along a stringer or beam caused by the uniform (live and dead) load may be obtained by applying formula **95,** Art. **1317**; the bending moment due to the concentrated load is then found and combined with the preceding, in order to obtain the resultant bending moment. For stringers and beams of uniform cross-section, it is necessary to find the *maximum* bending moment only. The method of obtaining the maximum bending moment produced by concentrated loads will now be explained.

A single load upon the span will first be considered. In Fig. 301 is represented a single load W in any position upon any span l. By Art. **1143,** $R_1 = \frac{Wb}{l}$ and $R_2 = \frac{Wa}{l}$. For any position of the load, the greatest bending moment is under the load, and its value is

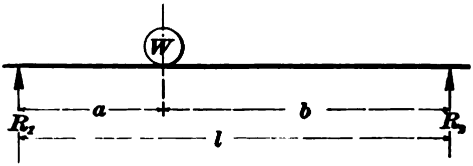


FIG. 301.

$$M = R_1 a = R_2 b = \frac{W a b}{l}. \qquad \textbf{(105.)}$$

The position of W that produces the greatest bending moment is at the center of the beam. In that case, $a = b = \frac{l}{2}$; $a b = \frac{l^2}{4}$; and formula **105** becomes

$$M = \frac{W l^2}{4 l} = \frac{W l}{4}. \qquad \textbf{(106.)}$$

NOTE.—By comparing formula **105** with formula **94**, Art. **1316**, it will be noticed that a single load concentrated at the center of the span will produce just double the bending moment at the center of the span that is produced at that point by the same amount of load uniformly distributed upon the same span.

1372. If two or more concentrated loads are situated in any position upon a stringer, the bending moment at each point along the stringer can readily be obtained by drawing the moment diagram. By constructing the shear diagram also, the point of maximum bending moment will be located by the point where the shear line crosses the shear axis. Or the bending moment at any point may be computed by simply finding the resultant moment of the forces acting on either side of that point.

The maximum bending moment in a simple beam, produced by any system of quiescent loads, occurs at a point where the sum of the loads at the left (including, if necessary, a portion of the load directly at the point) equals the left reaction; or, in other words, where the shear is zero.

1373. But the system of wheel loads under ordinary conditions of traffic, as an electric railway car, a road roller or heavily loaded truck, comes upon one end of the stringer, and, proceeding along its entire length, passes off at the opposite end, producing bending moments of varying intensity, which at some point during the passage of the loads become maximum. To find where this maximum bending moment occurs, and the position of the system of loads producing it, the two following principles must be observed:

I. The maximum bending moment will occur under a load.

II. The center of the span will be midway between the point of maximum bending moment and the center of gravity of the system of loads.

If all the loads belonging to the system are not upon the span at the same time, the center of gravity of those loads which are upon the span must be taken. When the position of the loads producing the maximum bending moment is found, the bending moment can be obtained either by calculation or by drawing the moment diagram.

In the case of two unequal loads upon a stringer (or any other simple beam), the maximum bending moment will occur under the heavier load. Let R_1 be the reaction on the same side as that on which the heavier load rests, W_1 the weight of this load, W_2 the weight of the smaller load, a the distance between the loads, and l the length of the stringer (or beam). Thus the distance x from R_1 to W_1 is given by the formula

$$x = \frac{l}{2} - \frac{a}{2\left(1 + \frac{W_1}{W_2}\right)}. \qquad \textbf{(107.)}$$

If $W_1 = W_2$, then

$$x = \frac{l}{2} - \frac{a}{4}. \qquad \textbf{(108.)}$$

In the latter case it will be noticed that *the position of the loads is independent of those loads;* that is, whatever the loads may be, if they are equal, the *position* for maximum bending moment is always the same, so long as the length of the beam and the distance between the loads remain unchanged.

EXAMPLE.—It is assumed that the floor system of the bridge truss, shown in Figs. 297 and 298, carries an electric railway track, and that stringers are so arranged that each track rail is supported directly on top of a stringer. The car and its entire load, assumed to weigh 16 tons, is supported upon four wheels, each of which is assumed to carry one-fourth of the load, or 8,000 pounds· the two wheels upon either track are 7.5 feet apart, center to center. What is the position of the wheels upon the stringer which will produce the maximum bending moment?

Solution.—The span of the stringer is equal to the panel length of the truss, or 16.5 feet. Here, $l = 16.5$, $a = 7.5$, and $W_1 = W_2 = 8{,}000$ lb. Then, formula **108** gives $x = \frac{16.5}{2} - \frac{7.5}{4} = 6.375$ ft. Distance of W_1 from center $= 7.5 - 6.375 = 1.875$, which is $= \frac{1}{2} \times \frac{7.5}{2}$, or one-half the distance between W_1 and the center of gravity of the system, as stated in II. See Fig. 302.

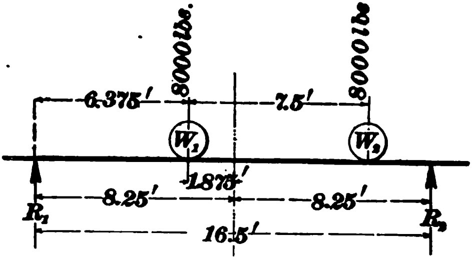


Fig. 302.

1374. With three loads upon the span, if the loads are equal, or if the center load is the greatest, the maximum bending moment will occur under the center load. If the two outer loads are equal and are at equal distances from the center load, the maximum bending moment will occur when the center load is at the center of the span.

As the number of loads increases, the problem becomes more complicated, especially if the magnitudes of the loads vary. It may always be solved, however, by applying the above conditions to the several wheels, the maximum bending moment being usually found under one of the heavier loads near the center of the system. In case of a stringer or floor beam of uniform cross section it is necessary to obtain only the one absolutely maximum moment; in such a case little, if any, difficulty will be experienced.

It must be noticed that formula **107** (and the same applies to **108**) gives the position for maximum bending moment when the two loads *are actually upon the span;* but, when a is greater than $\frac{l}{2}$, it may happen that this bending moment is less than $\frac{W_1 l}{4}$, or the center bending moment with only W_1 upon the span. In such a case, both moments should be calculated to see which is the real maximum.

Furthermore, the value of x given by formula **107** may be so great as to make it impossible for W_2 to be upon the span when W_1 is at the distance x from R_1. This happens whenever a is greater than $l\left(1 - \frac{1}{2 + \frac{W_2}{W_1}}\right)$. In this case, the center bending moment $\frac{W_1 l}{4}$ is to be taken as the maximum.

EXAMPLES FOR PRACTICE.

1. With the loads of the above example in the positions shown in Fig. 302, what is the value of R_1? Ans. 6,182 lb.

2. What is the maximum bending moment produced by the same? Ans. 39,410 ft.-lb.

3. With the loads in the same position, find the bending moment under the right wheel W_2. Ans. 25,773 ft.-lb.

4. Assuming the span of the stringer to be 20 feet, what is the position of the same loads producing maximum bending moment?

Ans. With either load 8.125 feet from the corresponding end of the stringer.

5. What is the maximum bending moment for the conditions of the preceding example? Ans. 52,813 ft.-lb.

6. Assuming the span of a stringer to be 18 feet, the two loads upon it to be 9,000 and 6,000 pounds, respectively, and the distance between the loads to be 11 feet, find the position at which the maximum bending moment will occur, both wheels being upon the span.

Ans. Under the heavier load at a distance of 6.8 feet from the nearer end, or 2.2 feet from the center of the span.

7. For the conditions of the preceding example, what is the maximum bending moment? Ans. 38,533 ft.-lb.

8. With the same span and loads as in the two preceding examples, and with the heavier load at the center of the span, what is the maximum or center bending moment? Ans. 40,500 ft.-lb.

9. With the same loads, but with a span 24 feet in length, what is the bending moment with the heavier load at the center of the span? Ans. 57,000 ft.-lb.

10. With the same span and loads as the preceding example, what is the maximum bending moment? Ans. 60,025 ft.-lb.

CONCLUDING REMARKS.

1375. There are several forms of bridge trusses which have not been noticed in the preceding pages. The forms of trusses for which the stress diagrams have been explained are those modern forms of simple trusses most commonly constructed in this country at the present time; they are thought to be representative and typical of approved modern practice.

Other methods are employed for obtaining the stresses in framed structures; the method which has been explained and generally followed in the preceding pages is believed to be the most systematic, flexible, and accurate, as well as the most clear and popular graphical method. The principles on which the method is founded are general, and the student will have no difficulty in applying them to any ordinary truss.

The examples that have been explained have been given solely for the purpose of illustrating the application of general principles and the methods to be employed. The design of a bridge must, of course, depend largely upon the physical conditions peculiar to the location and other special circumstances; so that, in order to obtain the highest possible degree of economy and efficiency in each case, it is usually necessary to treat each design separately and independently, especially if the bridge be one of considerable length of span. In doing this the designer must rely upon his knowledge of the underlying principles, and upon his own judgment and experience.

PROPORTIONING THE MATERIAL.

THE MATERIALS USED FOR SUPERSTRUCTURES OF BRIDGES.

1376. The materials commonly used in the construction of the superstructures of bridges are **structural steel, wrought iron,** and **wood.**

Owing to the increasing scarcity of suitable timber and to the cheapness of iron and steel, together with the fact that metal bridges are more durable than those constructed of wood, the latter material is now little used for bridge trusses.

In this country bridges were never constructed entirely of cast iron, although at one time this material was used to a considerable extent for compression members. On account of the unreliable character of this metal, however, it is no longer employed for the superstructure of fixed bridges, except for such purposes as bed-plates and small and unimportant details.

1377. A few years ago wrought iron had become the material used exclusively for the main parts of metal bridges; but, although it is still used to some extent, it is being largely superseded by structural steels, which are somewhat stronger and better materials than wrought iron, and cost about the same. The name *structural steels* is applied to these materials in order to distinguish them from other kinds of steel, such as cutlery steel and tool steel, which are very different. The use of structural steels is becoming more and more general, and the time is probably not far distant when wrought iron will no longer be used for

For notice of copyright, see page immediately following the title page.

structural purposes. This is due to two reasons: First, the manufacture of steel is being constantly perfected and cheapened. Second, less attention is being given to the manufacture of wrought iron, and on this account it is not possible to obtain as good qualities of this material as formerly. Structural steels are without doubt the best of known materials for bridges.

1378. When steel was first used for this purpose its properties were not thoroughly understood, and it was not as judiciously used as at present. The mistake of using grades of steel containing too high percentages of carbon was quite commonly made. Steel high in carbon possesses high ultimate strength and elastic limit, but is hard and brittle, and not uniform; it does not weld readily, and is very liable to injury in forging or punching. In the earlier use of structural steels, the impression that all steel possessed these unreliable properties became common; the brittleness of steel was also believed to be to some extent due to the fact that it is not fibrous. All this gave rise to considerable prejudice against the use of steel for structural purposes, and by many engineers it was considered inferior to wrought iron.

But from further experience in the use of steel and more accurate knowledge concerning its properties, it is known that steel low in carbon and reasonably free from certain injurious elements, noticeably sulphur and phosphorus, can be produced that is exceedingly tough and ductile, and in most respects superior to wrought iron. It has also been satisfactorily ascertained that the fiber existing in wrought iron is simply a circumstance of its manufacture rather than any inherent property of the material itself; that it does not make the iron materially stronger in the direction of the fiber, but merely weaker in a direction across the fiber. Steel, having no fiber, is of nearly uniform strength in all directions. Knowledge of these facts has largely removed the objections to the use of steel, and to-day this material is very extensively employed.

Structural steels are produced by both the Bessemer and the Open Hearth processes. The latter process is generally preferred, as it is believed to give better results.

QUALITY OF THE MATERIALS.

1379. Wrought Iron.—Specifications for the quality of the wrought iron used in bridges commonly require an elastic limit of not less than 26,000 pounds per square inch for all grades, and an ultimate tensile strength varying from 50,000 pounds per square inch, for bars of not more than 4½ square inches of sectional area, down to 48,000 pounds per square inch, for bars of 8½ square inches of sectional area, or for larger bars, for shaped iron, and for plates from 8 to 24 inches in width; and for wider plates, 46,000 pounds per square inch.

1380. They also require that the specimen pieces tested shall before rupture elongate, in a length of 8 inches, 18 per cent., if from bars of not more than 4½ square inches section; 15 per cent., if from larger bars, shaped iron, or plates less than 24 inches wide; 10 per cent., if from plates more than 24 inches and less than 36 inches wide; and 8 per cent., if from plates more than 36 inches wide.

1381. It is usually required that the tensile strength, limit of elasticity, and ductility shall be determined from a standard test piece of as near ½ square inch sectional area as possible. The elongation shall be measured on an original length of 8 inches.

1382. The following cold bending tests are also required: All iron for tension members and specimens from shaped iron must bend cold, without cracking, through 90° to a curve whose diameter is not more than twice the thickness of the piece. Samples from plate iron must bend cold, without cracking, through 90° to a curve whose diameter is not more than three times the thickness of the piece. Rivet iron must bend cold through 180° to a curve whose diameter is equal to the thickness of the rod tested, without sign of fracture on the convex side. When nicked and bent cold, all iron must show a fracture mostly fibrous.

1383. All iron must be tough, ductile, fibrous, and of uniform quality. Finished bars must be thoroughly welded during rolling, and must be straight, smooth, and free from injurious seams, blisters, buckles, cracks, or imperfect edges.

1384. Although the conditions of some specifications are rather more rigid, the above conditions represent a good quality of iron and contain no excessive requirements. It will be noticed that the elastic limit specified is slightly more than one-half the ultimate strength. This is commonly found to be the case for wrought iron.

1385. In some specifications the ultimate tensile strength per square inch required for bar iron is written

$$52{,}000 - \frac{7{,}000 \times \textit{area of original bar}}{\textit{circumference of original bar}} \text{ (all in inches).}$$

1386. The modulus, or coefficient, of elasticity of a good quality of double refined bar iron, as determined from tests made on finished eye-bars, is from 25,000,000 to 27,000,000 pounds per square inch. It often falls much below these amounts, and often runs as high as 32,000,000; it is usually taken at 27,000,000.

1387. Steel.—As commonly manufactured, there are three grades of structural steel, namely, **soft steel,** called also **mild steel; medium steel,** and **high steel,** called also **hard steel.** The following requirements are from the "Specifications for Constructional Steel," published by the Carnegie Steel Company, Limited. They plainly indicate the quality of each grade of steel as commonly specified, and the properties specified for each grade may be considered as fairly representative of that grade:

1388. "*Soft Steel.*—Specimens from finished material for tests shall have an ultimate strength of from 54,000 to 62,000 pounds per square inch; an elastic limit of one-half the ultimate strength; a minimum elongation of 26 per cent.

in 8 inches, and a minimum reduction of area at fracture of 50 per cent. This grade of steel to bend cold 180° flat on itself, without sign of fracture on the outside of the bent portion.

1389. "*Medium Steel.*—Specimens from finished material for tests shall have an ultimate strength of from 60,000 to 68,000 pounds per square inch; an elastic limit of one-half the ultimate strength; a minimum elongation of 20 per cent. in 8 inches, and a minimum reduction of area at fracture of 40 per cent. This grade of steel to bend cold 180° to a diameter equal to the thickness of the piece tested, without crack or flaw on the outside of the bent portion.

1390. "*High Steel.*—Specimens from finished material for test shall have an ultimate strength of from 66,000 to 74,000 pounds per square inch; an elastic limit of one-half the ultimate strength; a minimum elongation of 18 per cent. in 8 inches; a minimum reduction of area at fracture of 35 per cent. This grade of steel to bend cold 180° to a diameter equal to three times the thickness of the test piece, without crack or flaw on the outside of the bent portion."

1391. It will be noticed that no definite line of distinction exists between the three grades, but that the different grades blend into each other. Many specifications now allow a variation of 10,000 pounds per square inch in the ultimate strength of each grade.

1392. It will also be noticed that for each grade of steel the percentage of elongation specified is practically one-half the specified percentage of reduction of area at fracture, and that the elastic limit specified is one-half the ultimate strength. This requirement for the elastic limit is easily fulfilled. Recent investigations have shown that structural steel having an elastic limit considerably greater than one-half its ultimate strength can readily be obtained.

1393. The modulus, or coefficient, of elasticity of structural steel does not differ greatly from that of wrought iron; 29,000,000 pounds per square inch is probably a fair average value of this modulus for both soft and medium steel, though it is sometimes taken at 28,000,000 pounds per square inch.

1394. On account of its extreme toughness and ductility, soft steel is especially suitable for rivets, and is largely used for this purpose, although it is also very suitable for the riveted members of bridges for which it is now much used.

Medium steel is the material now most commonly used in the construction of the main members of bridges.

High steel is somewhat brittle, and is not very extensively used in bridge construction.

Intermediate and extreme grades of steel are sometimes specified, as **medium soft, medium hard, very soft,** or **very hard.**

THE USE OF WROUGHT IRON.

FACTORS OF SAFETY AND UNIT STRESSES.

1395. The nature and use of the **factor of safety** have been explained in Arts. **1214** to **1216.** At one time it was not an uncommon practice to construct bridges with certain factors of safety, based upon the ultimate strength of the material; it was customary to use a factor of safety of 4 for highway bridges and 5 for railroad bridges. This practice is still followed to a limited extent, but the more modern and now quite general practice is to use certain allowed stresses, called **working stresses** or **unit stresses,** whose values vary according to circumstances. The unit stresses allowed for members which receive their maximum stresses often or suddenly, such as web members, and especially counters, are less than those allowed for members which receive their maximum stresses seldom or gradually, such as chords.

SPECIFICATIONS AND HAND-BOOKS.

1396. In designing a bridge, the values given to the unit stresses are usually in accordance with certain specifications which are accepted as representing reliable engineering practice. Several such specifications for highway bridges have been published by prominent American bridge engineers, notably G. Bouscaren, Theodore Cooper, Edwin Thacher, and J. A. L. Waddell.

The bridge specifications most widely known, and which have obtained most popular recognition in this country, are those of Mr. Theodore Cooper, a prominent bridge engineer of New York City. These specifications may be said to have become a standard for American bridge practice.

1397. The stresses for the various members of the bridge shown in detail in Mechanical Drawing Plates, Titles: Highway Bridge: Details I, II, III, IV, and Highway Bridge: General Drawing, were obtained in Arts. **1292** and following, and are shown written along the several members upon the stress sheet in Fig. 283.

The student should, for his own convenience, make a copy of this stress sheet upon a sheet of paper, cap size, designating the joints by the system of notation shown in Fig. 76 of Mechanical Drawing. This need not be a neat copy, but simply a working copy, and is as well made in pencil.

The material for this bridge will now be proportioned of wrought iron, according to Cooper's "General Specifications for Highway Bridges," the necessary portions of which will be quoted in Art. **1399.**

NOTE.—As a copy of these specifications may be obtained for twenty-five cents, the student will find it to his advantage to provide himself with one.

In bridge designing, one of the valuable hand-books of structural shapes published by several of the large iron and steel manufacturing companies will be found of material assistance. Such a book gives the sizes, weights, sectional areas, and various properties of the structural shapes manufactured by the company by which it is published, besides convenient tables and other valuable information. In practice, a structural hand-book is found indispensable. These hand-books may be obtained from the various iron and steel manufacturers at a small cost.

UNIT STRESSES ALLOWED FOR TENSION MEMBERS.

1398. The stresses are found in the various members of a bridge simply for the purpose of ascertaining the amount of strength required for each member, which is given such size and form as to best resist its stress.

The material for tension members is more easily proportioned than for those which bear compressive stress; therefore, in the example chosen, the tension members will be first considered.

1399. *The tensile stresses for wrought iron, in pounds per square inch of sectional area, allowed by Cooper's General Specifications for Highway Bridges, are as follows:*

		Pounds per Square Inch.
(*a*)	"On lateral bracing	15,000
(*b*)	"On solid rolled beams, used as cross floor beams and stringers	12,000
(*c*)	"On bottom flange of riveted cross-girders, net section	12,000
(*d*)	"On bottom flange of riveted floor stringers, net section	12,000
(*e*)	"On floor beam hangers, and other similar members liable to sudden loading (bar iron with forged ends)	9,000
(*f*)	"On floor beam hangers, and other similar members liable to sudden loading (plates or shapes), net section	7,000

		For Live Loads.	For Dead Loads.
(*g*)	"Bottom chords, main diagonals, counters, and long verticals (forged eye-bars)	10,000	20,000

	For Live Loads.	For Dead Loads.
(*h*) "Bottom chords and flanges, main diagonals, counters and long verticals (plates or shapes), net section	9,250	18,500"

The following clauses from the specifications also have direct reference to the tension members. As is not uncommon, the word *strain* is used in these specifications in the sense of *stress.*

(*i*) "The *areas* obtained by dividing the live load strains by the live load unit strains will be added to the areas obtained by dividing the dead load strains by the dead load unit strains, to determine the required sectional area of any member."

(*j*) "*Single angles subject to direct tension* must be connected by both legs, or the section of one leg only will be considered as effective."

(*k*) *Net Section.*—"In members subject to tensile strains, full allowance shall be made for reduction of section by rivet holes, screw threads, etc.

"In deducting the rivet holes to obtain net sections in tension members, the diameter of the rivet hole will be assumed as ⅛ inch larger than the undriven rivets."

(*l*) *Effect of Wind on Chords and End Posts.*—"The strains in the chords and end posts from the assumed wind forces need not be considered, except as follows:

"1st. When the wind strains per square inch on any member exceed one-quarter of the maximum strains per square inch due to the dead and live loads upon the same member. The section shall then be increased until the total strain per square inch shall not exceed by more than one-quarter the maximum fixed for dead and live loads only.

"2d. When the wind strain alone, or in combination with a possible temperature strain, can neutralize or reverse the tension in any part of the lower chord, from dead load only."

(*m*) *Lateral Rods.*—"In no case shall any lateral or diagonal rod have a less area than ¾ of a square inch."

Each of the above items, quoted from the specifications, is here designated by a letter for convenience of reference. It will be noticed that in items (*a*) to (*f*), inclusive, the unit stresses are for the combined live and dead load stresses, while in items (*g*) and (*h*) the stresses per square inch allowed for the dead load are just double those allowed for the live load. As the trusses for the example are pin-connected, the tension members will be of bar iron. The tension members of this structure will be proportioned by items (*a*), (*c*), (*e*), and (*g*).

1400. If P_l is the total live load stress in pounds;
P_d is the total dead load stress in pounds;
S_l is the live load unit stress in pounds per square inch;
S_d is the dead load unit stress in pounds per square inch, and
A is the total area of the cross-section in square inches;
then, item (*i*) above may be concisely expressed by the formula

$$A = \frac{P_l}{S_l} + \frac{P_d}{S_d}. \qquad \textbf{(109.)}$$

Since, according to items (*g*) and (*h*), the allowed dead load unit stress is double the allowed live load unit stress, we have $S_l = \frac{1}{2} S_d$, and formula **109** may be written

$$A = \frac{P_l}{S_l} + \frac{P_d}{2 S_l} = \frac{P_l + \frac{1}{2} P_d}{S_l}. \qquad \textbf{(109}a.\textbf{)}$$

For clearness, the operations indicated by formula **109** will here be followed. The operations indicated by formula **109***a* are, however, slightly shorter, and the student may employ them if he chooses.

1401. Members of a pin-connected bridge, which are designed to resist tensile stresses only, are commonly made of flat, square, or round bars, and are usually placed in pairs. Main tie bars and lower chord bars are usually flat bars in pairs; hip vertical bars are usually flat or square bars, but are sometimes round bars. In order that they may be easily fitted with turnbuckles, counters are usually made of square or round bars (preferably square), and are in pairs or single, while lateral rods are usually single round rods.

In trusses having riveted connections, the tension members, as well as the compression members, are formed of plates or shape iron, or of the two combined.

PROPORTIONING THE MATERIAL FOR TENSION MEMBERS.

1402. According to item (*i*) of Art. **1399,** the sectional area for the main tie, required by item (*g*) (see Fig. 283), is $\frac{27,500}{10,000} + \frac{9,800}{20,000} = 3.24$ sq. in. The section is made up of two bars $2'' \times \frac{13}{16}'' = 3.25$ sq. in. As the material composing the required section is determined for each member, the student should note it upon his copy of the stress sheet; it should be marked along the member in the *right* half of the diagram of the truss. According to the same items of the Specifications, the sectional area required for the hip vertical Bb is $\frac{16,200}{10,000} + \frac{4,600}{20,000} = 1.85$ sq. in. It is made up of two bars, each 1 inch square, making a total area of 2 sq. in.

1403. The sectional area required for the counter Cc' is $\frac{13,750}{10,000} = 1.38$ sq. in. It is desirable that the diagonal members in the center panel should each be made up of two bars, rather than one bar. As item (*m*) specifies that no diagonal rod shall have a less area than $\frac{3}{4}$ square inch, which is practically to the effect that no diagonal rod shall be less than $\frac{7}{8}$ inch square (= .766 sq. in.) or 1 inch round (= .785 sq. in.), the section of the counter Cc' is made up of two bars $\frac{7}{8}$ inch square $= 1.53$ sq. in. The sectional area required by the live and dead load stresses in the panel *a b* or *b c* of the lower chord is $\frac{32,400}{10,000} + \frac{13,900}{20,000} =$ 3.94 sq. in.

1404. In highway bridges the lower chords are often proportioned with reference to the live and dead load stresses only; if this were done in the present case, the section could be made up of two bars $2\frac{1}{2}'' \times \frac{13}{16}'' = 4.06$ sq. in. Likewise, for the center panel *c c'* of the lower chord, the sectional area required by the live and dead load stresses is $\frac{48,600}{10,000} + \frac{20,800}{20,000} = 5.90$ sq. in. Without reference to any other stresses, the section could be made up of two bars $4' \times \frac{3}{4}' = 6$ sq. in. But the effect of the wind stresses upon the chords and posts should always be considered, especially in the case of a short and light structure. In order to comply with the requirements of item (*l*), it will be found necessary to considerably modify the section of the panels *a b* and *b c* of the lower chord. But this will be taken up further along, after the design of the compression members has been considered.

1405. The sectional area required for the diagonal rod in the end panel of the lower lateral system, item (*a*), is $\frac{14,900}{15,000} =$ .99 sq. in. The section is made up by a round rod $1\frac{1}{8}''$ in diameter $= (1\frac{1}{8})^2 \times .7854 = .99$ sq. in. The area of section for the diagonal in the second panel of the lower lateral system, as required by the stress, is $\frac{8,200}{15,000} = .55$ sq. in. In compliance with item (*m*), a round rod 1" in diameter $=$ .79 sq. in. is used for this member. The area of section for the diagonal rod in the middle panel of the lower lateral system, as required by the stress, is $\frac{2,200}{15,000} = .15$ sq. in. A round rod 1" in diameter is used for this member also. For the diagonal rod in the end panel of the upper lateral system, the section required by the stress is $\frac{3,700}{15,000} = .25$ sq. in. No stress was obtained for either diagonal in the center panel of the upper lateral system. In compliance with item *m*, each diagonal rod in the upper lateral system will be a round rod 1″ in diameter.

NOTE.—Having obtained the sectional area required for a tension member, the dimensions of the bars giving the required area are quickly determined from a table of areas; such tables are given in structural hand-books.

1406. It will be noticed that, in proportioning the material for the tension members, the amount of stress and the allowed unit stress are the only quantities considered. Bars for tension members should be of sizes commonly rolled and suitable for the practical requirements of the shop work; otherwise (with one exception, which will be hereafter noticed), the form of a tension member is unimportant.

COMPRESSION MEMBERS. RADII OF GYRATION.

1407. The form of a member which resists compressive stress bears a very important relation to its strength. Failure of a strut or column is not usually the result of direct crushing, but of bending or buckling. The capacity of a column to resist compression depends upon its length and form, as well as upon the area of its section. Various forms used for compression members are shown in Fig. 303. Of these, forms *A*, *B*, *C*, *D*, and *E* are used for top chords and end posts; forms *F*, *G*, *H*, *I*, *K*, and *L* are used for intermediate posts; although forms *F* and *H* are also sometimes used for top chords and end posts; forms *M*, *N*, *O*, and *P* are sometimes used for lateral struts. Forms *G* and *I* are seldom used on account of the difficulty of riveting; as one head of each rivet is within the assembled piece, it is not easily accessible for machine riveting. Form *E* is used for very heavy chord and end post sections. Forms *A*, *B*, *C*, *D*, and *E* are very economical.

1408. In Arts. **1240,** and following, the nature and derivation of the **moment of inertia** and the **radius of gyration** were explained. In the accepted formulas for

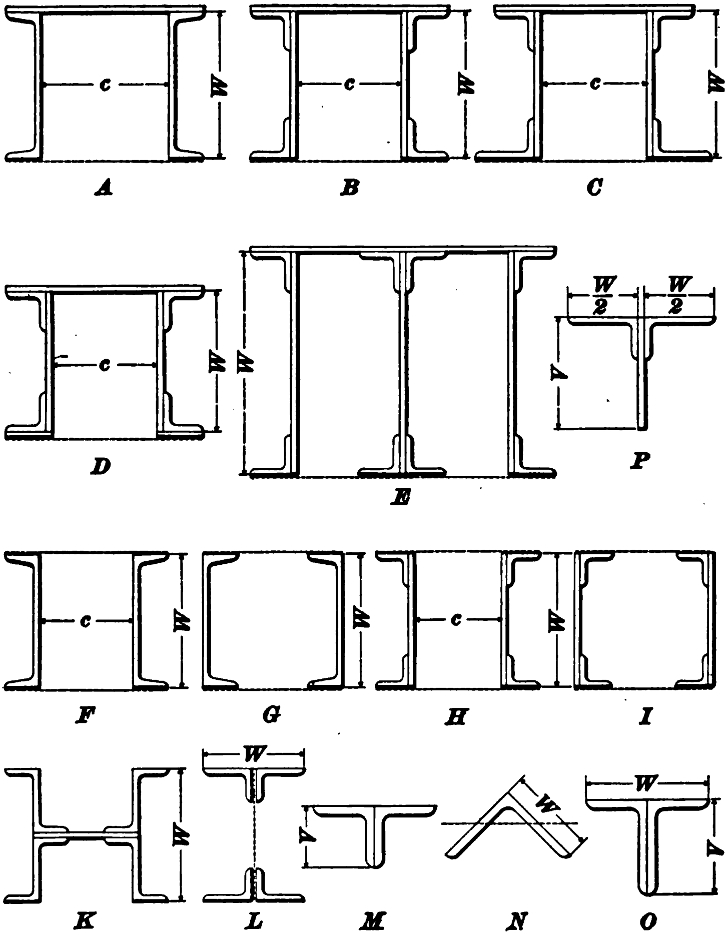


FIG. 303.

determining the required area for a strut of given external dimensions, the least radius of gyration of the section is the only variable quantity, when the length of the strut and the character of its end connections (i. e., whether flat-ended or pin-ended) are constant.

The radii of gyration of the various structural shapes commonly rolled may be found in any of the manufacturers' hand-books previously mentioned, and, although the bridge designer is constantly making use of the radius of gyration, he seldom finds it necessary to compute it. The exact formulas for determining the radii of gyration of the sections shown in Fig. 303 are often of a complicated character. The following are approximate formulas, which may be used where great accuracy is not required, and also for finding trial values; r is the least radius of gyration, and W the depth of the section, as shown in the figure.

For forms A, B, C, D, and E,

$$r = \frac{4}{10} W. \qquad \textbf{(110.)}$$

For forms F, G, H, and I,

$$r = \frac{3}{8} W. \qquad \textbf{(111.)}$$

For form K,

$$r = \frac{5}{16} W. \qquad \textbf{(112.)}$$

For form L,

$$r = \frac{24}{100} W. \qquad \textbf{(113.)}$$

For form M (equal legs),

$$r = \frac{31}{100} V. \qquad \textbf{(114.)}$$

For form N (equal legs),

$$r = \frac{1}{5} W. \qquad \textbf{(115.)}$$

For form *O* (longer legs back to back),

$$r_w = \frac{1}{5} W. \qquad \textbf{(116.)}$$

$$r_h = \frac{32}{100} V. \qquad \textbf{(117.)}$$

For form *P*,

$$r = \frac{1}{5} W. \qquad \textbf{(118.)}$$

These formulas are very convenient to use, and obviate the constant reference to structural hand-books. Although in reality they are only approximate formulas, yet the values for the radius of gyration given by most of them are believed to be sufficiently close to the correct values for many practical purposes. The results obtained by formulas **110, 111, 112, 114,** and **115** are quite reasonably accurate; those given by **113, 116,** and **117** are somewhat less accurate, while results obtained by **118** are but liberal approximations. On account of their convenience, some engineers use values for the radius of gyration obtained by such formulas in preference to the exact values. It is, however, better to take the correct values given in the hand-books.

FORMULAS FOR COMPRESSION.

1409. Formula **76,** of Art. **1256,** is

$$W = \frac{S_2 A}{f\left(1 + \frac{A l^2}{g I}\right)},$$

in which *W* is the load upon the column, S_2 is the ultimate strength of the material in pounds per square inch, *A* is the sectional area of the column in square inches, *f* is a factor of safety, *l* is the length of the column in inches, *I* is the least moment of inertia of the cross-section, and *g* is a constant.

As $I = A r^2$ (Art. **1241**) this formula may be written,

$$W = \frac{S_1 A}{f\left(1 + \frac{A l^2}{g A r^2}\right)} = \frac{S_1 A}{f\left(1 + \frac{l^2}{g r^2}\right)}.$$

Omitting the factor safety f and dividing by the area A, the formula for the *ultimate strength per square inch* of a column becomes

$$\frac{W}{A} = \frac{S_1}{1 + \frac{l^2}{g r^2}}, \qquad \textbf{(119.)}$$

which corresponds to the form in which this formula for columns is commonly written. In this form it is known as **Rankine's modification of Gordon's formula,** or simply as **Rankine's formula.** In Bridge Engineering the following forms of Rankine's formula are generally used:

(120.)	**(121.)**	**(122.)**
For two square bearings:	For pin and square bearings:	For two pin bearings.
$\frac{40{,}000}{1 + \frac{l^2}{36{,}000\, r^2}}$	$\frac{40{,}000}{1 + \frac{l^2}{24{,}000\, r^2}}$	$\frac{40{,}000}{1 + \frac{l^2}{18{,}000\, r^2}}$

in which l is the length of the column and r is the least radius of gyration, both in inches.

These formulas have been extensively used for the ultimate strength of wrought-iron columns or struts, the results given being usually divided by a factor of safety of either 4 or 5 for a unit stress or safe working stress per square inch. They are of the kind known as **curve formulas,** because, as they contain the squares of l and r, if the results are plotted for various consecutive values of those quantities, a curve will be obtained. Of recent years, however, some simpler formulas, known as **straight-line formulas,** have come into quite general use. They are so called because, as they contain only the first powers of l and r, the results of any one formula, if plotted, will form a straight line.

1410. *For wrought iron the compressive stresses per square inch of section, allowed by Cooper's General Specifications for Highway Bridges, are given by the following straight-line formulas:*

(*n*) *Chord Segments.*

For live load strains:

$$P = 10{,}000 - 40\frac{l}{r}. \qquad \textbf{(123.)}$$

For dead load strains:

$$P = 20{,}000 - 80\frac{l}{r}. \qquad \textbf{(124.)}$$

(*o*) *All Posts.*

For live load strains:

$$P = 8{,}750 - 50\frac{l}{r}. \qquad \textbf{(125.)}$$

For dead load strains:

$$P = 17{,}500 - 100\frac{l}{r}. \qquad \textbf{(126.)}$$

For wind strains:

$$P = 13{,}000 - 75\frac{l}{r}. \qquad \textbf{(127.)}$$

(*p*) *Lateral Struts.*

For assumed initial strain:

$$P = 11{,}000 - 60\frac{l}{r}. \qquad \textbf{(128.)}$$

In all of which P is the allowed compression per square inch of cross-section, l is the length of the compression member in inches, and r is the least radius of gyration in inches. As before noticed, the word *strain* is used in the sense of *stress*.

1411. In using compression formulas, and in all computations relating to the proportioning of the material for the members of a bridge, the length of any member taken is always its theoretical length, i. e., its length center to center of connections. The value of l is given to the nearest inch.

1412. The following clauses of the Specifications also refer to the compression members:

(q) "*No compression member*, however, proportioned by the above formula, shall have a length exceeding 45 times its least width. For ratios greater than 45 the constants in the above formula will be reduced proportionately."

(r) "*The lateral struts* shall be proportioned by the above formula to resist only the resultant due to an assumed initial strain of 10,000 pounds per square inch upon all rods attaching to them, assumed to be produced by adjusting the bridge or towers."

(s) "*Members subjected to alternate strains* of tension and compression shall be proportioned to resist each kind of strain. Both of the strains shall, however, be considered as increased by an amount equal to $\frac{8}{10}$ of the least of the two strains, for determining the sectional areas by the above allowed unit strains."

(t) "*The unsupported width of a plate* subjected to compression shall not exceed thirty times its thickness, except cover plates of top chords and end posts, which will be limited to forty times their thickness."

(u) "No iron shall be used less than $\frac{1}{4}$ inch thick, except for lining or filling vacant spaces."

(v) "*In compression chord sections*, the material must mostly be concentrated at the sides, in the angles, and vertical webs. Not more than one plate, and this not exceeding $\frac{5}{8}$ inch in thickness, shall be used as a cover plate, except when necessary to resist bending strains."

DIMENSIONS OF COMPRESSION MEMBERS.

1413. By reference to the formulas given in the preceding articles, it will in each case be noticed that the greater the radius of gyration of the section the greater will be the unit stress, and hence the less will be the area required to resist the given stress. In determining the dimensions of a compression member by any such formulas, therefore, the natural course would be to select a size and form of section giving the radius of gyration as large as possible, which would also require the width and depth of the member to be as great as possible. If this idea were carried to extremes it would result in large members of very thin metal, and there is evidently a limit beyond which the metal would be too thin to be of value as a strut. To obviate this it is the practice to limit the ratio between the minimum thickness of the material and the dimensions of the piece. Thus, item (*t*), Art. **1412,** limits the ratio between the thickness and width of any plate in a member subjected to compression. In accordance with this specification, the thickness of the side plates in forms *H* and *B*, Fig. 304, must not be less than one-thirtieth the distance between the two lines of rivets connecting them to the angles, while in form *B* of that figure the thickness of the cover plate must not be less than one-fortieth the distance between the two lines of rivets connecting it to the angles.

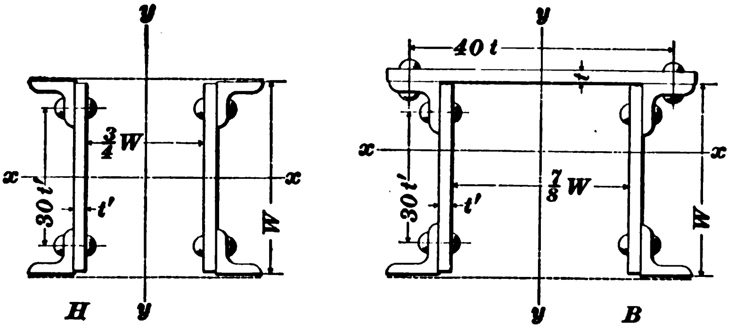


FIG. 304.

1414. In order that the radius of gyration obtained according to the formula given in Art. **1408,** for either section shown in Fig. 304 (i. e., the radius of gyration about the axis $x\,x$), shall be the *least* radius of gyration of the section; the clear distance between the side plates (or channels, if channels are used) must not be less than that given by the following formulas. The formulas apply also to those similar forms of sections shown at *A* and *F*, Fig. 303, in which channels are used. If *W* represents the width of the member, as shown in Figs. 303 and 304, and *c* represents the clear distance between side plates or backs of channels, then,

For post sections, forms *F* and *H*, Fig. 303:

$$c = \frac{3}{4} W. \qquad \textbf{(129.)}$$

For chord sections, forms *A*, *B*, *C*, and *D*:

$$c = \frac{7}{8} W. \qquad \textbf{(130.)}$$

For a section of the form shown at *L*, Fig. 303, the radius of gyration given by formula **113,** Art. **1408,** will be the least radius of gyration of the section when the two pairs of angles which are connected by the lattice bars are separated by a reasonable distance. This distance varies from zero in angles having both legs of nearly equal length (as 3″ × 2½″) to about 1¾″ in angles having the longer leg double the length of the shorter leg (as 7″ × 3½″). Practical considerations generally require them to be separated more than this. Each pair of angles is usually separated ½″ by the lattice bars. Hence, for *W* add ½″ to the dimension given by the two angles.

For a section of the form *O*, Fig. 303, formulas **116** and **117** must both be applied; the *smaller* value of *r* given by these formulas must be used.

For form *P*, the dimension *V* must be large enough so that the radius of gyration obtained (in terms of *W*) by formula **118** will be the *least* radius of gyration of the section. It is impossible to give a general rule for the comparative dimensions necessary to satisfy this condition; but the radius of gyration given by formula **118** will usually be the least radius of gyration of the section when the value of V is equal to or greater than that given by the formula

$$V = \frac{8}{10} W. \qquad \textbf{(131.)}$$

In order to be on the safe side, the value of V should always be taken somewhat greater than given by the above formula.

Forms *M*, *O*, and *P* are not very satisfactory forms of sections to use for compression members, and should not be used except for lateral struts, and not for this purpose when better forms can be as expediently used. It is generally much more satisfactory and economical to use symmetrical sections, i. e., those in which the centers of gravity are at the centers of the sections.

THE WEIGHT OF WROUGHT IRON.

1415. Before proceeding further it will be well to notice how the weight of a piece of wrought iron may be estimated. By reference to Table 19, Art. **1184,** it will be found that the average weight per cubic foot of wrought iron is 480 pounds. For rolled sections of large dimensions the average weight per cubic foot is slightly less than this amount, but 480 pounds per cubic foot is the weight invariably used in estimating wrought iron for bridge purposes. A bar of iron 1 foot long and 1 inch square weighs, therefore, $\frac{480}{144} = 3\frac{1}{3}$ lb. $= \frac{10}{3}$ lb. If the bar is one yard long its weight is $3 \times \frac{10}{3}$, or just 10 pounds.

Therefore, given the sectional area of a wrought-iron bar of any form, its weight per foot can readily be found by the following formula, in which A is the sectional area of the bar in square inches, and w is its weight in pounds per foot of length:

$$w = \frac{10}{3} A. \qquad (132.)$$

If w, the weight per lineal foot of a wrought iron bar of any form, is known, its sectional area may be found by the formula

$$A = \frac{3}{10} w. \qquad (133.)$$

1416. Wrought-iron structural shapes are rolled in various sizes, and each size may have any weight between certain maximum and minimum limits, according to the practical limits for rolling and the allowed minimum thickness of the metal. The variation in weight for each size is, for most shapes, effected by simply changing the distance between the rolls.

In specifying any structural shape other than a plate, or a round, square, or flat bar, for which the exact dimensions can be readily stated, it is necessary to specify both the size and the weight. If the weight of a shape is not stated, the lightest weight rolled of the specified size is always understood. For some shapes the thickness of metal is sometimes specified instead of the weight, though not commonly. Thus, a 4″ × 3″ angle bar having $\frac{3}{8}$″ thickness of metal is usually specified as a 4″×3″ L @ 8.4 lb., but it may be specified as a 4″ × 3″ × $\frac{3}{8}$″ L. The weight and the thickness of metal are never *both* specified, as the one determines the other.

1417. The maximum and minimum weights of the various sizes of wrought-iron channels, as commonly rolled, are given in the following table. All mills, however, do not roll channels as light as the minimum weights here given, nor do the maximum weights here given correspond to the maximum weights as rolled by all mills. The weights in the table, however, represent closely the least and greatest weight of each size as rolled by any mill. Each size can usually be rolled of any weight between the limits given:

TABLE 30.

MAXIMUM AND MINIMUM WEIGHTS OF WROUGHT IRON CHANNELS.

Size of Channel in Inches.	Minimum Weight, Pounds per Foot.	Maximum Weight, Pounds per Foot.
3	5.0	6.0
4	5.0	10.5
5	5.7	14.0
6	7.5	19.4
7	8.5	24.3
8	10.0	28.0
9	12¾	35.0
10	16.0	37.5
12	20.0	53.5
13	29.5	60.3
15	32.0	69.0

1418. The maximum and minimum weights of the most common sizes of wrought-iron angles are given in the following tables. Each size can be rolled of any weight between the limits given:

TABLE 31.

MAXIMUM AND MINIMUM WEIGHTS OF WROUGHT-IRON ANGLES WITH EQUAL LEGS.

Size of Angle in Inches.	Minimum Thickness of Metal in Inches.	Minimum Weight, Pounds per Foot.	Maximum Weight, Pounds per Foot.
2 × 2	$\frac{3}{16}$	2.4	4.8
2¼ × 2¼	$\frac{3}{16}$	2.6	6.0
2½ × 2½	$\frac{3}{16}$	3.0	8.0
2¾ × 2¾	¼	4.4	8.8
3 × 3	¼	4.8	11.4
3½ × 3½	$\frac{5}{16}$	6.8	14.0
4 × 4	$\frac{5}{16}$	8.0	19.0
5 × 5	⅜	12.0	32.8
6 × 6	⅜	14.5	38.5

TABLE 32.

MAXIMUM AND MINIMUM WEIGHTS OF WROUGHT-IRON ANGLES WITH UNEQUAL LEGS.

Size of Angle in Inches.	Minimum Thickness of Metal in Inches.	Minimum Weight, Pounds per Foot.	Maximum Weight, Pounds per Foot.
2¼ × 1½	$\frac{3}{16}$	2.2	5.4
2½ × 2	$\frac{3}{16}$	2.6	7.1
3 × 2	$\frac{7}{32}$	3.5	8.0
3 × 2½	¼	4.4	9.3
*3¼ × 2	¼	4.2	8.8
3½ × 2	$\frac{15}{64}$	4.0	6.6
3½ × 2½	¼	4.8	10.2
3½ × 3	$\frac{5}{16}$	6.4	15.4
4 × 3	$\frac{5}{16}$	6.8	16.8
4 × 3½	$\frac{5}{16}$	7.6	18.1
4½ × 3	$\frac{5}{16}$	7.6	18.1
5 × 3	$\frac{5}{16}$	8.0	19.5
5 × 3½	$\frac{5}{16}$	8.5	22.3
5 × 4	⅜	10.8	23.7
5½ × 3½	⅜	10.8	17.8
6 × 3½	⅜	11.3	31.3
6 × 4	⅜	12.0	32.7
6½ × 4	⅜	12.6	33.6
7 × 3½	½	16.7	31.7
8 × 3½	$\frac{7}{16}$	16⅓	27.0

NOTE.—As herein used, the weight of an angle corresponding to any specified thickness of metal will be the weight as estimated from the exterior dimensions of the angle. Thus, the weight per foot of a 3″ × 3″ angle of ¼″ thickness of metal is $(3 + 3) \times \frac{1}{4} \times \frac{10}{3} = 5$ lb. As actually rolled, however, the weights per foot are slightly less than the weights as thus estimated, as will be noticed in the preceding tables. In estimating the thickness of an angle from the weight, the thickness will usually be taken at the full sixteenth of an inch nearest to the thickness, as estimated on the above basis.

---

* Now rolled only in steel.

EXAMPLE.—What is the minimum area of section given by two 6-inch channels?

SOLUTION.—By Table 30, Art. **1417,** the minimum weight of a 6-inch channel is 7½ pounds. Therefore, by formula **133,** Art. **1415,** the minimum area of section given by two 7-inch channels is $7.5 \times 2 \times \frac{3}{10} = 4.5$ sq. in. Ans.

EXAMPLE.—A section of 10.2 square inches is to be made up of a 12" × ¼" plate and two 8-inch channels. What is the weight per foot of each channel?

SOLUTION.—The area given by the plate is $12'' \times \frac{1}{4}'' = 3$ sq. in., and, therefore, the area to be given by the two channels is $10.2 - 3 = 7.2$ sq. in. By formula **132,** Art. **1415,** the weight per foot of each channel is $7.2 \times \frac{10}{3} \times \frac{1}{2} = 12$ lb. Ans.

EXAMPLES FOR PRACTICE.

1. What is the least area of section that can be made up of two 9-inch channels without wasting material? Ans. 7.65 sq. in.

2. It is required to make up a section of 16 square inches of two 10-inch channels and a 14" × ⅜" cover plate. What is the weight of each channel? Ans. 17.92 lb.

3. Can a section of 7 square inches be made up of two 4-inch channels?

4. How large channels can be used in making up a section of 6.2 square inches with two channels? Ans. 8 in.

5. A section is composed of two 6-inch channels weighing 8 pounds per foot each, and a 10" × ¼" cover plate. What is the area of the section? Ans. 7.3 sq. in.

6. What is the minimum section given by four 3½" × 2" angles? Ans. 4.8 sq. in.

7. What is the weight per foot of each of four 4" × 3" angles of which a section containing 8.4 square inches is composed? Ans. 7 lb.

8. A section of 4.8 square inches is made up of two 5" × 3" angles. What is the weight per foot of each angle? Ans. 8 lb.

9. A section of 21 square inches is made up of two 16" × ⅜" plates and four 3" × 3" angles. What is the weight per foot of each angle? Ans. 7.5 lb.

PROPORTIONING THE MATERIAL FOR THE COMPRESSION MEMBERS.

1419. Chords and End Posts.—For the upper chords and end posts of the bridge shown in Mechanical Drawing Plate, Title: Highway Bridge: General Drawing, the effective section is of the form shown at *A*, Fig. 303, being made up of two channels and a cover plate. In selecting the size of channel it is not always possible to choose at first trial the size most suited to the required section, two or more trials being often necessary. But after a certain amount of experience the designer is able to judge quite correctly of the size of channel most suitable to use. In the present case, 7-inch channels will first be tried for the top chord.

1420. The width of the flange of a channel is given approximately by the formula

$$F = .6 + .2\,W, \qquad \textbf{(134.)}$$

in which F is the width of the flange and W is the depth of the web of the channel, both in inches. By combining formula **134** with formula **130,** of Art. **1414,** it is evident that when chord sections are made up of two channels and a cover plate, the width T of the cover plate or top plate in inches must somewhat exceed the value

$$T = \frac{7}{8}\,W + 2\,(.6 + .2\,W) = 1.2 + 1.28\,W. \qquad \textbf{(135.)}$$

The width of the top plate should always be slightly more than the width given by this formula, because the width of the flange of the channel is liable to vary considerably from the width given by formula **134,** and it is essential that the flange of the channel should not project beyond the edge of the cover plate.

1421. With 7-inch channels, then, the width of the top plate must be somewhat more than $1.2 + 1.28 \times 7 = 10.16$ inches. A width of 11 inches will be taken.

The general dimensions of a section are often spoken of as the **size** of the section. For instance, in the present case, the section may be called a 7″ × 11″ section, without reference to the weight of the channels or thickness of the cover plate.

By formula **110,** Art. **1408,** the least radius of gyration of a section made up of two 7-inch channels and an 11-inch cover plate is $\frac{4}{10} \times 7 = 2.8$ in. $= r$. In the example the length of a top-chord member is 18 ft. = 216 in. $= l$. Therefore, with 7-inch channels the live load unit stress allowed for this member by formula **123,** Art. **1410,** is $10{,}000 - 40 \times \frac{216}{2.8} = 6{,}910$ lb., and the dead load unit stress allowed by formula **124** of the same article is $20{,}000 - 80 \times \frac{216}{2.8} = 13{,}820$ lb. The dead load unit stress is most quickly obtained by simply doubling the live load unit stress. Thus, $6{,}910 \times 2 = 13{,}820$ lb. The amount of section required for the upper chord (see stress sheet) is, therefore, $\frac{48{,}600}{6{,}910} + \frac{20{,}800}{13{,}820} = 8.54$ square inches.

This section can be made up by a plate 11″ × ¼″ = 2.75 square inches, and two 7-inch channels, each weighing $\frac{(8.54 - 2.75)}{2} \times \frac{10}{3} = 9.65$ lb. per ft. (See formula **132,** Art. **1415.**)

1422. The size of channels and width of cover plate for the end post should be the same as for the upper chord. The length of the end post, center to center of connections, is 25′ 5⅝″ = 305.6″ $= l$. By formula **125,** Art. **1410,** the live load unit stress that may be allowed, using 7-inch channels for the end post, is $8{,}750 - 50 \times \frac{305.6}{2.8} = 3{,}290$ lb., and the dead load unit stress is $3{,}290 \times 2 = 6{,}580$ lb. Therefore, with 7-inch channels the amount of section required for the end post (see stress sheet) is $\frac{45{,}800}{3{,}290} + \frac{19{,}600}{6{,}580} = 16.90$ sq. in. If a 11″ × $\frac{7}{16}$″ (= 4.81 sq. in.) cover plate is used, the weight per foot of each channel will be

$\frac{(16.90 - 4.81)}{2} \times \frac{10}{3} = 20.15$ pounds per foot. With these sections for the chord and end post required by the use of 7-inch channels, it will be well to compare the amount of section required by using 8-inch channels.

By formula **135,** Art. **1420,** the width of cover plate required with 8-inch channels must exceed $1.2 + 1.28 \times 8 = 11.44''$; a 12-inch cover plate will be used. By formula **110,** Art. **1408,** the least radius of gyration of a section composed of two 8-inch channels and a cover plate is $\frac{4}{10} \times 8 = 3.2''$. The live load unit stress allowed by the specifications (formula **125,** Art. **1410**) is $8,750 - 50 \times \frac{305.6}{3.2} = 3.975$ lb. The dead load unit stress is $3,975 \times 2 = 7,950$ lb. Therefore, with 8-inch channels, the amount of section required for the end post is $\frac{45,800}{3,975} + \frac{19,600}{7,950} = 13.99$ sq. in. The section given by a $12'' \times \frac{3}{8}''$ cover plate is 4.5 sq. in., and by using this size of plate the weight per foot required for each channel is $\frac{(13.99 - 4.5)}{2} \times \frac{10}{3} = 15.82$ lb.

With 8-inch channels the live load unit stress allowed for the upper chord (formula **123,** Art. **1410**) is $10,000 - 40 \times \frac{216}{3.2} = 7,300$ lb., and the allowed dead load unit stress is $7,300 \times 2 = 14,600$ lb. Therefore, the section required for the upper chord (see stress sheet) is $\frac{48,600}{7,300} + \frac{20,800}{14,600} = 8.08$ sq. in. According to item (*u*), Art. **1412,** the thickness of the cover plate must not be less than $\frac{1}{4}$ inch. The section given by a $12'' \times \frac{1}{4}''$ cover plate is 3 square inches, and the weight per foot required for each channel is $\frac{(8.08 - 3)}{2} \times \frac{10}{3} = 8.47$ lb. According to Table 30, Art. **1417,** the minimum weight of 8-inch channel is 10 pounds per foot; therefore, if this size channels are used, the upper chord must have a section of $3 + 10 \times \frac{3}{10} \times 2 = 9$ sq. in.. which is somewhat in excess of the section required.

The section made up by using 8-inch channels can now be compared with that in which 7-inch channels are used, in order to determine which section is the more economical.

If l is the length in feet of a piece of iron of uniform cross-section A, then its weight is (formula **132,** Art. **1415**) $\frac{10}{3} A\, l$. For the upper chord, $l = 3 \times 18 = 54$; for the *two* end posts, $l = 2 \times 25.47 = 50.94$. With 7-inch channels the area (A) required for the upper chord $= 8.54$ sq. in.; for each end post, $A = 16.9$ sq. in. With 8-inch channels the corresponding values of A are 9 and 14 sq. in. (nearly). Therefore, the weight of iron required by the 7-inch channels $= \frac{10}{3} \times 8.54 \times 54 + \frac{10}{3} \times 16.9 \times 50.94 = \frac{10}{3}(8.54 \times 54 + 16.9 \times 50.94)$; and that required by the 8-inch channels $= \frac{10}{3}(9 \times 54 + 14 \times 50.94)$. The ratio of the former to the latter is

$$\frac{8.54 \times 54 + 16.9 \times 50.94}{9 \times 54 + 14 \times 50.94} = \frac{1,322}{1,199} = 1.10.$$

By using 7-inch channels, therefore, 10 per cent. more of iron is required than by using 8-inch channels. The latter will be used.

1423. For the top chord the section will be made up of two 8-inch channels, each weighing 10 pounds per foot, and a $12'' \times \frac{1}{4}''$ cover plate, giving a total area of section of 9 square inches. For the end posts the section will be made up of a $12'' \times \frac{3}{8}''$ cover plate and two 8-inch channels, each weighing 16 pounds per foot, giving a total sectional area of $4.5 + 16 \times \frac{3}{10} \times 2 = 14.1$ sq. in. Channels weighing 15.82 pounds per foot would give sufficient section, but the 16-pound channels can usually be more readily obtained, and the difference between the two weights is not material.

1424. The *intermediate posts* will be made of four angles connected in the form shown at L, Fig. 303. By reference to Fig. 3 of Mechanical Drawing Plate, Title:

Highway Bridge: Details I, it will be seen that the connected legs of the angles are separated ½ inch by the lattice bars. Therefore, if 3½″ × 2″ angles are used, the width W of the section equals $3\frac{1}{2}'' + \frac{1}{2}'' + 3\frac{1}{2}'' = 7\frac{1}{2}''$, and by formula **113,** Art. **1408,** the least radius of gyration of the section is $.24 \times 7.5 = 1.8'' = r$. The length of the post is 18 ft. $= 216'' = l$. By formula **125,** Art. **1410,** the allowed live load unit stress is $8{,}750 - 50 \times \frac{216}{1.80} = 2{,}750$ lb.; the dead load unit stress is $2{,}750 \times 2 = 5{,}500$ lb. Therefore, the section required for the intermediate post is $\frac{9{,}700}{2{,}750} + \frac{2{,}300}{5{,}500} = 3.95$ sq. in.

By formula **132,** Art. **1415,** the weight per foot of each one of the four angles necessary to make this section would be $\frac{3.95}{4} \times \frac{10}{3} = 3.29$ lb. But by reference to Table 32, of Art. **1418,** it is seen that the lightest weight of 3½″ × 2″ angles is 4 pounds per foot, and also that the thickness of metal for this weight is $\frac{15}{64}$ inch, or slightly less than ¼ inch.

According to item (*u*) of Art. **1412,** the metal in a member must not be less than ¼ inch thick. For ¼ inch thickness of metal the weight per foot of a 3½″ × 2″ angle, as computed, is $(3.5 + 2) \times \frac{1}{4} \times \frac{10}{3} = 4.6$ lb. The actual weight, however, is 4.4 pounds per foot.

The sectional area given by four 3½″ × 2″ angles weighing 4.4 pounds to the foot is $4.4 \times \frac{3}{10} \times 4 = 5.28$ sq. in., which is somewhat in excess of the area required.

By using 3″ × 2″ angles the radius of gyration would be $.24 \times 6.5 = 1.56$ inches. The allowed live load unit stress would be $8{,}750 - 50 \times \frac{216}{1.56} = 1{,}830$ lb., and the dead load unit stress would be $1{,}830 \times 2 = 3{,}660$ lb. The sectional area required for this size of angle would, therefore, be $\frac{9{,}700}{1{,}830} + \frac{2{,}300}{3{,}660} = 5.93$ sq. in., which is more than the section of the four 3½″ × 2″ × ¼″ angles. Four 3½″ × 2″ angles could have been used, however, with slightly better economy. The $3\frac{1}{2}'' \times 2''$ size of angles was used on account of showing the connections rather more plainly.

1425. The *intermediate lateral struts* are proportioned according to items (*p*) and (*r*) of Arts. **1410** and **1412.** By reference to item (*r*) it will be noticed that the lateral struts are not proportioned according to the stresses obtained for them, but are proportioned "to resist only the resultant due to an assumed initial strain of 10,000 pounds per square inch upon all rods attaching to them." For any strut, the resultant due to the assumed initial strain, or stress, may readily be found by simply drawing a force polygon for the joint at either end of the strut. The construction for finding the resultant of the assumed initial stress upon the rods attaching to the strut $C\ C_1$ is shown in Fig. 305. In the diagram of the upper lateral system, the strut $C\ C_1$ and the rods attaching to it are shown by full lines, while all other portions of the diagram are represented

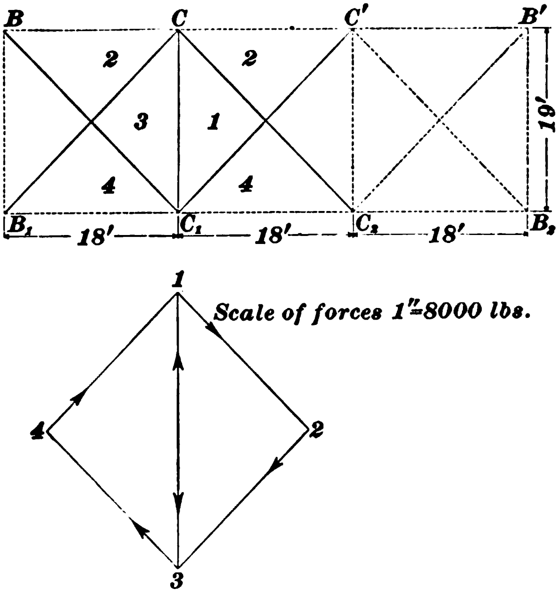


FIG. 305.

by dotted lines. Bow's notation is used with reference to the strut and the attaching rods. The rods are round and 1 inch in diameter. (See Art. **1405.**) The section of a round rod 1 inch in diameter is .785 sq. in., and, therefore, the amount of the initial strain assumed for each rod is 10,000 × .785 = 7,850 lb. This strain (stress) is evidently tension.

Now, considering the forces acting upon joint C, the line *1-2* is drawn parallel, and to any convenient scale is made equal to the action of the assumed tension in $C\ C_2 =$ 7,850 lb.; *2-3* is drawn equal and parallel to the tension in $C B_1 =$ 7,850 lb. Then, as *1-2* and *2-3* are equal, *3-1* will be parallel to and represent the compressive stress in $C\ C_1$, due to the initial stresses in $C B_1$ and $C C_2$; it scales 11,400 pounds. If the sectional area of $C\ C_2$ were less than that of $C\ B_1$, the line *1-2* would be shorter than the line *2-3*, and a line drawn from *3* to *1* would not be parallel to $C\ C_1$. In order that the closing line *3-1* may be parallel to $C\ C_1$, in such a case, it will be necessary to introduce in the force polygon a line representing the difference between the longitudinal components of the stresses in $C C_2$ and $C\ B_1$, which is taken by the chord $C C'$. The resultant of the initial stresses in the rods $C_1\ B$ and $C_1\ C'$ simply resists the resultant of those in $C\ B_1$ and $C\ C_2$, the chords not being required to act. By drawing the force polygon *3-4-1-3* for joint C_1, the same amount of stress, 11,400 lb., is obtained for $C\ C_1$ as before, while the positions and directions of the arrow-heads in the two polygons indicate that the resultant stress in $C\ C_1$ is compression.

1426. The amount of the resultant stress in a strut can also be obtained by applying the following general

Rule.—*Divide the length of the strut by the length of the rod, and multiply the quotient by the assumed stress in the rod. The result will be the stress in the strut due to the initial stress in that one rod. The sum of the results thus obtained for all rods attaching to one end of the strut will be the total resultant for the strut.*

The lengths used should be the lengths between the points where the center lines of the lateral struts intersect the center lines of the chords.

The resultant R of the assumed stress upon *each* rod, as given by the above rule, may be expressed by the equation

$$R = \frac{S_a l_s}{l_r}, \qquad \textbf{(136.)}$$

in which S_a is the assumed initial stress, l_r is the length of the rod on which the stress is assumed, and l_s is the length of the strut for which the resultant is obtained.

Thus, in the present case, the theoretical length of the strut CC_1 is 19 ft., and that of the rod CB_1 is $\sqrt{18^2 + 19^2} = 26.17$ ft. The amount of initial stress assumed for the rod is 7,850 lb. Hence, the resultant stress in the strut CC_1 due to the initial stress in the rod CB_1 is $7{,}850 \times \frac{19}{26.17} =$ 5,699 lb. The resultant due to the assumed initial stress in CC_2 is the same as found for CB_1. The resultant due to both rods, therefore, is $5{,}699 + 5{,}699 = 11{,}398$, or practically 11,400 lb. All conditions for the lateral strut $C'C_2$ are the same as for CC_1; hence, each of the intermediate lateral struts is to be proportioned to resist a resultant compressive stress of 11,400 lb.

1427. The section of two 5″ × 3″ angles with the longer legs riveted together, back to back, in the form shown at O, Fig. 303, will first be tried. This is not a symmetrical section, but it is probably the best form to be used for the lateral struts when the lateral rods connect on top of the chords. For this section the dimension W will be $3 + 3 = 6$ in., and the dimension V will be 5 in. By applying formulas **116** and **117,** of Art. **1408,** it is found that $r_w = \frac{1}{5} \times 6 = 1.2$ in., and $r_h = \frac{32}{100} \times 5 = 1.6$ in. Therefore, the least radius of gyration of the section is 1.2 in. The length of the strut is 19 ft. = 228 in. By formula **128,** Art. **1410,**

the allowed unit stress for this strut is $11{,}000 - 60 \times \frac{228}{1.2}$, which gives no unit stress. It is, therefore, necessary to try angles of larger size.

Two $5'' \times 3\frac{1}{2}''$ angles, with the longer legs back to back, will next be tried. For this section the dimension W will equal $3\frac{1}{2} + 3\frac{1}{2} = 7$ inches. Formulas **116** and **117,** Art. **1408,** give $r_w = \frac{1}{5} \times 7 = 1.4$ in., and $r_h = \frac{32}{100} \times 5 = 1.6$ inches; 1.4 inches is, therefore, the least radius of gyration of the section. The allowed unit stress for this strut is $11{,}000 - 60 \times \frac{228}{1.4} = 1{,}230$ lb. The sectional area required is $\frac{11{,}400}{1{,}230} = 9.27$ square inches, making the weight per foot of each angle $\frac{9.27}{2} \times \frac{10}{3} = 15.45$ lb.

By reference to Table 32, Art. **1418,** it is found that this weight is in excess of the minimum weights of angles of larger size, and it will, therefore, be more economical to use larger size angles.

By using two $5'' \times 4''$ angles, with the 5-inch legs back to back, the dimension W will equal $4 + 4 = 8$ inches, and the dimension V will remain as before. By applying formulas **116** and **117,** $r_w = \frac{1}{5} \times 8 = 1.6$ in., and $r_h = \frac{32}{100} \times 5 =$ 1.6 in. When the radius of gyration of a section about each of two perpendicular axes is the same, as in the present case, the section is said to be a **balanced section.** It is evident that a balanced section is the most economical section to use for a compression member. The allowed unit stress for this section is $11{,}000 - 60 \times \frac{228}{1.6} = 2{,}450$ lb., and the area required for the section is $\frac{11{,}400}{2{,}450} = 4.65$ sq. in. Hence, the weight per foot required for each angle is $\frac{4.65}{2} \times \frac{10}{3} = 7.75$ lb. Table 32, Art. **1418,** gives 10.8 lb.

as the minimum weight per foot of 5″ × 4″ angles, giving for two angles a sectional area of $10.8 \times 2 \times \frac{3}{10} = 6.48$ sq. in. Although the sectional area given by the minimum weight of two angles of this size is found to be considerably in excess of the area required for them, it has also been found to be less than the area required for angles of smaller size. Therefore, each intermediate lateral strut will be made up of two 5″ × 4″ angles, each weighing 10.8 pounds per foot.

1428. The Portal Bracing.—The design of the portal bracing is largely a matter of judgment; the investigation of the stresses which has been made will, however, be found a very helpful guide. In proportioning the material for this member the methods of engineers differ. The method here explained and followed will give very reliable results when a straight-line compression formula is used. It does not apply so well to curve formulas, but may be used without material error. Substantially the same results are obtained by other methods.

The stresses at intervals of one foot along each flange of the portal are shown in Fig. 278, to which the student is referred. The stress at each point along the upper flange is somewhat greater than the corresponding stress in the horizontal portion of the lower flange. On account of the inclinations of the brackets the stresses as found along those members are greater than in the corresponding portions of the upper flange. But, as the horizontal portion of the lower flange extends to and connects with each end post, it probably relieves the stresses in the flanges of the brackets to a very considerable extent. To what extent it does so, it is, of course, impossible to say, but the actual conditions will undoubtedly be fully provided for if the upper flange be proportioned to resist its stresses as found, and the lower flange be given the same section as the upper flange. By reference to the figure it will be noticed that, where the depth of the portal is uniform, the stresses at the various points along the upper flange vary by a practically constant difference. Now, neglecting the fact that the stresses in the upper flange decrease at the left of c, by reason of the increased depth of the portal, and assuming that the stress at the left of c is uniform and equal to the stress at $c = +23,520$ pounds, that portion of the upper flange which is in compression may be considered to be divided into a series of (seven) elementary struts of varying lengths. Designating each elementary strut by the letter at its right extremity, the length of each elementary strut and the amount of stress borne by it are as follows:

Strut c: Stress $= +23,520 - 19,820 = +3,700$ lb. Length $= 3.5$ ft.
Strut d: Stress $= +19,820 - 16,130 = +3,690$ lb. Length $= 4.5$ ft.
Strut e: Stress $= +16,130 - 12,430 = +3,700$ lb. Length $= 5.5$ ft.
Strut f: Stress $= +12,430 - 8,740 = +3,690$ lb. Length $= 6.5$ ft.
Strut g: Stress $= +8,740 - 5,050 = +3,690$ lb. Length $= 7.5$ ft.
Strut h: Stress $= +5,050 - 1,350 = +3,700$ lb. Length $= 8.5$ ft.
Strut i: Stress $= +1,350 = +1,350$ lb. Length $= 9.5$ ft.
23,520

The increment of the flange stress, or difference between the stresses at two adjacent points on the flange, is actually about 3,695 lb. The increment, or amount of stress borne by each elementary strut, as obtained above is, in each case, either 3,700 or 3,690 lb. This slight irregularity is due simply to the fact that each stress is written to the nearest ten pounds.

As the amount of stress is the same upon each elementary strut (except strut i), therefore, in order to resist the same amount of total stress, the total area of section required for the seven elementary struts will be the same as the sectional area required for a single equivalent strut of the average length, assuming the radius of gyration to be the same. This will be the case when a straight-line compression formula is used, but it will be only approximate for a curve formula. The slight error due to the fact that the stress upon strut i is less than upon each of the other struts may be neglected; this small error in the length of the equivalent strut will be on the side of safety; that is, the equivalent strut will be slightly too long.

The average length of the seven elementary struts is

$\frac{3.5 + 9.5}{2} = 6.5$ ft. $= 78$ in. $= l$. The strut is supported in one direction by the lattice bars, and, therefore, its radius of gyration in that direction (about an axis perpendicular to the plane of the lattice bars) need not be considered. The two flanges of the portal, connected by the lattice bars, form a section similar to that shown at L, Fig. 303, and its least radius of gyration is found by formula **113**, Art. **1408**. Using $3\frac{1}{2}'' \times 2\frac{1}{2}''$ angles, with the shorter legs back to back, separated $\frac{1}{2}''$ by the lattice bars, the dimension W will equal $3\frac{1}{2} + \frac{1}{2} + 3\frac{1}{2} = 7\frac{1}{2}$ in., and the least radius of gyration will be $\frac{24}{100} \times 7.5 = 1.8$ in. Hence, the allowed unit stress is $11,000 - 60 \times \frac{78}{1.8} = 8,400$ lb., and the sectional area required is $\frac{23,520}{8,400} = 2.80$ sq. in. In Table 32, Art. **1418,** the minimum weight per foot of a $3\frac{1}{2}'' \times 2\frac{1}{2}''$ angle is given at 4.8 lb. For the upper flange two of these angles will be used, giving a sectional area of $4.8 \times 2 \times \frac{3}{10} = 2.88$ sq. in. The lower flange and the flange of the bracket will be made the same.

1429. It will be noticed that in proportioning the material for the flanges, two stresses only have been used; namely, the greatest stress, which, with a straight bracket, as in the figure, occurs at a point opposite to where the bracket connects to the horizontal flange, and the least stress, which may be considered to occur at the center.

For appearance, the portal brackets of a bridge are often curved and otherwise ornamented. In case of a curved bracket, the greatest stress in the upper flange will occur vertically opposite that point where a line drawn through the center of the upper flange becomes tangent to the curve of the bracket.

1430. Let us now determine whether the area of the section as obtained above is sufficient to resist the greatest tensile stress in compliance with item (*k*), Art. **1399.** By reference to Mechanical Drawing Plate, Title: Highway Bridge: Details II, it is found that $\frac{5}{8}''$ rivets are used. At each point where lattice bars connect, one rivet passes through each angle, and, therefore, according to item (*k*), two rivet holes $\frac{5}{8}'' + \frac{1}{8}'' = \frac{3}{4}''$ in diameter must be deducted from the section. Table 32, Art. **1418,** gives $\frac{1}{4}$ inch as the thickness of metal for the minimum weight (4.8 pounds per foot) of $3\frac{1}{2}'' \times 2\frac{1}{2}''$ angles. Hence, the section to be deducted for the two rivet holes is $\frac{3}{4} \times \frac{1}{4} \times 2 = .375$ sq. in., and the net section remaining is $2.88 - .38 = 2.50$ sq. in. By item (*a*), of Art. **1399,** the tensile stress per square inch allowed for lateral bracing is 15,000. Therefore, the net section remaining after deducting the rivet holes will bear an amount of stress equal to $2.5 \times 15{,}000 = 37{,}500$ lb., or more than the greatest tensile stress occurring in either flange of the portal. It is thus found that the section given by two $3\frac{1}{2}'' \times 2\frac{1}{2}''$ angles, each weighing 4.8 pounds per foot, fulfils the necessary conditions for either compression or tension in the flange of the portal.

It is to be observed that, in proportioning the material for the flanges of the portal, the condition required by item (*s*), of Art. **1412,** has not been fulfilled. Any portion of either flange of the portal is subject to alternate stresses of tension and compression, and, therefore, according to item (*s*), each stress should be increased by an amount equal to $\frac{8}{10}$ of the smaller stress. This condition, however, may be neglected, as is often done.

1431. The size of the lattice bars remains to be considered. It is not customary to proportion the lattice bars according to the stresses; indeed, it is very seldom that the stresses are found for the lattice bars. Relying upon their judgment and experience, bridge engineers are usually able to decide upon proper dimensions for lattice bars without finding the stresses.

In the present case, however, attention will be given to the stresses in proportioning the lattice bars. By the method explained in connection with Fig. 280, of Art. **1314,** stress was found in a lattice bar on the assumption that the web stress due to the increment of the flange stress is borne entirely by the tensile lattice bar connecting at the given point on the flange.

This assumption was made for convenience in finding the stress; the total amount of stress in the bars connecting at one point is the same, whether it be borne entirely by one bar or divided between the two bars, and it can always be correctly found in this manner, so long as both bars have the same inclination. In proportioning the material, however, it is reasonable to assume that the total web stress due to the increment in the flange stress at any one point is resisted in equal parts by the two lattice bars connecting at the point, one bar being in tension and one bar in compression. The strength of the lattice bars will be investigated on this assumption. The increments in the flange stress, and hence the stresses in the lattice bars, are uniform where the depth of the portal is uniform. Where the depth of the portal increases, the stresses generally decrease, and may be neglected.

1432. Where the portal is of uniform depth, the web stress due to the increment of the chord stress at any point was found to be 5,230 lb. This total stress is assumed to be equally divided between the two lattice bars connecting at the point, tension in one bar and compression in the other. The tension bar will first be considered; its stress is $\frac{5,230}{2} =$ 2,620 lb. The size of the lattice bars will be taken at $2'' \times \frac{1}{4}''$; it is to be determined whether the sectional area given by this size of bar will be sufficient to resist the stress.

The sectional area of a lattice bar of this size is $2 \times \frac{1}{4} = .5$ sq. in., while the area to be deducted for the rivet hole is $\frac{3}{4} \times \frac{1}{4} = .19$ sq. in., leaving for tension a net sectional area of $.5 - .19 = .31$ sq. in. By item (*a*), Art. **1399,** the amount of tensile stress which can be borne by this section is $15,000 \times .31 = 4,650$ lb., which is greater than the amount of tension (2,620 lb.) that it is required to bear.

1433. The compression bar will now be considered; its stress is 2,620 lb., the same as the tension bar. By reference to Table of Moments of Inertia, it is found that the value of the moment of inertia I of a rectangular section is given by the formula

$$I = \frac{b\,d^3}{12}, \qquad \textbf{(137.)}$$

in which b is the width of the section, or dimension parallel to the axis, and d is the depth, or dimension perpendicular to the axis. By reference, also, to formula **72,** of Art. **1241,** it is found that for any section the moment of inertia $I = A\,r^2$, in which A is the area of the section and r is the radius of gyration. By equating these two values of I for a rectangular section, we get

$$A\,r^2 = \frac{b\,d^3}{12}. \qquad \textbf{(138.)}$$

But the area $A = b\,d$; by dividing both terms of equation **138** by the area, we have for the square of the radius of gyration

$$r^2 = \frac{d^2}{12}. \qquad \textbf{(139.)}$$

By extracting the square root,

$$r = \frac{d}{3.464} = .289\,d. \qquad \textbf{(140.)}$$

In applying this formula, d must always be the dimension *perpendicular to the axis.* If the strut is unsupported in either direction, as in the present case, the *least* radius of gyration of the section must be obtained; in order to obtain the *least* radius of gyration of a rectangular section, the least dimension must be taken for the value of d.

In the present case, for the least radius of gyration,

$d = \frac{1}{4}''$, and $r = .289 \times .25 = .0723$ in. Each compression lattice bar is supported by its connection with each tension lattice bar, and, as each compression bar is riveted at its supports, each separate strut length of the bar partakes somewhat of the nature of a strut having fixed ends; therefore, the clear distance between its connections may be taken as the column length of the compression bar. This distance is about 6.36 inches. By formula **128,** of Art. **1410,** the allowed unit stress is $11{,}000 - 60 \times \frac{6.36}{.0723} = 5{,}720$ lb., and the sectional area required for the bar is $\frac{2{,}620}{5{,}720} = .46$ sq. in., or a little less than the area of a $2'' \times \frac{1}{4}''$ bar. It is thus found that $2'' \times \frac{1}{4}''$ lattice bars will fulfil the conditions for both tension and compression.

1434. The Shoe Strut.—A strut should always be placed between the two shoes at the expansion end of a bridge; in important structures a strut is often placed between the shoes at the anchored end also. It is impossible to determine definitely the maximum amount of stress that may come upon a shoe strut. Under certain conditions it could equal the total amount of wind pressure against the windward shoe, but the presence of such conditions is so exceedingly improbable as to be practically impossible. It is a not uncommon practice to design the shoe strut to resist one-half the total wind pressure against the windward shoe, and this is a reasonable practice.

As, however, the shoe strut is a lateral strut, in compliance with item (*r*), of Art. **1412,** it must be proportioned to resist the resultant due to an assumed initial stress of 10,000 pounds per square inch upon the lateral rod attaching at either end. The size of each diagonal rod in the end panels of the lower lateral system is $1\frac{1}{8}$ inch round, having a sectional area of practically one square inch. (Art. **1405.**) Therefore, the amount of initial stress assumed upon each rod is 10,000 pounds. In Art. **1426,** the theoretical length of the diagonal lateral rods was found to be 26.17 feet, and, according to the rule there given, the resultant stress in the shoe strut is $10,000 \times \frac{19}{26.17} = 7,260$ lb. In order to illustrate the application of the compression formulas to as large a variety of sections as possible, a section of the form shown at *P*, Fig. 303, will be used. The section given by two $3\frac{1}{2}'' \times 2\frac{1}{2}''$ angles having the shorter legs riveted together, with a $6'' \times \frac{1}{4}''$ plate between them, will be tried.

For this section the dimension W is equal to $3\frac{1}{2} + 3\frac{1}{2} = 7$ in., and the dimension V equals 6 in. According to formula **131,** Art. **1414,** the radius of gyration given by formula **118,** Art. **1408,** will be the least radius of gyration of the section when the dimension V is not less than $\frac{8}{10}\,W$. In the present case, $V = \frac{6}{7}\,W = \frac{86}{100}\,W$; therefore, the least radius of gyration of the section is $\frac{1}{5} \times 7 = 1.4$ inches, as given by formula **118.** As the actual length of the shoe strut, as given in the detail drawings, is not known when the material is proportioned, the length to be used in proportioning the material will be taken equal to the clear distance between the end posts. In the present case, this distance is 18 ft. = 216 in., which is the value that will be given to l in the compression formula. By formula **128,** Art. **1410,** the allowed unit stress is $11,000 - 60 \times \frac{216}{1.4} = 1,740$ lb.; hence, the sectional area required is $\frac{7,260}{1,740} = 4.17$ sq. in. By Table 32, Art. **1418,** the minimum weight per foot of a $3\frac{1}{2}'' \times 2\frac{1}{2}''$ angle is 4.8 lb.; hence, the total sectional area given by two $3\frac{1}{2}'' \times 2\frac{1}{2}''$ angles and a $6'' \times \frac{1}{4}''$ plate is $4.8 \times \frac{3}{10} \times 2 + 6 \times \frac{1}{4} = 4.38$ sq. in. This sectional area, though somewhat in excess of the area required, will be used. The connections for this strut are eccentric, i. e., not symmetrical with reference to the center of gravity of the strut. As this eccentricity tends to weaken the strut, some excess of section is very desirable. A symmetrical strut would have been better for the purpose.

THE EFFECT OF WIND STRESSES UPON THE CHORDS AND END POSTS.

1435. The Upper Chord.—The first condition of item (*l*) of Art. **1399** may be more clearly stated as follows:

Let W be the computed wind stress in any chord member or end post, L the live load stress, and D the dead load stress in the same. Also let A be the sectional area of the member, computed to provide for dead and live load stresses only. Then,

(*a*) *If* W *is smaller than* $\frac{D+L}{4}$, *no change need be made in the sectional area* A.

(*b*) *If* W *is greater than* $\frac{D+L}{4}$, *the sectional area of the member should be recalculated by the following formula:*

$$A_1 = .8\,A\left(1 + \frac{W}{D+L}\right), \qquad \textbf{(141.)}$$

in which A_1 *is the new sectional area required to provide for wind stress.*

It is very seldom necessary, if indeed it is ever necessary, to increase the sectional area of the upper chord of a through bridge, in order to provide for the wind stress in compliance with this requirement of the specifications. In the structure under consideration, the amount of wind stress upon the upper chord (omitted from the stress sheet) is 2,560 lb. This amount of stress may exist throughout the entire chord as compressive stress, or as tensile stress in the center panel. As this is much less than one-fourth of the combined live and dead load stresses in the same $= \frac{48,600 + 20,800}{4} = 17,350$ lb., no provision need be made for the wind stress in the upper chord.

1436. The End Posts.—The first condition of item (*l*) applies also to the end posts. The amount of wind stress in the end post is equal to the reaction in the direction of the length of this member caused by the wind pressure against the portal; that is, the so-called vertical reaction at the foot of either end post. This reaction reduces the compression in the windward end post and increases the compression in the leeward end post. By reference to Fig. 278, it is found that in the structure under consideration this reaction, and, therefore, the wind stress in each end post, are 7,390 lb. As this is less than one-fourth the combined live and dead load stress in the end post, or $\frac{45,800 + 19,600}{4} =$ 16,350 lb., the wind stress in the same need not be considered.

It is evident that the wind pressure also produces bending moment in each end post; this bending moment is greatest at the point at which the lower flange of the portal connects. In Cooper's specifications, however, the bending moments upon the end posts are provided for in a general way by the small unit stresses allowed for those members, and, therefore, need not here be further considered.

1437. The Lower Chord.—In considering the effect of the wind stresses upon the lower chord, it will be well to determine first the amount of stress produced in the lower chord by the wind pressure against the upper chord. The so-called vertical reaction at the foot of each end post is really a reaction exerted *in the direction of the end post*—that is, it is a force *whose line of action is represented by the center line of the end post*, and which resists the force produced in the end post, or along the same line of action, by the wind pressure against the portal. If the end post is inclined, this force has a horizontal component as well as a vertical component. The vertical component simply increases or diminishes the vertical pressure upon the support, and is resisted by the vertical reaction of the support, but the horizontal component must be resisted by stress in the lower chord. As the amount of stress in the lower chord is in direct ratio to the amount of stress in the end post, it is evident that the wind pressure against the portal will diminish not only the compression in the windward end post, but also the tension in the windward lower chord, while the same wind pressure will increase both the compression in the leeward end post and the tension in the leeward lower chord. In other words, the horizontal component of the wind stresses in the end posts tends to produce compression in the windward lower chord and tension in the leeward lower chord. The amount H of the stress developed in each lower chord by the wind pressure against the portal is given by the formula

$$H = \frac{P_w p}{b + c}, \qquad \textbf{(142.)}$$

in which P_w is the *total* lateral wind pressure against the portal, p is the panel length, b is the clear width of roadway, and c is the width of one chord; i. e., $b + c$ is the width center to center of chords. The stress H is constant throughout the entire length of each lower chord; it increases the tension in the leeward chord and tends to cause compression, i. e., diminishes the tension in the windward chord. When the compression H in the windward lower chord is combined with the compressive stresses produced in the same by the wind pressure against the lower chord (i. e., as the windward chord of the lower lateral system), a reversal of stress sometimes occurs and compression is actually produced in the end panels of the windward chord. In the present case, P_w, the total lateral wind pressure against the portal, is equal to $4{,}050 + 1{,}350 = 5{,}400$ lb. (See Fig. 278.) Hence, by formula **142,** the amount of stress $= H = \frac{5{,}400 \times 18}{18 + 1} =$ 5,100 lb. This stress is tension in the leeward chord and compression in the windward chord.

1438. *Panel a b.*—By combining this wind stress H, as obtained above, with the wind stresses in the lower chords, as found for the lower lateral system and shown on the stress sheet, the various stresses that may come upon the end panel *a b* of the lower chord are found to be as follows:

Live load	$-$ 32,400 lb.
Dead load	$-$ 13,900 lb.
Wind load	$-$ 5,100 lb. $+ 10{,}200 + 5{,}100 = + 15{,}300$ lb.

Of these stresses it will be noticed that the dead load stress is always present in the chord; the other stresses are occasionally present or may be wholly absent.

The compressive stress upon this panel of the chord produced by the wind is equal to 15,300 pounds, and as this is greater than the dead load stress upon the same (13,900 pounds), compression will actually obtain, and, according to the second condition of item (*l*), (Art. **1399**), this stress must be considered. A portion of this compressive stress equal to the dead load tensile stress, simply neutralizes the effect of the latter stress, so that the resultant, or net compressive stress, upon this panel of the chord is + 15,300 − 13,900 = + 1,400 lb. By item (*s*), of Art. **1412,** the member must be proportioned to resist each kind of stress, but each stress must be considered to be increased by eight-tenths of the smaller stress. In this case, $\frac{8}{10} \times 1,400 = 1,100$ lb. which added to + 1,400 and − 5,100 gives + 2,500, and − 6,200, respectively, for the wind compressive and tensile stress. As 6,200 is less than $\frac{32,400 + 13,900}{4}$, the sectional area of the member need not be increased. The section will be made up of four 3″ × 2½″ angles riveted in the form shown at *L*, Fig. 303.

1439. By reference to the details of the connections shown in Mechanical Drawing Plate, Title: Highway Bridge: Details III, Fig. 1, it will be found that the section cut out by two rivet holes must be deducted from the section given by each angle. It will be noticed that the lines of rivets in the two legs of each angle are staggered; that is, the rivet spacing is so arranged that two rivets do not come opposite each other in the same angle. When this is done it is not uncommon to deduct but one rivet hole from each angle or piece, assuming that as the section of the same is reduced by but one rivet hole at any normal cross-section its strength is reduced only by the same amount. But, unless the rivet spacing is exceedingly liberal, the section will be weakened by more than this amount, and it is a much better practice to deduct one rivet hole for each line of rivets through each piece. This practice will here be followed, and two rivet holes will be deducted from each angle.

1440. When a portion of the sectional area of a tension member is cut away by rivet holes, the remaining portion of the uninjured metal in the section, which is available to resist tensile stress, is called the **net area** or **net section.** The entire section, with no deduction of area for rivets, is called the **gross area** or **gross section.** As the rivet holes are thoroughly filled in driving the rivets, the gross area of a member is counted to resist compression, while the net area only is counted for tension.

1441. According to Table 32, Art. **1418,** the minimum weight per foot of a 3″ × 2½″ angle is 4.4 lb., and the thickness of metal for this weight is ¼ inch. Assuming the metal in the angles used to be of this thickness, the sectional area to be deducted for eight rivet holes (two in each angle) is $\frac{3}{4}'' \times \frac{1}{4}'' \times 8 = 1.5$ sq. in. (See item (*k*) of Art. **1399.**) This area must be added to the area required by the stresses to give the gross area required for the section. Therefore, $3.94 + 1.5 = 5.44$ sq. in. is the gross sectional area required, making the weight per foot of each angle $\frac{5.44}{4} \times \frac{10}{3} = 4.5$ lb. This weight is very little in excess of minimum weight of 3″ × 2½″ angles (4.4 pounds), and, hence, the thickness of metal for this weight of 3″ × 2½″ angle is so very slightly more than ¼ inch that the area deducted for the rivet holes will be considered correct. If the weights per foot obtained for the angles were such that the thickness of metal would be materially more than the thickness assumed in deducting the rivet holes, it would be necessary to correct the areas deducted to correspond with the greater thickness, and this would again somewhat increase the weights of the angles.

The amount of compressive stress to be resisted by the section is $1,400 + 1,100 = 2,500$ lb. The dimension W of the section is $3 + \frac{1}{2} + 3 = 6\frac{1}{2}$ in., and by formula **113,** Art. **1408,** the least radius of gyration of the section is $\frac{24}{100} \times 6.5 = 1.56$ in. As $l = 18 \times 12 = 216$ in., by taking the unit stress for the wind load midway between the live and dead load unit stresses, it will equal $15,000 - 60 \times \frac{216}{1.56} = 6,690$ lb. The compressive stress upon this member is thus found to require a sectional area of $\frac{2,500}{6,690} = .37$ sq. in.; it is abundantly provided for.

The specifications give no compression formula for wind stresses upon the chords, but in item (*o*) the compression formula for the allowed wind stress upon the posts gives a value substantially midway between the value given by the formula for the allowed live load stresses and the value given by the formula for the allowed dead load stresses. Hence, a formula may be used giving a corresponding value for the allowed wind stress upon the chord.

1442. *Panel b c.*—By combining the stress H with the wind stresses given by the stress sheet for the panel *b c* of the lower chord, it is found that the various stresses which can come upon this member are as follows :

$$\begin{array}{ll}
\text{Live load} & -\,32,400 \text{ lb.} \\
\text{Dead load} & -\,13,900 \text{ lb.} \\
\text{Wind load} & \left\{\begin{array}{l} -\,10,200 - 5,100 = -\,15,300 \text{ lb.} \\ +\,15,300 + 5,100 = +\,20,400 \text{ lb.} \end{array}\right.
\end{array}$$

The net or resultant compressive wind stress in this panel of the chord is $+\,20,400 - 13,900 = +\,6,500$ lb. As this is less than the tensile stress, both the tensile and compressive stresses must be increased by eight-tenths of this amount, or $\frac{8}{10} \times 6,500 = 5,200$ lb. This makes the tensile wind stress $W = -\,15,300 - 5,200 = -\,20,500$ lb. As this is numerically greater than $\frac{D + L}{4} = \frac{13,900 + 32,400}{4} = 11,575$, the sectional area must be recalculated by formula **141.** The sectional area (A) required by the dead and live load stresses was found to equal 3.94 sq. in. (Art. **1403.**) Therefore, the corrected area (formula **141**) is

$$A_1 = .8 \times 3.94\left(1 + \frac{20,500}{13,900 + 32,400}\right) = 4.55 \text{ sq. in.}$$

The section will consist of four 3¼" × 2" angles riveted together in the form shown at *L*, Fig. 303, or in the same general form as the member *a b*. As the required section is greater, the thickness of the metal will be somewhat greater than in the member *a b*. A thickness of ⅜" = .37" will be tried. As the rivets are ⅝" in diameter, the area to be deducted for eight rivet holes (see item (*k*) of Art. **1399**) is ¾ × ⅜ × 8 = 2.25 sq. in., and the total gross area required is 4.55 + 2.25 = 6.8 sq. in. For this amount of sectional area made up of angles of this size, the thickness of metal is, quite closely, $\frac{6.8}{4 \times (3.25 + 2)} = .32$ in., which is near enough to the thickness assumed (.37 in.) to consider the area deducted for the rivet holes to be correct. The weight per foot of each angle necessary to give this amount of section is $\frac{6.8}{4} \times \frac{10}{3} = 5.7$ lb.

The amount of compression which the section is to be proportioned to resist is 6,500 + 5,200 = 11,700 lb. The dimension *W* is equal to 3¼ + ½ + 3¼ = 7 in., and by formula **113,** Art. **1408,** the least radius of gyration of the section is $\frac{24}{100} \times 7 = 1.68$ in. Using the same compressive formula as for *a b*, the allowed unit stress is $15,000 - 60 \times \frac{216}{1.68} = 7,290$ lb. The sectional area required by the compressive wind stress is, therefore, $\frac{11,700}{7,290} = 1.6$ sq. in., which is less than the area given by the section.

1443. *Center Panel c c'*.—By combining the wind stress *H* with the wind stresses given on the stress sheet for the center panel of the lower chord, the various stresses for that panel are found to be as follows:

Live load $-$ 48,600 lb.
Dead load $-$ 20,800 lb.
$$\text{Wind load}\begin{cases} -15{,}300 - 5{,}100 = -20{,}400 \text{ lb.} \\ +15{,}300 + 5{,}100 = +20{,}400 \text{ lb.}\end{cases}$$

The compressive wind stress is less than the dead load tensile stress. Hence, it can not neutralize the latter or produce a resultant compressive stress, and, according to the second condition of item (*l*), it need not be considered.

But the tensile wind stress ($W = 20{,}400$, disregarding the sign) is greater than one-quarter of the combined live and dead load tensile stress $\left(\frac{D+L}{4} = \frac{20{,}800 + 48{,}600}{4} = 17{,}350\right)$, and, according to Art. **1435,** the section must be recalculated by formula **141.** In Art. **1404,** the sectional area A was found to equal 5.9 sq. in. Hence, the corrected area is $A_1 = .8 \times 5.9\left(1 + \frac{20{,}400}{20{,}800 + 48{,}600}\right) = 6.11$ sq. in.

1444. As this member is not required to resist compressive stress, its section could be made up simply by two $4\frac{1}{2}'' \times 1\frac{1}{8}''$ bars, giving a sectional area of 6.19 square inches. But it is desired to illustrate another expedient often employed when a tension member is required to resist a small amount of compressive stress also, namely, that of staying or latticing the chord bars together by bent lattice bars. The chord bars as thus latticed are shown in Mechanical Drawing Plate, Title: Highway Bridge: Details III, Fig. 3, from which the manner of latticing the bars will be readily understood. In order to avoid cutting out too much of the section of the chord bar, small size rivets are generally used to rivet the lattice bars. In order to avoid injury to the metal in the chord bars, the rivet holes are drilled in them. When the rivet holes are drilled, it is necessary to deduct only the actual size of the rivet hole, which is drilled one-sixteenth of an inch larger in diameter than the undriven rivet. In the present case the rivets will be $\frac{1}{2}''$ in diameter, and the rivet hole in the bars will be drilled $\frac{9}{16}''$ in diameter. The thickness t of the bars may be found by applying the following formula:

$$\frac{S}{n} = t\,(w - d), \text{ or } t = \frac{S}{n\,(w - d)}, \qquad \textbf{(143.)}$$

in which S is the sectional area, n is the number of bars used, w is the width of one bar, and d is the diameter of the rivet hole, or the sum of the diameters, if more than one rivet hole is to be deducted. If the value of t is known, and the value of any other one quantity is unknown, that value may be readily found by substituting the known values in the equation. In the present case, if two bars $4\frac{1}{2}''$ in width are used, by formula **143,** the thickness required for each bar will be $t = \frac{6.11}{2 \times (4.5 - .56)} = .78$ in. The thickness used will be the nearest even sixteenth of an inch greater than this amount, or $\frac{13}{16}$ of an inch; two bars $4\frac{1}{2}'' \times \frac{13}{16}''$ will be used. By substituting these dimensions in the first form given for formula **143,** we have $\frac{S}{2} = \frac{13}{16}\,(4.5 - .56)$. By solving this equation it is found that the actual net sectional area S given by two bars of these dimensions is 6.4 sq. in.

1445. If a member of this form is required to resist compression also, the radius of gyration is found by formula **140,** Art. **1433,** d being always the width of one bar. This value will always be the *least* radius of gyration of the section when y, the perpendiculars between the centers of the two bars, is greater than $.4\,d$.

The value of y is usually so much greater than $.4\,d$, however, that it is seldom necessary to apply the preceding rule. As stated above, this member is not required to resist compression, and could as well be composed of two ordinary chord bars of such size as would give the required section, without being latticed; they are proportioned with lattice bars simply as an illustration. Lower chord members designed to resist compression as well as tension are often called **stiffened lower chords.**

EXAMPLES FOR PRACTICE.

1. What is the least radius of gyration of the lower chord member *cc'*? Ans. 1.3 in.

2. What would be (*a*) the live load unit stress, and (*b*) the dead load unit stress allowed upon this member in compression?

Ans. { (*a*) 3,350 lb.
(*b*) 6,700 lb.

PROPORTIONING THE MATERIAL FOR FLOOR-BEAMS.

1446. The tensile unit stress allowed in proportioning the material for the bottom flanges of riveted floor-beams or cross girders is given in item (*c*), of Art. **1399.** The bottom flanges of simple beams are in tension, while in cantilever beams the upper flanges are the tension flanges. Item (*c*) applies to the *tension* flange. In almost all beams the tension flange is the bottom flange. The tension flange of a riveted beam must be proportioned in compliance with item (*k*), Art. **1399.** To items (*c*) and (*k*), Art. **1399,** and (*u*), Art. **1412,** the following must be added; those that have reference to plate girders being applicable to riveted floor-beams also:

(*w*) "In beams and plate girders the compression flanges shall be made of same *gross* section as the tension flanges."

(*x*) "Plate girders shall be proportioned upon the supposition that the bending or chord strains are resisted entirely by the upper and lower flanges, and that the shearing or web strains are resisted entirely by the web-plate; no part of the web-plate shall be estimated as flange area.

"The distance between centers of gravity of the flange areas will be considered as the effective depth of all girders."

(*y*) "The iron in the web-plates shall not be subjected to a shearing strain greater than 5,000 pounds per square inch, but no web-plates shall be less than three-eighths of an inch in thickness."

(*z*) "The webs of plate girders must be stiffened at intervals, about the depth of the girders, wherever the shearing strain per square inch exceeds the strain allowed by the following formula:

$$\text{Allowed shearing strain} = \frac{15{,}000}{1 + \frac{H^2}{3{,}000}}, \qquad \textbf{(144.)}$$

where H = ratio of depth of web to its thickness."

1447. By reference to the stress sheet, the maximum flange (designated in item (x) as "bending or chord strain") in the floor-beam of the example is found to be 53,700 lb., requiring a net sectional area, item (c), Art. **1399,** of $\frac{53{,}700}{12{,}000} = 4.48$ sq. in. Assuming the metal in the flange angles to be $\frac{3}{8}$ of an inch in thickness, the amount of sectional area to be deducted from the gross section for two rivet holes (one in each angle) is $\frac{3}{8} \times \frac{3}{4} \times 2 = .56$ sq. in. Therefore, the gross sectional area of the bottom flange must be equal to $4.48 + .56 = 5.04$ sq. in., making the weight of each angle equal to $\frac{5.04}{2} \times \frac{10}{3} = 8.4$ lb. If two $4'' \times 3''$ angles are used, the thickness of metal will be very closely $\frac{5.04}{(4 + 3) \times 2} = .36$ in., which is so near to $\frac{3}{8}$ of an inch (.38) that the sectional area deducted for the rivet holes may be considered correct. The bottom or tension flange of the floor-beam will consist of two $4'' \times 3''$ angles, each weighing 8.4 pounds per foot; and in compliance with item (w), Art. **1446,** the top or compression flange will be the same.

1448. The latter portion of item (y) specifies that no web-plate shall be less than three-eighths of an inch in thickness. If found to fulfil all other requirements, a web-plate $\frac{3}{8}$ of an inch in thickness will be used.

According to Art. **1321,** the maximum vertical shear in each end of the floor-beam = R_1 = 20,600 pounds. According to item (x) of Art. **1446,** this shear must be resisted entirely by the web-plate, while in compliance with item (y) the iron in the web-plate must not be subjected to a shearing stress greater than 5,000 pounds per square inch. The maximum shearing stress is a vertical stress of 20,600 pounds, and the sectional area given by a vertical section of the web-plate, after deducting the holes for the two lines of rivets connecting the flanges, is $22.5 \times \frac{3}{8} = 8.44$ sq. in. The maximum shearing stress to which the metal in the web-plate is subjected is $\frac{20,600}{8.44} = 2,440$ lb. per sq. in., which is well within the limit of 5,000 pounds per square inch fixed by item (*y*). A web-plate $\frac{3}{8}$ of an inch in thickness will, therefore, be amply sufficient to resist the shear.

According to item (*z*) the web of the floor-beam must be stiffened if the shearing stress per square inch exceeds the stress allowed by formula **144,** Art. **1446.** In the present case, H, the ratio of the depth of web to its thickness, is equal to $\frac{24}{.375} = 64$, and $H^2 = 64 \times 64 = 4,096$. Therefore, the allowed shearing stress per square inch is equal to

$$\frac{15,000}{1 + \frac{4,096}{3,000}} = 6,340 \text{ pounds.}$$

The shearing stress to which the web-plate is subjected has been found to be equal to 2,440 pounds per square inch, which is well within the limit of 6,340 pounds per square inch fixed by the same formula. It is thus found that a web-plate $\frac{3}{8}$ of an inch in thickness will fulfil all requirements of the specifications without stiffeners being used. But it is desired to illustrate the use of stiffeners, and, therefore, though not required on this beam, they will be used. A web-plate for this floor-beam $\frac{1}{4}$ of an inch thick would probably require stiffeners. This thickness is not uncommonly used for web-plates of light floor-beams.

THE FLOOR.

1449. The following items of the specifications have reference to the general features of the structure, but especially to the floor system:

(*A*) "All parts of the structures shall be of wrought iron or steel, except the flooring, floor joists, and wheel guards, when wooden floors are used. Cast iron or steel may be used in the machinery of movable bridges, and, in special cases, for bed-plates."

(*B*) "For all through bridges there shall be a clear head-room of 14 feet above the floor."

(*C*) "The floor joists will generally rest upon transverse iron floor-beams. They will be spaced not over 2 feet centers, and will lap by each other, so as to have full bearings on the floor-beams, and will be separated ½ inch for free circulation of air. Their scantling will vary in accordance with the length of panels selected, but shall never be less than 3 inches wide."

(*D*) "The floor plank shall be — inches thick, laid with ¼ inch openings, and spiked to each supporting joist. When this is to be covered with an additional wearing floor it must be laid diagonally; all plank shall be laid with the heart side down."

(*E*) "For bridges of classes *A* and *B* an additional wearing floor 1½ inches thick of white oak plank shall be placed over the above."

(*F*) "Where a foot-walk is required it will generally be placed outside of the trusses, and formed by longitudinal wooden joists, supported on wrought-iron overhanging brackets. The plank will be 2 inches thick and not over 6 inches wide, spaced with ¾ inch openings."

(*G*) "There will be a wheel-guard, of a scantling not less than 4″ × 6″, on each side of the roadway to prevent the hubs of wheels striking any part of the bridge. The roadways will be 12 feet wide for a single track, and multiples of 10 feet for a greater number of tracks."

(*H*) "A substantial railing will be placed at each side on all deck bridges. In through bridges the openings in the trusses must be closed by an iron railing of a suitable form;

there must be a substantial hand rail on the outside of the foot-walks."

(*I*) "The maximum strain allowed upon the extreme fiber of the joist will be 1,200 pounds per square inch on yellow pine and white oak, and 1,000 pounds per square inch on white pine and spruce."

(*K*) "When iron or steel joists are used they must be securely fastened to the cross floor-beams. The floor plank must have a thickness, *in inches*, at least equal to the distance apart of these joists, *in feet*. The floor plank must bear firmly upon the iron joist and be securely fastened to the same."

(*L*) "In calculating strains, the length of span shall be understood to be the distance between centers of end pins for trusses, and between centers of bearing plates for all beams and girders."

It will be noticed that in the design which has been taken as an example, item (*E*) and the latter portions of items (*G*) and (*H*) have not been complied with, while items (*F*) and (*K*) do not apply to this design. Item (*E*) is not very generally complied with. Many engineers consider that the tendency of the two layers of plank to retain moisture and thereby hasten decay, is greater than the advantage derived from the additional wearing floor. Probably the plank floor most commonly used for the roadways of bridges consists of a single layer of plank 3 inches in thickness. This thickness for the roadway floor plank has been found to give very satisfactory results in ordinary highway bridges, and for such bridges has been very extensively used. The roadway floor plank, if of a single layer, should not usually be less than 3 inches in thickness, and should not under any circumstances be less than 2½ inches thick. When of the latter thickness, hard lumber should always be used. In the example, the floor plank is 3 inches thick and the wearing floor is omitted.

1450. The dimensions of the joists vary with their span; that is, with the length of the panels. The most common varieties of timber used for joists are long leaf yellow pine (known also as southern pine and hard pine), white oak, white pine, and spruce. A very excellent timber known as Oregon fir is also being extensively used in the West. This timber may be considered as equivalent to long leaf yellow pine. It is generally accepted as good engineering practice to allow for timber in flexure a modulus, or assumed stress in the extreme fiber, of 750 pounds for white pine or spruce, 1,000 pounds for white oak, and 1,200 pounds for long leaf yellow pine.

Timber joists are usually spaced 2 feet apart from center to center. This is the common practice, and it is also in accordance with item (*C*). Table 33 gives the dimensions for joists, spaced 2 feet apart, center to center, required to sustain a total uniform load of 100 pounds per square foot, assuming the extreme fiber stresses to be 1,200 pounds, 1,000 pounds, and 750 pounds for long leaf yellow pine, white oak, and white pine, respectively. It will be noticed that in this table two widths are given in each case. The upper dimension is the calculated width, while the lower is the practical width to be used. For loads greater or less than 100 pounds per square foot, or for spacing greater or less than 2 feet, center to center, use widths proportionally greater or less than the computed (upper) width in each case given.

Item (*C*) of Art. **1449** requires that joists shall never be less than three inches wide. It is also not considered to be good engineering practice to use joists having widths less than one-quarter of their depths. It will be noticed that in a few cases both these conditions are violated in Table 33. When joists having depths greater than four times their widths are used, it is customary to brace them laterally by means of small pieces connecting the bottom of each joist with the top of each adjacent joist. Such bracing is called **bridging**; it is placed at the centers of the joists and at intervals of a few feet along their length.

TABLE 33.

Dimensions of Timber Joists or Stringers, spaced 2′ 0″ apart, center to center, required to support a uniformly distributed load of 100 lb. per square foot.

Panel Length.	Yellow Pine.		White Oak.		White Pine.	
Feet.	Inches.		Inches.		Inches.	
	Width.	Depth.	Width.	Depth.	Width.	Depth.
12	2.16 2.50	× 10	2.58 2.50	× 10	2.40 3.00	× 12
13	2.52 2.50	× 10	3.06 3.00	× 10	2.80 3.00	× 12
14	2.95 3.00	× 10	2.46 3.00	× 12	3.25 3.50	× 12
15	2.35 3.00	× 12	2.80 3.00	× 12	2.75 3.00	× 14
16	2.67 3.00	× 12	3.20 3.50	× 12	3.14 3.50	× 14
17	3.01 3.00	× 12	2.66 3.00	× 14	3.54 3.50	× 14
18	3.37 3.50	× 12	2.98 3.00	× 14	3.96 4.00	× 14
19	2.75 3.00	× 14	3.30 3.50	× 14	....	
20	3.05 3.50	× 14	3.67 4.00	× 14	....	
22	3.70 4.00	× 14	....		....	

1451. The number of lines of timber joists spaced 2 feet apart, center to center, required to support a roadway floor of any width is given by the formula

$$n = \frac{b}{2} + 1, \qquad \textbf{(145.)}$$

in which n is the number of lines of joists, and b is the clear width of roadway in feet.

In the example the floor is assumed to be of yellow pine timber. From Table 33 it is found that, with panel lengths of 18 ft., $3\frac{1}{2}'' \times 12''$ are the dimensions required for joists of this material.

For a clear width of roadway of 18 feet, according to formula **145,** there will be $\frac{18}{2} + 1 = 10$ lines of joists, as shown in Mechanical Drawing Plate, Title: Highway Bridge: General Drawing. These joists are spaced nominally 2 feet apart between centers, but really somewhat less than this.

Example.—For a width of roadway of 16 feet and a panel length of 14 feet, what are (*a*) the number of lines, and (*b*) the dimensions required for white oak joists spaced 2′ 0″ apart, center to center, to support a uniformly distributed load of 120 pounds per square foot?

Solution.—(*a*) According to formula **145,** there will be $\frac{16}{2} + 1 =$ 9 lines of joists. Ans.

(*b*) From Table **33,** the computed dimensions of white oak joists to sustain a uniform load of 100 pounds per square foot upon a panel length of 14 feet are $2.46'' \times 12''$. For a uniform load of 120 pounds per square foot the required width of the joists will be $2.46 \times 1.20 = 2.95$ in. The dimensions required for the joists will be $3'' \times 12''$. Ans.

EXAMPLES FOR PRACTICE.

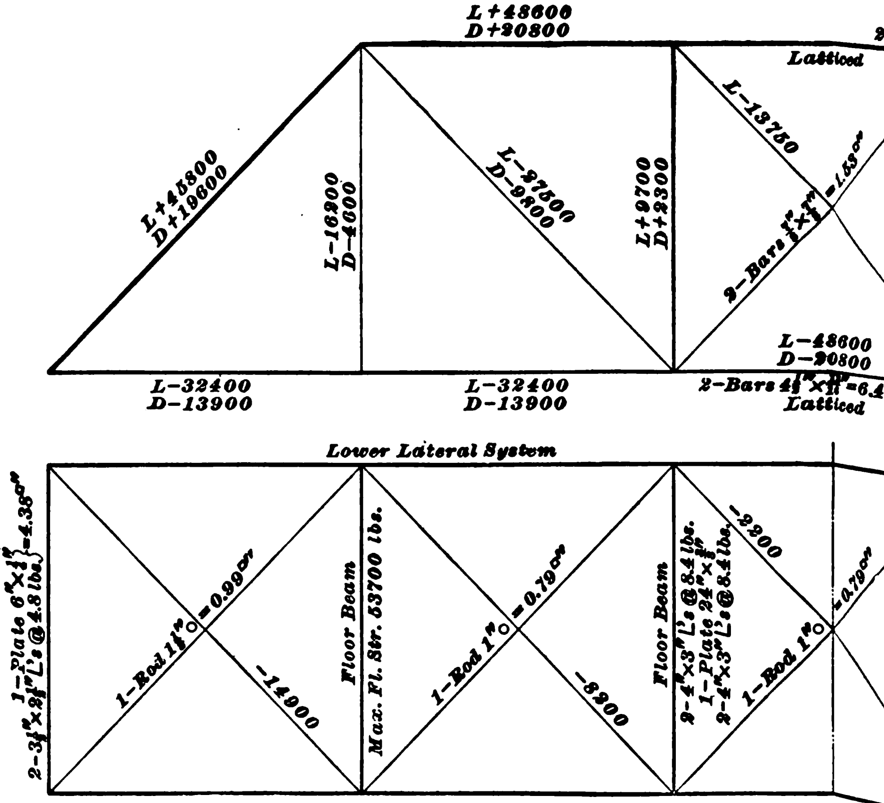
1. For a width of roadway of 14 feet, how many lines of joists will be required, if spaced 2′ 0″, center to center? Ans. 8 lines.

2 If the panel length is 18 feet, and long leaf yellow pine is used, what will be the dimensions of the joists required to support a uniform load of 90 pounds per square foot? Ans. $3'' \times 12''$.

3. The width of roadway in a bridge is 16 feet, and the panel length is 15 feet. (*a*) How many lines and (*b*) what dimensions of yellow pine joists spaced 2′ 0″, center to center, will be required to support a uniform load of 125 pounds per square foot? Ans. $\begin{cases} (a) \text{ 9 lines.} \\ (b) \ 3'' \times 12''. \end{cases}$

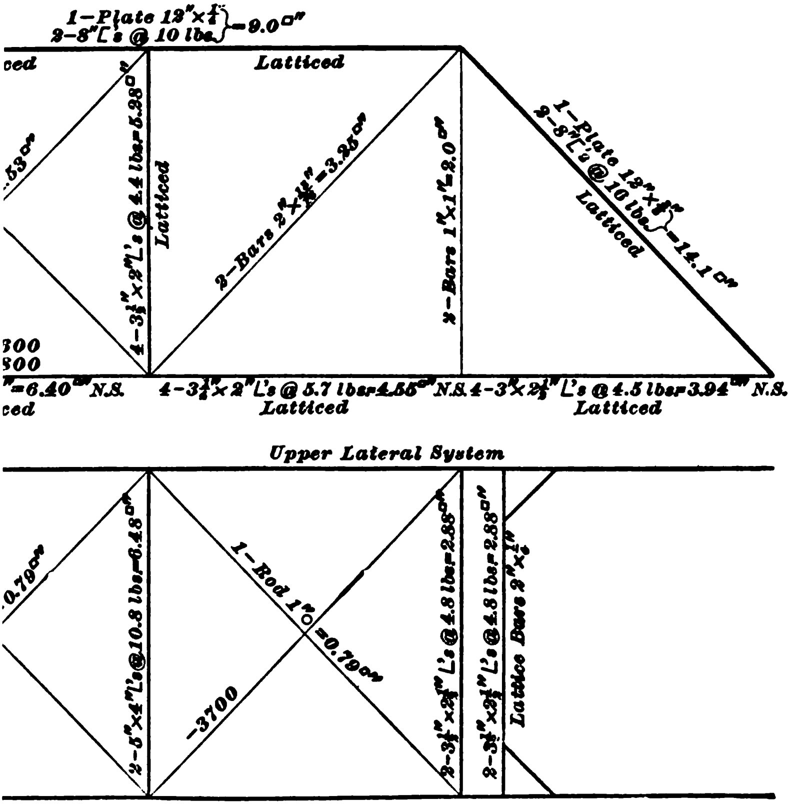
COMPLETING THE STRESS SHEET.

1452. The form and size of the material have now been determined for each member of the bridge, taken as an example to illustrate the general method of proportioning the material. In Fig. 306 is shown a complete stress sheet for the bridge. It is similar to Fig. 283, Art. **1322;** but it gives in addition the sections and general dimensions of the



Length of Span, C. to C. End Pins............. = 90′ 0″
Height of Trusses, C. to C. Chord Pins......... = 18′ 0″
Number of Panels = 5. Length of Each Panel.. = 18′ 0″
Clear Width of Roadway...................... = 18′ 0″
Material: Wrought Iron, proportioned according to Cooper's Specifications for Highway Bridges.

FIG. 306.


1–Plate 12″×¼″
2–8″[′s @ 10 lbs.} = 9.0□″
ced
Latticed
.53□″
4–3½″×2″L′s @ 4.4 lbs = 5.28□″
Latticed
2–Bars 2″×1⅝″ = 3.25□″
2–Bars 1″×1″ = 2.0□″
1–Plate 12″×⅜″
2–8″[′s @ 10 lbs.} = 14.1□″
Latticed
300
800
″ = 6.40□″ N.S.
ced
4–3½″×2″L′s @ 5.7 lbs = 4.55□″ N.S.
Latticed
4–3″×2½″L′s @ 4.5 lbs = 3.94□″ N.S.
Latticed
Upper Lateral System
0.79□″
2–5″×4″L′s @ 10.8 lbs = 6.48□″
1–Rod 1″○ = 0.79□″
–3700
2–3½″×2½″L′s @ 4.8 lbs = 2.88□″
2–3½″×2½″L′s @ 4.8 lbs = 2.88□″
Lattice Bars 2″×¼″

Live Load per sq. ft. . . . . = 100 lb.

Live Load per lin. ft. . . . . = 1,800 lb.

Dead Load per lin. ft. . . . . = 770 lb.

Roadway Floor Plank, 3″ thick, Yellow Pine.

10 lines Roadway Joists, 3½″ × 12″, Yellow Pine.

2 lines Wheel-Guards, 4″ × 6″, Yellow Pine.

16.

material. The material for each respective member is shown written along the member in the *right* half of the diagram of the truss and along each web member in the diagram of the lateral systems.

It will be noticed that in this stress sheet the stresses are omitted from all members of the lateral systems except the diagonal members. The effects of the lateral wind stresses upon the chords of the lateral systems are considered in connection with the live and dead load stresses in the chords of the truss; they are, therefore, sometimes written with the live and dead load stresses, but are much more commonly omitted. As the lateral struts are proportioned by the resultant of the initial stresses assumed upon the attaching rods, the wind stresses in those members need not be considered, and are usually omitted from the stress sheet.

The complete data for the bridge are written below the diagrams, as in the former stress sheet, with additional information designating the kind of material used and the specifications by which the material is proportioned; also, the kind and dimensions of the material for the floor. Sometimes, also, the location of the bridge and the name of the designer or builder are placed in the lower right-hand corner of the sheet.

The stress sheet as thus completed is intended to convey all necessary information with reference to the general design of the superstructure of the bridge ; it may be taken as a basis for receiving or submitting proposals or making the contract for the construction of the same. To a bridge engineer it gives nearly all the information relating to the superstructure that is necessary for making an estimate of the cost. It does not, however, give any information in regard to the details and connections, other than to designate the specifications by which they are to be proportioned, which is sufficient for the purposes of an estimate. Sometimes the stress sheet does not designate any general specifications, but is attached to, or accompanied by, a set of specifications drawn especially to suit the requirements of the structure for which the design is made.

1453. In making the complete stress sheet the student should follow the general arrangement shown in Fig. 306. The stress sheet should always show a side elevation of the truss and a half plan of each lateral system. Sometimes, for important structures, larger stress sheets are made, upon which are also drawn end views, showing the portal bracing, and intermediate lateral sections, showing the intermediate sway bracing, floor-beams, and connections.

In completing this stress sheet, write correctly, to one decimal place, the weight per foot of each rolled shape used, and write the sectional area of each entire section, using two decimal places, except in case of riveted tension members, for which write the *net* sectional area, following it by the letters N. S.

The weights per foot of plates and of round, square, or flat rectangular bars are not usually written. When the different portions of a built member are connected by means of lattice bars, the word "*latticed*" should be written beneath the member.

The thickness of the floor plank, the material of which it is composed, the number of lines, dimensions, and material of the joists, and the material and dimensions of the wheel guard should be given on the stress sheet, and also all other important conditions or data relating to the design.

THE USE OF STEEL.

1454. The members of a bridge are proportioned in steel in the same general manner as when proportioned in wrought iron. The unit stresses and compression formulas are somewhat different, but, otherwise, the process is substantially the same. The observations concerning the forms of tension members (Arts. **1399** to **1401**), and the formulas for the relative dimensions of compression members and for the radii of gyration of the different forms of sections (Arts. **1407, 1408, 1413, 1414, 1419,** and following) apply to steel as well as to iron.

Soft steel (Art. **1388**) is probably the best and most thoroughly reliable material for bridges. But medium steel is somewhat stronger than soft steel, and, being allowed higher unit stresses, is, therefore, more economical. As medium steel is also quite reliable, it is the material commonly used for bridges at the present time.

Specifications for the use of soft steel and medium steel vary even more than those for iron. The use of high steel in bridges is not usually either specified or allowed. A few of the common and prominent features of the specifications for the use of structural steel will now be noticed.

COOPER'S SPECIFICATIONS.

1455. In Cooper's General Specifications for Highway Bridges, the use of soft steel and medium steel is based upon the practice specified for wrought iron. The specifications referring to the use of steel are brief, and are here quoted in full. They are as follows:

"Medium steel may be used for tension members, plate girders, rolled beams, and top chord sections with an allowance of 20 per cent. increase above allowed working strains on wrought iron; and for all posts by use of the following formulas, in place of those given for wrought iron:

$$P = 10{,}000 - 70\frac{l}{r} \text{ for live load strains.} \qquad \textbf{(146.)}$$

$$P = 20{,}000 - 140\frac{l}{r} \text{ for dead load strains.} \qquad \textbf{(147.)}$$

$$P = 16{,}000 - 105\frac{l}{r} \text{ for wind load strains.} \qquad \textbf{(148.)}$$

"Provided, that, in addition to the previous details of construction (i. e., those given for wrought iron), all sheared edges of plates and angles be planed off to a depth of one-quarter of an inch. All punched holes be reamed to a diameter of $\frac{1}{8}$ inch larger, so as to remove all the sheared surface of the metal."

1456. "No sharp or unfilleted re-entrant corners to be allowed."

1457. "All rivets to be of steel."

1458. "Any piece which has been partially heated or bent cold be afterwards fully annealed."

1459. "Soft steel may be used under the same conditions as wrought iron for all *riveted* work.

"Provided, that any rivet hole punched, as in ordinary practice, will stand drifting to a diameter 25 per cent. greater than the original hole without cracking, either in the periphery of the hole or on the external edges of the piece, whether they be sheared or rolled."

1460. It will be noticed that while by the above specifications medium steel in tension members and in compression chord members is allowed unit stresses 20 per cent. greater than those allowed wrought iron, in posts it is quite otherwise. For short struts the unit stresses allowed by the compression formulas for steel posts are greater than those allowed by the compression formulas for iron posts, while for long struts the formulas give less unit stresses for steel than for iron. The live and dead load unit stresses allowed by the post formulas for steel are greater than those allowed for iron when the strut length is less than 62½ times its radius of gyration, but they are less than those allowed for iron when the strut length exceeds 62½ radii.

For the value given by the iron formula $8,750 - 50\frac{l}{r}$ becomes equal to the value given by the steel formula $10,000 - 70\frac{l}{r}$, when the quotient $\frac{l}{r}$ equals 62.5; and the former value exceeds the latter when $\frac{l}{r}$ exceeds this amount. Many engineers consider this specification for medium steel to be unnecessarily severe. Most specifications allow greater unit stresses for soft steel than for wrought iron.

EXAMPLES FOR PRACTICE.

See stress sheet, Fig. 306.

1. If medium steel is used, what is the sectional area required for the main tie bar *Bc*? Ans. 2.7 sq. in.

2. What size of bars will give the required area for this member?

3. What is the sectional area required for the counter *Cc'*, if composed of medium steel? Ans. 1.15 sq. in.

4. Of what should this member consist in order to have the required area?

5. Using 8-inch channels of medium steel, what is (*a*) the live load unit stress and (*b*) the dead load unit stress allowed for the upper chord? Ans. $\begin{cases} (a) \text{ 8,760 lb.} \\ (b) \text{ 17,520 lb.} \end{cases}$

6. What is the sectional area required for the upper chord? Ans. 6.74 sq. in.

7. What is (*a*) the live load unit stress and (*b*) the dead load unit stress allowed for the end post, using the same size of steel channels? Ans. $\begin{cases} (a) \text{ 3,315 lb.} \\ (b) \text{ 6,630 lb.} \end{cases}$

8. What is the sectional area required for the end post? Ans. 16.78 sq. in.

9. What is (*a*) the live load unit stress and (*b*) the dead load unit stress allowed for the intermediate post, using 3¼″ × 2″ angles of medium steel? Ans. $\begin{cases} (a) \text{ 1,600 lb.} \\ (b) \text{ 3,200 lb.} \end{cases}$

10. What sectional area is required for the intermediate post? Ans. 6.78 sq. in.

REMARKS.

1461. A comparison of the preceding results with the corresponding results as previously obtained for iron sections will be found instructive. It will be noticed that the sectional areas obtained above for the tension member and for the upper chord are less than obtained for the corresponding members when proportioned in iron, while those obtained for the posts are greater than for the iron posts. This feature of these specifications is by many engineers considered to be inconsistent and objectionable.

1462. In designing bridges, according to Cooper's specifications for medium steel, it is always advantageous to make the heights of truss somewhat lower than would ordinarily be used in designing by most other specifications. This statement applies to iron as well as to steel, though in a less degree. When designing a bridge having parallel chords, according to Cooper's specifications for medium steel, a height of truss equal to about one-seventh the span will usually afford good economy.

In the example for which the material has been proportioned in iron, if the height of truss had been taken as low as the required head-room would permit, say 15 ft., the sections of the top chord and end post would have been more nearly uniform. The sections required for the chords would be increased by reason of the increased chord stresses, while the sections of the posts would be diminished by reason of the diminished strut length. The dead load stresses in the lower chord would be increased, while the wind stresses would be somewhat diminished.

THE FATIGUE OF METALS.

1463. What is ordinarily called the *ultimate strength* of a material is the greatest stress to which it can be subjected by a force applied *gradually* and for *a moderate length of time;* it occurs at or just before rupture. Experiments, however, have shown that when forces below the ultimate strength are constantly or repeatedly applied, they may finally produce rupture; their effect seems to be to exhaust the material, or to *tire it out*, so to speak; whence the name *fatigue of metals* is applied to this phenomenon. Later experiments have shown that the true cause of the phenomenon lies in the imperfections and lack of homogeneity of the material.

1464. The following law was discovered by A. Woehler, after a series of experiments, and is known as **Woehler's law:**

Rupture may be caused not only by a force exceeding the ultimate strength, but by the repeated or prolonged action of forces below the ultimate strength. When these forces are alternately applied, the unit stress that finally causes rupture *depends upon the range of stress, that is, upon the difference between the alternately applied forces. As this difference increases, the number of applications necessary to produce rupture becomes less.*

Thus, wrought iron was found to fail after 800 applications of a tensile force of 52,000 pounds per square inch; also, after 409,000 applications of a tensile force of 39,600 pounds per square inch. With forces alternating between 22,000 and 48,400 pounds per square inch, the number of applications required for rupture was nearly 2,400,000; whereas, with a range between 26,400 and 48,400 pounds per square inch, 4,000,000 applications were made *without producing rupture.* In the two cases last mentioned, although the higher limit of stress was the same (48,400), the effects were different, owing to the difference in the *range*, 48,400 — 26,400 being less than 48,400 — 22,000. In some cases rupture (by bending) has been produced by the repeated application of a load of only two-fifths the breaking load calculated from the ultimate strength; a bar that could stand a gradually applied load of 26,000 pounds broke under a load of 10,480 pounds after 5,200 applications.

1465. The preceding facts are of great importance in the design of such structures as bridges which are subjected to varying stresses, whether of the same or of opposite kinds; and it will be readily seen that the resistance of a member can not be accurately determined from considerations of *ultimate strength* only, but the phenomenon of fatigue must be taken into account.

When a member is subjected alternately to stresses of the same kind (all tension or all compression), the allowed or working unit stress P (pounds per square inch) is commonly taken as given by the following formula:

$$P = a\left(1 + \frac{\textit{minimum stress}}{\textit{maximum stress}}\right). \qquad \textbf{(149.)}$$

When the alternating stresses are of different kinds,

$$P = a\left(1 - \frac{\textit{maximum stress of smaller kind}}{2 \times \textit{maximum stress of greater kind}}\right). \qquad \textbf{(150.)}$$

In both of these formulas a is a constant depending on the material and on the kind of stress. The value of a is not necessarily the same for the two cases considered.

Other and more accurate formulas have been constructed, but those here given are the ones required by some of the most modern specifications.

1466. Of specifications founded on the theory of the fatigue of metals, those of Mr. Edwin Thacher for highway bridges may be mentioned. In them formula **149** is used for tension members, with the following values for a:

Member.	Values of *a*.		
	Wrought Iron.	Soft Steel.	Medium Steel.
Chords, ties, counters, long suspenders	9,400	10,800	11,700
Plates and shapes	8,500	9,700	10,500

For members in which tension and compression alternate, formula **150** is used, with

$$a = \begin{cases} 9{,}400 \text{ for wrought iron,} \\ 10{,}800 \text{ for soft steel,} \\ 11{,}700 \text{ for medium steel.} \end{cases}$$

Other members are proportioned by ordinary formulas, which, although giving somewhat different values from those we have used, involve no new principles.

EXAMPLE.—What are, according to Thacher's specifications, the unit stress allowed for hip vertical $B\,b$ (Fig. 306), and the required area (a) when the member is to be of wrought iron, and (b) when the member is to be of medium steel?

SOLUTION.—Since the hip vertical is a long suspender, formula **149** must be used, with the values of a given above for tension members. By reference to Fig. 306, it is found that the minimum stress to which the member is subjected is 4,600 pounds (when the truss is unloaded), while the maximum stress $= 16{,}200 + 4{,}600 = 20{,}800$ lb. (when the truss is loaded). These values in formula **149** give, calling the required area A, and remembering that required area = total stress ÷ allowed unit stress :

(*a*)
$$P = 9{,}400\left(1 + \frac{4{,}600}{20{,}800}\right) = 11{,}480 \text{ lb.}$$

$$A = \frac{20{,}800}{11{,}480} = 1.81 \text{ sq. in.}$$

(*b*)
$$P = 11{,}700\left(1 + \frac{4{,}600}{20{,}800}\right) = 14{,}290 \text{ lb.}$$

$$A = \frac{20{,}800}{14{,}290} = 1.46 \text{ sq. in.}$$

SOME ITEMS FROM THACHER'S AND LEWIS'S SPECIFICATIONS.

1467. The requirements of the many specifications in existence are very different; but, if the student has mastered the *general principles* on which they are founded, and the *general methods* by which they are applied, he will experience no difficulty in adapting his knowledge to the different requirements; the question is simply one of substituting different values for the constant quantities contained in a few *general* formulas.

Some items not before mentioned are of importance, and they are here quoted from the specifications of E. Thacher, and those of F. H. Lewis.

1468. From Thacher's Specifications for Highway Bridges.—"*Clearance.* All through bridges shall have a clear height of not less than 14 feet."

1469. "The width from *c* to *c* of trusses shall not be less than one-twentieth of the span from *c* to *c* of pins."

NOTE.—*c* stands for center.

1470. "*Flanges.*—The compressed flanges (of girders) will be stayed transversely when their length is more than thirty times their width."

1471. "*Webs.*—One-sixth of the web will be considered as available area in each flange, except at web splices, where the full section will be provided by extending the flange plates, or by the addition of separate cover plates. All joints will be spliced by a plate on each side of web."

1472. "*Stiffeners.*—All web-plates shall be stiffened at the inner edges of end bearing, and at all points of local concentrated loadings. Intermediate stiffeners will be used if the shearing stress per square inch exceeds 12,000 $\div \left(1 + \frac{d^2}{3,000\, t^2}\right)$; in which d = clear distance between flange angles or stiffening angles and t = thickness of web, both in inches."

1473. *Timber.*—For beams, allowed fiber stress for yellow pine and white oak, 1,200 pounds per square inch; for white pine or hemlock, 900 pounds per square inch.

1474. For timber struts, the allowed unit stress shall be determined by the following formulas:

	Yellow Pine.	White Pine.
Flat ends....................	$1,075 - 112\frac{L}{d}$	$675 - 94\frac{L}{d}$
One flat and one pin end......	$1,075 - 125\frac{L}{d}$	$675 - 100\frac{L}{d}$
Pin ends......................	$1,075 - 138\frac{L}{d}$	$675 - 107\frac{L}{d}$

In which L = length of member *in feet.*
d = least dimension of member *in inches.*

1475. The following are the allowed unit stresses for shearing and bending:

	Yellow Pine.	White Pine.
Shearing—Sliding on grain...	130	100
Bearing — Direction of grain..	1,200	900
Perpendicular to grain............	300	200

1476. From Lewis's Specifications for Railroad Bridges.—"*Superstructure.*—For spans of 16 feet or less, rolled beams will be used, and from 16 to 100 feet, riveted plate girders. All spans over 100 feet will be pin-connected trusses."

1477. *Material Used.*—It is required:

"1. That all eye-bars and pins shall be of medium steel.

"2. That all web-plates shall be of steel.

"3. That loop rods and all other devices which are welded shall be of wrought iron.

"These requirements are common to all bridges, whether built of wrought iron, soft steel, or medium steel. The other parts of bridges, however, may be built of such grades of material as the contractor may elect, provided only that each member and each set of members performing similar functions must be of the same grade of material throughout."

1478. "*Dead Load.*—The dead load shall consist of the entire structure. The load of the structure when complete shall not exceed the dead load used in calculating the stresses."

1479. "In through bridges, two-thirds ($\frac{2}{3}$) of the dead load shall be assumed as concentrated at the joints of the bottom chord, and one-third ($\frac{1}{3}$) at the joints of the upper chord."

1480. "In deck bridges, two-thirds ($\frac{2}{3}$) of the dead load shall be assumed as concentrated at the joints of the upper chord, and one-third ($\frac{1}{3}$) at the joints of the bottom chord."

1481. "*Plate girders* shall be proportioned upon the supposition that the bending or chord strains are resisted entirely by the upper and lower flanges, and that the shearing or web strains are resisted entirely by the web-plate."

1482. "*Stiff suspenders* must be able to carry a compressive strength equal to six-tenths ($\frac{6}{10}$) the maximum tensile stress."

1483. "Stiffened chords will be proportioned to take compression equal to $60\frac{T}{L}$, in which T is the maximum tension in pounds in the chord, and L is the span in feet."

1484. "*Coefficients of friction* will be used as follows:

"Wrought iron or steel on itself	.15
"Wrought iron or steel on cast iron	.20
"Wrought iron or steel on masonry	.25
"Masonry on itself	.50"

COMPARATIVE WEIGHT AND SPECIFIC GRAVITY OF WROUGHT IRON AND STEEL.

1485. The specific gravity, and, therefore, the weight of both wrought iron and steel, vary according to the purity of the metal, and also according to the density imparted to it by the process of rolling.

As a rule, soft steel possesses somewhat higher specific gravity than high steel, while both are denser than iron.

The following tables give fair average values of the specific gravity and weight of wrought iron and these two grades of steel:

TABLE 34.

ORDINARY SMALL ROLLED BARS.

Material.	Specific Gravity.	Weight per Cubic Foot, Pounds.	Weight per Cubic Inch, Pounds.
Soft Steel	7.86	490.1	.2836
High Steel	7.83	488.2	.2825
Wrought Iron	7.70	480.1	.2778

TABLE 35.

LARGE ROLLED SECTIONS.

Material.	Specific Gravity.	Weight per Cubic Foot, Pounds.	Weight per Cubic Inch, Pounds.
Soft Steel	7.84	489.0	.2830
High Steel	7.80	486.6	.2816
Wrought Iron	7.67	478.3	.2768

1486. As stated in Art. **1415,** it is customary to estimate the weight of rolled iron at 480 pounds per cubic foot, which is believed to be practically correct for the average material. No allowance is made for the slight decrease in the weight of large sections.

Medium steel, having properties about midway between those of soft steel and high steel, will average about 2 per cent. heavier than iron. It is customary to estimate all grades of structural steel 2 per cent. heavier than iron.

Upon this basis the following values are obtained for steel:

By adding 2 per cent. to the value of w in formula **132,** Art. **1415,** the following value of W, the weight per lineal foot of a bar of steel, is obtained:

$$W = \frac{10}{3} A \times 1.02 = 3.4 A. \qquad \textbf{(151.)}$$

Similarly, $$A = \frac{W}{3.4}. \qquad \textbf{(152.)}$$

In both of the above formulas, A is the sectional area of a steel bar of any shape having a uniform cross-section, i. e., the sectional area of any rolled steel bar.

Example.—A bar of steel has a sectional area of .4418 of a square inch; what is its weight per foot?

Solution.—By applying formula **151,** the weight per foot is found to be $.4418 \times 3.4 = 1.502$ lb. Ans.

Note.—Some engineers prefer to estimate the weight of steel in the same manner as wrought iron and increase the result by 2 per cent. Thus, in the above example, the weight per foot of the bar, if of wrought iron, would be $.4418 \times \frac{10}{3} = 1.473$ lb., which, when increased by 2 per cent. equals $1.473 \times 1.02 = 1.502$ lb., as above.

Example.—What is the sectional area given by two 6-inch channels, each weighing 8 lb. per foot, and a $10'' \times \frac{1}{4}''$ cover plate?

Solution.—The area given by the cover plate is $10'' \times \frac{1}{4}'' = 2.50$ sq. in. The area given by the two channels equals $\frac{8 \times 2}{3.4} = 4.71$ sq. in. Therefore, the total area given by the section equals $2.50 + 4.71 = 7.21$ sq. in. Ans.

1487. The weight per foot of a *round* bar of steel may readily be obtained by applying the following convenient

Rule.—*Square the diameter of the bar, expressed in quarter inches, and divide by 6.*

This may also be expressed by the following formula:

$$W = \frac{d^2}{6}, \qquad \textbf{(153.)}$$

in which W is the weight per foot of bar in pounds, and d is the diameter of the bar *in quarter inches.*

Example.—What is the weight per foot of a round bar of steel ¾ inch in diameter?

Solution.—Expressing the diameter of the bar in quarter inches and applying formula **153,** the weight per foot is $\frac{3 \times 3}{6} = 1.50$ lb. Ans.

ECCENTRICITY. POSITION OF PINS. BENDING STRESSES DUE TO WEIGHT OF MEMBER.

1488. The bending moment upon a member under direct compressive stress, due to the weight of the member itself, should always be taken into consideration in fixing the positions of the pins. Some specifications also require the unit stress allowed upon the member to be reduced by the amount of stress per square inch required to resist this bending moment; or, what amounts to the same thing, require the sectional area to be correspondingly increased. Many engineers, however, consider this latter to be an unnecessary degree of refinement for highway bridges. But it is very essential that the pins be placed as nearly as practicable in their proper positions.

1489. *Cooper's specifications contain the following:*

"If the fiber strain resulting from the weight only of any member exceeds ten per cent. of the allowed unit strain on such member, such excess must be considered in proportioning the areas."

1490. *The following are from Thacher's specifications:*

"For top chords, the stresses per square inch due to weight of member will be deducted from the above unit stresses, the reduction for chords flat at one end being one-half, and for chords flat at both ends one-third the amount for members with pin ends.

"Unsymmetrical sections, composed of two rolled or riveted channels and one plate, shall in chords be so proportioned that the centers of pins shall be in the same line and in the centers of gravity of sections. In web members eccentricity may be made sufficient to counteract the bending stress due to weight of member under maximum load."

1491. *The following are from Lewis's specifications:*

"The effect of the weights of horizontal or inclined members in reducing their strength as columns must be provided for. It will also be considered in fixing the positions of pin centers.

"All eccentricity of stress shall be avoided. Pin centers will be in the center of gravity of the members, less the eccentricity required to provide for their own weight; and in continuous chords, pin centers must be in the same plane."

In these last specifications the conditions are very clearly stated, and the provisions there indicated will be quite fully noticed. Bending stresses in tension members, produced by their own weight, are not usually provided for, as their only effect is to slightly increase the stress in the lower portion of the member. Furthermore, as the member deflects under its own weight, the bending stress becomes somewhat relieved by the deflection. But with a compression member the case is quite different. The bending moment produces eccentricity of stress; the effect of the bending stress combined with the direct stress is practically the same as that of an eccentric load upon a column, i. e., it diminishes the efficiency of the member as a column. And this weakened condition is further augmented by the deflection of the member due to its own weight. It is thus evident that the bending stress upon a compression member, due to its own weight, materially impairs its strength and should be provided for.

1492. The **position of the center of gravity** of a member is found by a process very similar to that explained in Art. **1373** for finding the center of gravity of a system of wheel loads. The position of a horizontal line passing through the center of gravity of a section may be readily found by applying the following

Rule.—*Multiply the area (or weight) of each separate piece composing the section by the distance of its center of gravity from some given or assumed horizontal line, and divide the sum of the products by the sum of the areas (or weights). The quotient will be the distance of the center of gravity of the entire section from the assumed line.*

NOTE.—The assumed horizontal line from which distances are measured may be *any* horizontal line, but should be preferably so situated that the center of gravity of each piece composing the section should be *on the same side of the line*. Thus, it is convenient to assume the line to pass through the center of the cover-plate, the top of the cover-plate, or the lower edge of the channels. All distances must be *from the same line.*

EXAMPLE.—For the section of the upper chord given in Fig. 306 (see also Mechanical Drawing Plate, Title: Highway Bridge: Details I, Fig. 1), find the distance of the center of gravity of the section below the tops of the channels.

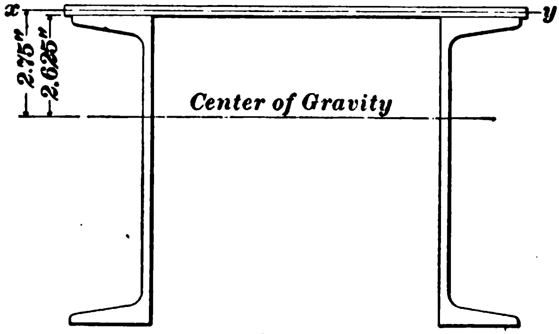


FIG. 307.

SOLUTION.—The form of the section is shown in Fig. 307; the effective section is composed of two 8-inch channels weighing 10 pounds per foot and a 12" × $\frac{1}{4}$" cover-plate. The sectional area of each channel is 10 × $\frac{3}{10}$ = 3 sq. in., and that of the cover-plate is 12" × $\frac{1}{4}$" = 3 sq. in. If all distances be taken from a horizontal line through the center of the cover-plate, as xy in the figure, the distance of the center of gravity of the section from that line may be found by the following computation:

	Area Sq. In.		Distance.	
Cover-plate..................	3.00	×	0.000 =	0.00
Two channels...............	6.00	×	4.125 =	24.75
	9.00			24.75

Hence, $\frac{24.75}{9} = 2.75$ in. is the distance of the center of gravity of the section below the horizontal line xy through center of cover-plate. As this line is $\frac{1}{8}$ inch above the tops of the channels, the center of gravity of the section is $2.75 - 0.125 = 2.625$ in. below the tops of the channels. Ans.

In sections similar to the above, the distance from the center of gravity of the section to the center of the channels or vertical plates is called the **eccentricity** of the section. Thus, the eccentricity of the above section is $\frac{8}{2} - 2.625 = 1.375$ inches.

If a pin-connected member were subjected to direct longitudinal stress (tension or compression) only, the pins should pass exactly through its center of gravity.

But, as a horizontal or inclined member is always subjected to a bending stress due to its own weight, this condition never fully obtains. As noticed above, it is seldom necessary to provide for the bending stress in a tension member, due to its own weight, and, consequently, the centers of the pins are usually located at the center of gravity of the member. But in a compression member the effect of the bending stress should be considered in fixing the positions of the pins.

1493. Bending Moment on a Member Due to Its Own Weight.—Fig. 308 represents the entire upper chord, of which a section is shown in Fig. 307. The chord is shown in Mechanical Drawing Plates, Titles: Highway Bridge: Details I, and Highway Bridge: General Drawing. It will be noticed that in supporting its own weight the chord acts as a beam continuous over two supports. For compressive stresses some specifications consider each end panel of the upper chord as having one flat and one pin end, and all intermediate panels as having flat ends. For direct bending, the end panels are considered as beams having one end fixed and the other simply supported; and all intermediate panels as beams fixed at both ends.

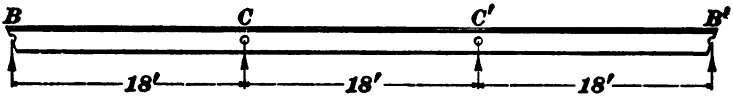


FIG. 308.

In a uniformly loaded beam having one or both ends fixed, the greatest bending moment occurs at the fixed end. Formulas for the maximum bending moments in such beams are given in the Table of Bending Moments. As, however, the compression members of bridges resist principally direct compressive stress as columns, and as columns do not commonly fail at the ends, but near the center, the bending moments at the ends may be neglected. It will, therefore, be necessary to consider only the maximum values of those bending moments which occur along the central portion of the member.

Neglecting, then, the bending moments at the ends, the greatest bending moment M_o which can occur along the central portion of a uniformly loaded beam fixed at both ends will occur at the center; its amount is given by the formula

$$M_o = \frac{w\, l_o\, l_h}{24}. \qquad \textbf{(154.)}$$

1494. The greatest bending moment M_1 which can occur along the central portion of a uniformly loaded beam fixed at one end and simply supported at the other will occur at a distance of $\frac{3}{8}\, l_o$ from the free end; its amount is given by the formula

$$M_1 = \frac{9\, w\, l_o\, l_h}{128}. \qquad \textbf{(155.)}$$

1495. In a uniformly loaded beam simply supported at both ends, the maximum bending moment M occurs at the center; its amount is given by the formula

$$M = \frac{w\, l_o\, l_h}{8}. \qquad \textbf{(156.)}$$

In all of the above formulas w is the load, or weight of member per unit of length, l_o is the length of the beam or member, from center to center of supports or connections, and l_h is the horizontal projection of l_o, which is usually simply the panel length. The unit of length must be the same for l_o as for w. It will be found most convenient to take w as the weight per foot of member and l_o as the length of member in feet. In order that the bending moment shall be in inch-pounds, l_h should be expressed in inches.

1496. If the member is horizontal, then the distance l_o will equal the distance l_h. Calling $l_o = l_h = l$, formula **156** becomes

$$M = \frac{w\, l^2}{8}, \qquad \textbf{(157.)}$$

which is the same as formula 7 of the Table of Bending Moments.

1497. In applying any one of these formulas to obtain the bending moment in a built member, due to its own weight, the weight per foot, as estimated from the effective section, should be increased about 15 per cent. to provide for the weight of lattice bars, rivet heads, etc.

EXAMPLE.—What is the bending moment in inch-pounds upon the end panel of the top chord designated in Fig. 306 and shown in Fig. 308?

SOLUTION.—The sectional area of the chord has been found to be 9 square inches; hence, the weight per foot of the effective section is $9 \times \frac{10}{3} = 30$ lb., and the total weight per foot about $30 + .15 \times 30 = 34.5$ lb. As this panel of the chord may be considered as a beam fixed at one end only, the bending moment is found by applying formula **155.** In this case, $w = 34.5$, $l_o = 18$, and $l_h = 18 \times 12$; therefore, the bending moment

$$M_1 = \frac{9 \times 34.5 \times 18 \times 18 \times 12}{128} = 9{,}431 \text{ in.-lb.} \quad \text{Ans.}$$

This bending moment is positive, and tends to produce compression in the upper portion of the chord and tension in the lower portion. It does not actually produce tension in any portion of the chord, because the direct compression in the chord, being much greater than the bending stress, entirely overcomes the tension, so that the effect of the bending stress is really to increase the compression in the upper portion of the chord and diminish it in the lower portion.

1498. The **position of the pins** may be so fixed that when the maximum direct compressive stress comes upon the chord, through the pins, it will produce an amount of bending moment equal to that produced by the weight of the member, but in the opposite direction (negative).

This may be more clearly understood by reference to Fig. 309, which is a somewhat distorted representation, in outline, of the end-panel member of a top chord, indicating in a general way the relative positions of the forces.

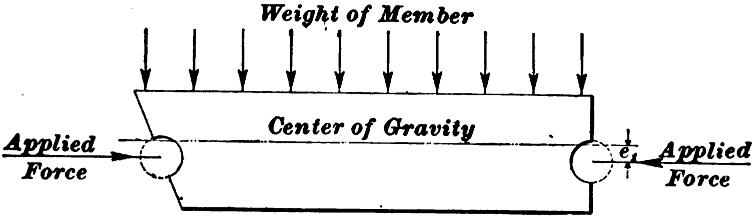


FIG. 309.

The two opposite bending moments will then balance each other, and the resultant stress will be direct compression uniformly distributed over the section. In the ordinary form of upper chord and end-post sections, this condition will be effected when the position of the pins is fixed at a distance e_1 perpendicularly below the center of gravity of the section given by the formula

$$e_1 = \frac{M}{S_c}, \qquad \textbf{(158.)}$$

in which e_1 is the distance in inches below the center of gravity of the section at which pins should be fixed, M is the bending moment in inch-pounds due to the weight of the member, and S_c is the total maximum direct compressive stress upon the member in pounds.

1499. In fixing the positions of the pins in compression members in which the center of gravity of the section lies above the center, the total eccentricity, as previously obtained, must be *reduced* by the amount e_1, which may be considered as **negative eccentricity.** The eccentricity, as thus reduced, is often called the **final,** or **net eccentricity.**

EXAMPLE.—At what distance below the center of gravity should the positions of the pins be fixed in the end panel of the upper chord, Figs. 306 and 308, in order that the stress be evenly distributed over the section?

SOLUTION.—The bending moment due to the weight of the member has been found to be 9,431 inch-pounds = M. By reference to Fig. 306, the total amount of direct compressive stress upon the member is known to be 48,600 + 20,800 = 69,400 lb. = S_c. Therefore, by applying formula **158,** e_1, or the distance below the center of gravity of the section at which the pins should be placed, is found to be $\frac{9,431}{69,400} =$ .136 in. Ans.

EXAMPLE.—What is the net eccentricity of this section?

SOLUTION.—The total eccentricity of the section has been found to be 1.375 inches. The eccentricity in the opposite direction due to the weight of the member, or the negative eccentricity, has been found to be .136 inch. Therefore, the net eccentricity is 1.375 − .136 = 1.24 in. Ans.

1500. Increase of Section to Provide for Bending Stress.—If in the above example the position of the pins be fixed at 1.24 inches, or practically $1\frac{1}{4}$ inches, above the center of the channels, the stress will presumably be distributed uniformly over the section. But in proportioning the material for this member, provision was made only for the direct compressive stress, while it has been shown that the member must also resist a bending stress due to its own weight. Therefore, the sectional area of the member should be increased an amount corresponding to the amount of this bending stress. As previously stated, this, by many engineers, is considered an unnecessary degree of refinement in designing the members of highway bridges; but as some specifications require provision to be made for this condition, a method of determining the required increase of section will be explained. The method given below, though not in strict accord with some elaborate formulas which have been deduced for this purpose, is believed to give results sufficiently accurate for all practical purposes, and has the advantage of being very simple and easily applied.

Formulas **72,** Art. **1241,** and **73,** Art. **1243,** give the following values for the resisting moment, which must be equal to the bending moment:

$$M = \frac{S\,I}{c} = \frac{S\,A\,r^2}{c}, \qquad \textbf{(159.)}$$

in which, as relating to the present case, M is the bending moment in inch-pounds, S may be the fiber stress per square inch or the assumed unit stress, A is the required or the assumed sectional area in square inches, r is the radius of gyration of the section in inches, and c is the distance in inches from the neutral axis to the most remote fiber.

In order to express the *sectional area* required to resist the bending moment, formula **159** may be given the form

$$A = \frac{M\,c}{S\,r^2}. \qquad \textbf{(160.)}$$

By taking S to represent the allowed unit stress, and bearing in mind that the formula is applied after the general dimensions of the section are determined, it will be noticed that the values of all the quantities in the second term of formula **160** are known, and the value of A can be found by simply substituting these values.

But some specifications require the fiber stress, produced by the bending moment, to be deducted from the allowed unit stress. To meet this requirement, a formula expressing the amount of fiber stress per square inch may be derived from formula **159,** as follows:

$$S = \frac{M\,c}{A\,r^2}. \qquad \textbf{(161.)}$$

As the bending stress due to the weight of the member is always very small as compared with the direct compressive stress upon the member, the value of A as determined from the latter will be sufficiently accurate to use in formula **161.**

As previously stated, the bending moment tends to increase the compression in the *upper* portion of the section; hence, the stress in the extreme upper fiber becomes the critical stress, and, in applying the above formulas, the value taken for c should be the distance from the neutral axis to this fiber, or, in other words, the distance from the center of gravity of the section to the upper side of the cover-plate.

EXAMPLE.—Referring to the examples previously given in this article, what is the sectional area required to provide for the bending stress upon the end panel of the upper chord, Figs. 306 and 308?

SOLUTION.—The bending moment due to the weight of the member has been found to be 9,431 in.-lb. $= M$. The distance from the center of gravity of the section to the top of the channels has been found to be 2.625 inches; hence, the distance from the center of gravity to the top of the cover plate is $2.625 + .25 = 2.875$ in. $= c$. The bending stress due to the weight of the member is a dead load stress; consequently, for the value of S in formula **160,** the dead load unit stress should be used. By reference to Art. **1422,** the dead load unit stress used for this section is known to be 14,600 lb. $= S$, and the radius of gyration of the section is 3.2 in. $= r$. Therefore, by applying formula **160,** the sectional area required to resist the bending moment due to the weight of the member is found to be $\frac{9,431 \times 2.875}{14,600 \times 3.2^2} = .18$ sq. in. Ans.

EXAMPLE.—For the same, what is the amount of stress per square inch of section produced by the bending moment, assuming the stress to be uniformly distributed over the entire section?

SOLUTION.—The same values are substituted in formula **161** that were substituted in formula **160** in the preceding example, except that the area of the section, as obtained for the direct compressive stresses, is used instead of the unit stress. The area of the section is 9 square inches, and the stress per square inch required to resist the bending moment is found to be

$$\frac{9,431 \times 2.875}{9 \times 3.2^2} = 294 \text{ lb. Ans.}$$

1501. In determining the final area of the section, the area required for the bending stress, as obtained by formula **160,** may be added to the area required for the direct compressive stress; or, what amounts to the same thing, the stress per square inch required to resist the bending moment, as determined by formula **161,** may be deducted from the unit stress.

The members of the bridge shown in Mechanical Drawing Plates, Titles: Highway Bridge: Details I, II, III, IV, and Highway Bridge: General Drawing, were proportioned according to Cooper's specifications, which do not require that the bending stress upon a member due to its own weight shall be considered in proportioning the areas, unless it "exceeds ten per cent. of the allowed unit strains on such member." As the bending stress per square inch (294 lb.), determined above, does not exceed ten per cent. of the allowed dead load unit stress (.10 × 14,600 = 1,460 lb.) it need not be considered in proportioning the area of the section.

EXAMPLES FOR PRACTICE.

1. In the end post shown in Mechanical Drawing Plate, Title: Highway Bridge: Details I, what is the distance of the center of gravity of the section above the lower edge of the channels? Ans. 5.34 in.

2. What is the total eccentricity of this section? Ans. 1.34 in.

3. What is the bending moment upon this member due to its own weight, adding 15 per cent. for weight of details? Ans. 37,170 in.-lb.

4. What is the "negative eccentricity," or distance below the center of gravity, at which the pins should be fixed in order to counteract this bending moment? Ans. .57 in.

5. What is the net eccentricity of the section? Ans. .77 in.

6. What is the sectional area required to resist the bending stress in the end post due to its own weight? Ans. 1.39 sq. in.

7. What is the fiber stress per square inch due to the same, if considered to be uniformly distributed over the entire section? Ans. 781 lb.

8. According to Cooper's specifications, is it necessary to consider this fiber stress in proportioning the area of the section?

GENERAL REMARKS.

1502. The metal for the individual members of any truss bridge is proportioned in substantially the same manner as explained in the preceding articles for the example chosen to illustrate the process. Different specifications have somewhat different requirements, but the general methods of proportioning the material are much the same for all. When the operations explained in the foregoing pages are thoroughly understood, no difficulty will be experienced in proportioning the material for a metal bridge according to any specifications.

1503. The relative dimensions of the usual forms of chord sections are determined by formula **135,** Art. **1420.** Post sections composed of two channels latticed, or of plates and angles latticed in the same general form, should comply with formula **129,** Art. **1414.** All modern compression formulas for wrought iron or steel columns make use of the radius of gyration; and for the forms of section shown in Fig. 303, this quantity is determined approximately, but near enough for most practical purposes, by formulas **110** to **118,** Art. **1408.**

1504. A tension member having a sectional area of more than about 3 square inches should consist of flat bars, which should always be placed in pairs.

A wrought-iron tension member having a sectional area of less than about 2 square inches may consist of square or round bars, preferably the former. They should always be in pairs except when but one bar is used. A single bar used as a tension truss member should generally be a square bar. Round bars are commonly used for lateral rods.

The thickness of flat bars used for tension members should always be between the limits of $\frac{1}{3}$ *and* $\frac{1}{8}$ *of their width, and preferably between* $\frac{1}{4}$ *and* $\frac{1}{6}$ *of their width.*

The widths of flat bars should be multiples of a half inch; multiples of a quarter inch may be used, but are not desirable. The diameters of round bars and the sides of square bars should generally be multiples of $\frac{1}{8}$ of an inch, though dimensions of sixteenths of an inch may be used.

1505. In the selection of the material for metal bridges the following important considerations should be kept in mind:

High steel is not a suitable material to be used in bridges.

Soft steel and medium steel are both superior to wrought iron for bridge purposes.

For the riveted members of short span bridges, soft steel is the best material.

For all members of long span bridges, medium steel is preferable, because it produces less dead load than soft steel or iron.

For forged eye-bars, medium steel is the best material. Eye-bars forged of medium steel should afterwards be annealed.

Medium steel is the best material for pins.

As steel is of nearly uniform strength in all directions, it is the best material to be used for web-plates.

All portions of the same built member should be of the same material.

Steel of a thickness greater than $\frac{9}{16}$ *of an inch is considerably injured by punching; and, therefore, the use of greater thicknesses for riveted members should be avoided so far as possible. When greater thicknesses are used, the rivet holes should usually be punched* $\frac{1}{8}$ *of an inch smaller than required, and then reamed to the required diameter. Steel of a thickness greater than* $\frac{3}{4}$ *of an inch should never be used without reaming. This refers to both soft and medium steel, but especially to the latter.*

1506. In proportioning the material for the members of a bridge, as well as in the calculations of stresses, unnecessary exactness is to be avoided, as involving unnecessary waste of time. For the sake of explicitness, many of the computations in this Course have been made unnecessarily exact.

In general, it may be stated that unit stresses should be obtained correctly to the nearest ten pounds. Areas of sections should be computed and expressed to the nearest hundredths of a square inch, and the weights per foot of channels, angles, or other shape iron to the nearest tenths of a pound.

1507. The student will find it to his advantage to closely observe and study the design of such metal bridges as may be situated in his vicinity, or as may come under his observation; he will thereby obtain many practical and valuable ideas of design, as well as a general knowledge of the usual practice. In doing this, however, he must bear in mind that the ordinary highway bridge does not always represent the best engineering practice, although it usually embodies principles of economical design.

Railroad bridges, though possibly of less bold and economical design, will be found generally to represent better and safer engineering practice than highway bridges. Although the amount and distribution of the applied loads are different in the two classes of bridges, the *principles* to be applied are the same for both.

DETAILS OF CONSTRUCTION.

GENERAL REQUIREMENTS.

1508. Thus far in the study of Bridge Engineering, attention has been given solely to the general design. The designing of the details will now be considered.

In America, metal bridges are manufactured at shops having special machinery and facilities for the purpose.

Each individual member of a metal bridge, and sometimes the entire bridge, if a small one, is manufactured complete at the shop. The various members of the structure, as thus manufactured, are shipped to the bridge site, where they are placed in position and properly connected.

1509. Before a bridge is manufactured, the practical constructive details of each member and its connections must be designed, and drawings must be made showing all such details. These drawings are known as **detail drawings.** When the bridge is designed by a consulting bridge engineer, complete detail drawings are usually made before the contract is let. But quite commonly, however, the drawings of the details are left to be made by the contracting company after the letting of the contract, the conditions for the details being usually covered by specifications. The latter practice is generally the more economical, as it allows the contracting company to use their customary details and such as are best suited to their equipment. But in every such case the drawings for the details should be submitted to, examined, and approved by the consulting engineer before any actual work on the structure is begun.

1510. The drawings of the practical constructive details, from which the different portions of a bridge are

For notice of copyright, see page immediately following the title page.

manufactured in the shop, are called **shop,** or **working drawings.** Each member of the bridge is manufactured strictly according to the shop drawings; the workmen are not allowed to vary from the drawings in any respect. Therefore, the shop drawings must not only represent good, practical, and workmanlike details and connections, but they must also be explicit in every particular, and absolutely accurate.

Upon the shop drawings all distances and dimensions are plainly marked in figures, and, although the drawings are usually made to scale and the scale is marked upon them, dimensions are never taken by scale from the drawings. *The dimensions as marked in figures govern in all cases.* Dimensions less than two feet are marked in inches and halves, quarters, eighths, sixteenths, thirty-seconds, and sixty-fourths of an inch, although the last two fractions are seldom used. Decimal fractions are never used in shop drawings. Dimensions greater than two feet are marked in feet and inches, using fractions of inches where necessary.

Any inaccuracy in the shop drawings will almost certainly result in a misfit between some parts of the structure. If discovered in the shop, the error can usually be corrected without very great expense or delay, but if not discovered until during the process of erection, the error will usually cause a large amount of both delay and expense. Hence, the necessity of absolute accuracy in shop drawings can not be too strongly emphasized. This statement is made especially pertinent by the fact that shop drawings are often required to be made hurriedly, so that the liability to error is great.

The shop drawings of the highway bridge of 90 feet span, the design of which has been chosen to illustrate the principles given in the preceding articles, are shown in Mechanical Drawing Plates, Titles: Highway Bridge: Details I, II, III, IV, and Highway Bridge: General Drawing. The manner in which the dimensions of the same are determined will now be studied.

SPECIFICATIONS.

1511. As the structure referred to above has, with slight variation, been designed according to Cooper's General Specifications for Highway Bridges, such portions of these specifications as relate to the proportioning of the details are quoted below:

(a_1) *Shearing and Bearing on Rivets.*—"The rivets and bolts connecting the parts of any member must be so spaced that the shearing strain per square inch shall not exceed 9,000 pounds, or $\frac{3}{4}$ of the allowed strain per square inch upon that member; nor the pressure upon the bearing surface per square inch of the projected semi-intrados (diameter × thickness of piece) of the rivet or bolt hole exceed 15,000 pounds, or one and a half times the allowed strain per square inch upon that member. In the case of field riveting, the above limits of shearing strain and pressure shall be reduced one-third part. Rivets must not be used in direct tension."

(b_1) *Shearing, Bearing, and Bending on Pins.*—"Pins shall be so proportioned that the shearing strain shall not exceed 9,000 pounds per square inch; nor the crushing strain upon the projected area of the semi-intrados of any member [other than forged eye-bars, see item (l_1)] connected to the pin be greater per square inch than 15,000 pounds, or one and a half times the allowed strain per square inch; nor the bending strain exceed 18,000 pounds per square inch when the centers of bearings of the strained members are taken as the points of application of the strains."

(c_1) *Strength and Character of Details.*—"All the connections and details of the several parts of the structures shall be of such strength that, upon testing, ruptures shall occur in the body of the members rather than in any of their details or connections.

"Preference will be had for such details as shall be most accessible for inspection, cleaning, and painting; no closed sections will be allowed."

(d_1) "*The pitch of rivets** in all classes of work shall never exceed 6 inches, or sixteen times the thinnest outside plate, nor be less than three diameters of the rivet."

"The pitch of rivets at the ends of compression members shall not exceed four diameters of the rivets for a length equal to twice the width of the member."

"The rivets used shall generally be ⅝, ¾, and ⅞ inch diameter."

(e_1) "The distance between edge of any piece and the center of a rivet hole must never be less than 1¼ inches, except for bars less than 2½ inches wide; when practicable it shall be at least two diameters of the rivet."

(f_1) "Wherever possible, all rivets must be machine driven. No hand-driven rivets exceeding ⅞-inch diameter will be allowed. Field riveting must be reduced to a minimum or entirely avoided where possible."

(g_1) *Joints and Splices.*—"All joints in riveted-tension members must be fully and symmetrically spliced."

"In compression members, abutting joints with planed faces must be sufficiently spliced to maintain the parts accurately in contact against all tendencies to displacement."

"The ends of all square-ended members shall be planed smooth, and exactly square to the center line of strain."

"In compression members, abutting joints with untooled faces must be fully spliced, as no reliance will be placed on such abutting joints. The abutting ends must, however, be dressed straight and true, so there will be no open joints."

"The sections of compression chords shall be connected at the abutting ends by splices sufficient to hold them truly in position."

(h_1) "*Web plates* of all girders must be arranged so as not to project beyond the faces of the flange angles, nor on the top to be more than 1/16 inch below the face of these angles, at any point."

---

* By the **pitch of rivets** is understood the distance from center to center of rivets in a line of rivets. In a long line of rivets the pitch is usually uniform.

(i_1) "In lattice girders, the web members must be double, and connect symmetrically to the web of the flanges."

(j_1) "*The heads of eye-bars* shall be so proportioned and made that the bars will preferably break in the body of the original bar rather than at any part of the head or neck. The form of the head and the mode of manufacture shall be subject to the approval of the engineer. The heads must be formed either by the process of upsetting and forging or by the process of upsetting, piling, and forging."

(k_1) "*The lower chord* shall be packed as narrow as possible.'

"*The pins* shall be turned straight and smooth, and shall fit the pin holes within $\frac{1}{50}$ of an inch for pins less than 4½ inches in diameter; for pins of a larger diameter the clearance may be $\frac{1}{32}$ inch."

(l_1) "*The diameter of the pin* shall not be less than two-thirds the largest dimension of any tension member attached to it. The several members attaching to the pin shall be so packed as to produce the least bending moment upon the pin, and all vacant spaces must be filled with wrought-iron filling rings."

(m_1) *Upset Ends.*—"All rods and hangers with screw ends shall be upset at the ends, so that the diameter at the bottom of the threads shall be $\frac{1}{16}$ inch larger than any part of the body of the bar.

"All threads must be of the United States standard, except at the ends of the pins."

(n_1) "*Floor-beam hangers* shall be so placed that they can be readily examined at all times. When fitted with screw ends they shall be provided with check nuts. Preference will be given to hangers without screw ends.

"When bent loops are used, they must fit perfectly around the pin throughout its semi-circumference."

(o_1) *Batten Plates and Lattice Bars.*—"The open sides of all compression members shall be stayed by batten plates at the ends and diagonal lattice work at intermediate points. The batten plates must be placed as near the ends as practicable, and shall have a length of 1½ times the width of the member. The size and spacing of the lattice bars shall be duly proportioned to the size of the member. They must not be less than

1½ × ¼ inch for	5 to 6 inch channels.
1¾ × ¼ inch for	7 to 8 inch channels.
2 × 5/16 inch for	9 to 12 inch channels.
2¼ × ⅜ inch for	13 to 16 inch channels.
2½ × 7/16 inch for	17 to 20 inch channels.
2½ × ½ inch for	21 and upwards.

"They shall be inclined at an angle of not less than 60° to the axis of the member. The pitch of the latticing must not exceed the width of the channel plus nine inches."

(p_1) *Reinforcing Plates.*—"Where necessary, pin holes shall be reinforced by plates, so the allowed pressure on the pins shall not be exceeded. These reinforcing plates must contain enough rivets to transfer their proportion of the bearing pressure, and at least one plate on each side shall extend not less than six inches beyond the edge of the batten plate."

(q_1) *Forked Ends.*—"Where the ends of compression members are forked to connect to the pins, the aggregate compressive strength of these forked ends must equal the compressive strength of the body of the members ; in order to insure this result the aggregate sectional area of the forked ends, at any point between the inside edge of the pin hole, and six inches beyond the edge of the batten plate, shall be about double that of the body of the member."

(r_1) "*The attachment of the lateral system* to the chords shall be thoroughly efficient. If connected to suspended floor-beams, the latter shall be stayed against all motion."

(s_1) *Portal Bracing and Transverse Diagonal Bracing.*—"All through bridges with top lateral bracing shall have wrought-iron latticed portals, of approved design, at each end of the span, connected rigidly to the end posts. They shall be as deep as the specified head-room will allow. Knee braces shall also be placed at each intermediate panel point, and connected to the vertical posts and top lateral struts, for trusses 20 feet and less in depth.

"When the height of the trusses exceeds 20 feet, an approved system of overhead diagonal bracings shall be attached to each post at an elevation sufficient to give the required head-room, and to the top lateral struts."

(t_1) *Expansion Rollers.*—"All bridges over 75 feet span shall have at one end nests of turned friction rollers, formed of wrought iron or steel, running between planed surfaces. The rollers shall not be less than 2 inches diameter, and shall be so proportioned that the pressure per lineal inch of rollers shall not exceed the product of the square root of the diameter of the roller in inches multiplied by 500 pounds ($500\sqrt{d}$).

"Bridges less than 75 feet span shall be secured at one end to the masonry, and the other end shall be free to move upon planed surfaces."

(u_1) *Anchors.*—"While the roller ends of all trusses must be free to move longitudinally under changes of temperature, they shall be anchored against lifting or moving sideways.

"Variations in temperature, to the extent of 150 degrees (Fahrenheit) shall be provided for."

(v_1) *Bed-Plates.*—"All the bed-plates and bearings under fixed and movable ends must be fox-bolted to the masonry; for trusses, these bolts must not be less than 1¼ inches diameter; for plate and other girders, not less than ⅞ inch diameter. The contractor must furnish all bolts, drill all holes, and set bolts to place with sulphur.

"All bed-plates must be of such dimensions that the greatest pressure upon the masonry shall not exceed 250 lb. per square inch."

(w_1) "*All bolts* must be of neat lengths, and shall have a washer under the heads, and nuts where in contact with wood."

(x_1) *Camber.*—" All bridges with parallel chords shall be given a camber by making the panel lengths of the top chord longer than those of the bottom chord, in the proportion of $\frac{3}{16}$ of an inch to every 10 feet."

1512. In connection with the above items quoted from the specifications, the following important conditions will also be noticed:

I. In chain-riveting, the distance between the center lines of adjacent rows should preferably not be less than three diameters of the rivet, and in no case less than two and one-half diameters.

II. In zigzag, or staggered, riveting, the distance between the center lines of adjacent rows should preferably not be less than two and one-half diameters, and never less than two diameters.

III. The distance from the center of the rivet hole to the edge of the plate (which should generally not be less than 1¼ inches) should in no case be less than one and one-half times the diameter of the hole for steel, nor less than one and five-eighths times the diameter of the hole for iron.

If less than this, allowance must be made for the reduced strength of the joint.

IV. The distance from the center of a rivet hole to the end of a wrought-iron plate (which should generally not be less than 1¼ inches) should in no case be less than one-half the diameter of the hole plus the thickness of the plate plus one-half inch.

V. The grip of a rivet, i. e., its length between heads when driven, should never exceed four times its diameter.

VI. Countersunk rivets should not be used in plates of less thickness than one-half the diameter of the rivet.

VII. The heads of steel eye-bars should not be made thicker than the body of the bar.

1513. It will be expedient to explain here the respective conditions distinguished by the terms *single shear* and *double shear*, as applied to rivets.

See Mechanical Drawing Plate, Title: Riveted Joints. It will be noticed that if the joint shown in Fig. 2, 3, or 4 should fail by shearing of the rivets, it would be necessary that each rivet should be sheared off but once; i. e., "cut once in two." A rivet in this condition is said to be in **single shear.** But in the joint shown in Fig. 5 or 6, it will be noticed that if the joint fails purely by shearing of the rivets, each rivet which fails must be sheared off twice; it must be sheared off between each two adjacent plates. The same would be true of the rivets in the joints shown in Figs. 7 and 8 when completed, and of those in the joint shown in Fig. 9. Rivets in such condition are said to be in **double shear.** It is evident that a rivet can not be in double shear unless it passes through more than two thicknesses of metal.

Rule.—*Rivets in double shear are allowed double the amount of shearing stress allowed upon the same size of rivets in single shear.*

CAMBER.

1514. As actually constructed, the so-called horizontal chords of bridges are not made perfectly straight from end to end, but are curved slightly upwards, in order that when the truss is heavily loaded the chord will not be deflected below a horizontal line connecting its ends, which would give it the unpleasing appearance of sagging. This slight curve or crown given to the chords is called **camber.** In making the working drawings for a bridge, the first things to be determined are the amount of camber and the exact lengths of the members.

The camber of a truss is shown in an exaggerated manner in Fig. 310. It will be noticed that each panel of the lower chord is the chord of the arc of a circle, and that each panel of the upper chord is the chord of the arc of a somewhat larger circle having the same center. It is from this fact that the upper and lower longitudinal members of a truss derive the name of chords. It will also be noticed that the vertical members are not truly vertical, but coincide with radii of the circle.

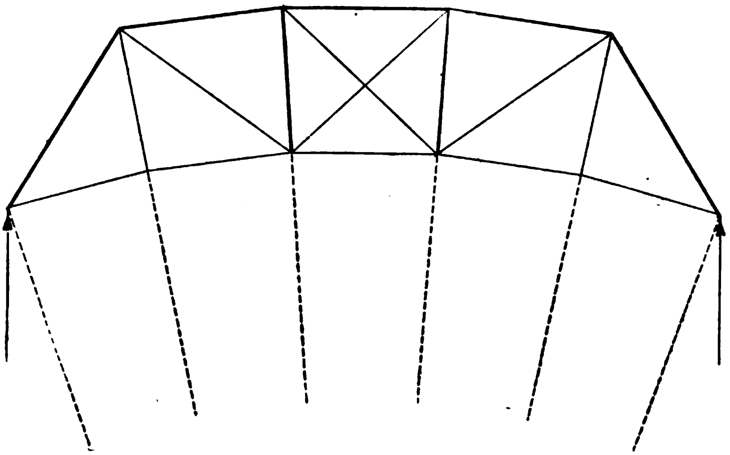


FIG. 810.

1515. What is spoken of as the **amount of camber** is the elevation of the center of the chord above its ends, i. e., the middle ordinate of the total arc, or the versed sine of the angle of one-half the arc. The amount of camber given to a bridge truss is usually from 1 to 2 inches for each 100 feet length of span; that is, from $\frac{1}{1200}$ to $\frac{1}{600}$ of the span, although it may be more or less than these limits.

1516. In through bridges the camber is always obtained by increasing the length of the upper chord, the length of the lower chord remaining unaltered. The span length is practically unaffected by the camber; the slight curvature affects the length of the chord to such a very small extent that it may be wholly neglected.

In changing the form of the truss, the values of the stresses are also changed, but this change is very small and is entirely neglected.

The amount i, which the length of each panel of the upper chord is to be increased in order to give the required amount of camber, is given approximately, but sufficiently close for all practical purposes, by the formula

$$i = \frac{8ch}{ns}, \qquad \textbf{(162.)}$$

in which c is the amount of camber in inches, h is the height of truss in feet, s is the length of span in feet, and n is the number of panels in the truss. The value obtained for i is in inches.

The definite amount of increase to be given to the upper chord, however, is commonly specified. In such cases the amount of camber may be obtained by substituting the value of i in formula **162,** and solving for c.

1517. According to item (x_1) of Art. **1511,** the panel lengths of the top chord must be increased $\frac{3}{16}$ of an inch for each ten feet of length. In complying with this specification, some latitude must be taken in nearly every case, in order to avoid unusual and inconvenient fractions. For instance, in the truss of the example the panel length is 18 feet, and if the upper chord be increased $\frac{3}{16}$ of an inch for each 10 feet, the increase in each panel would be $\frac{3}{16} \times \frac{18}{10} = \frac{27}{80}$ of an inch, which would be an inconvenient fraction to use. Increasing the panel length of the upper chord by $\frac{3}{8}$ of an inch will be sufficiently close to the requirement of the specifications.

EXACT LENGTH OF DIAGONAL MEMBERS.

1518. By increasing the panel length of the upper chord, the lengths of all diagonal members also are increased; this is shown in a somewhat exaggerated manner in Fig. 311. It will be noticed that the horizontal projection of each diagonal member is increased by one-half the panel increase of the upper chord. Hence, the length d from center to center of the diagonal is given by the formula

$$d = \sqrt{h^2 + \left(p + \frac{i}{2}\right)^2}. \qquad \textbf{(163.)}$$

in which h is the height of the truss, p is the panel length, and i is the increase in a panel length of the upper chord, all in feet or all in inches.

In applying the preceding formula, the operations may be somewhat facilitated by the use of logarithms, and still more by the use of a good table of squares. Buchanan's tables, giving the squares in feet for every foot, inch, and sixteenth of an inch between one-sixteenth of an inch and fifty feet, are especially adapted to finding the lengths of truss diagonals. In applying the formula without the aid of tables, it is usually expeditious to reduce all values to inches, or, in some cases, to convenient fractions of an inch.

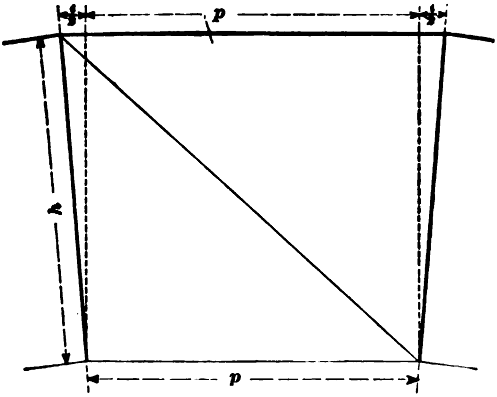


FIG. 811.

EXAMPLE.—In the truss shown in Mechanical Drawing Plates, Titles: Highway Bridge: Details I, II, III, IV, and Highway Bridge: General Drawing, the height of truss and panel length are each 18 feet, while the length of each panel of the upper chord is increased $\frac{3}{8}$ of an inch for camber. What is the length of the diagonal, center to center?

SOLUTION.—The height of truss is 18 ft. $= 216$ in. $= \frac{3,456}{16}$ in. The panel length is the same. The panel increase i in the upper chord is $\frac{3}{8}$ of an inch, and $\frac{i}{2} = \frac{3}{16}$ in.; hence, the expression $p + \frac{i}{2} = \frac{3,459}{16}$ in.

Formula **163** gives the center to center length of the diagonal $=$ $\sqrt{\frac{3,456^2}{16^2} + \frac{3,459^2}{16^2}} = \frac{4,889.64}{16}$ in. $= 25' \, 5\frac{19}{32}''$, very closely. Ans.

1519. The length of the diagonal, from center to center of *pins*, will be taken at 25′ $5\frac{19}{32}$″, but in giving the length from center to center of *pin holes*, allowance must be made for the clearance of pins. (See item (k_1) of Art. **1511.**) For this purpose the clearance of each pin may be taken at $\frac{1}{32}$ of an inch. If the actual clearance is $\frac{1}{50}$ of an inch the error will not be of consequence. It is evident that when a member is under stress, the center of neither pin upon which it connects will coincide with the center of the pin hole, but one side of the pin will be in contact with the pin hole, while the opposite side will be separated the full amount of the clearance. This is clearly shown in Fig. 312. If the clearance is $\frac{1}{32}$ of an inch, the center of the pin will be at a distance of $\frac{1}{2} \times \frac{1}{32} = \frac{1}{64}$ of an inch from the center of the pin hole. If connected upon a pin at each end, this tends to make a compression member $\frac{2}{64} = \frac{1}{32}$ of an inch too short, and a tension member the same amount too long.

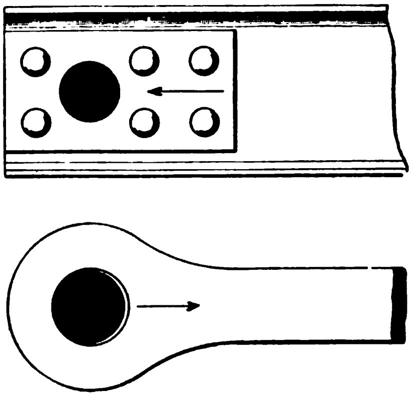


FIG. 312.

Rule.—*For all members connecting upon a pin at each end, the distance between centers of pin holes should be made $\frac{1}{32}$ of an inch longer in a compression member, and in a tension member $\frac{1}{32}$ of an inch shorter, than its calculated length.*

In pin-connected structures this applies to all diagonal and lower chord members, but not to upper chord members or vertical posts unless pin-connected at both ends.

In the preceding example, therefore, the length of the end post, center to center of pin holes, is 25′ $5\frac{19}{32}$″ + $\frac{1}{32}$″ = 25′ $5\frac{5}{8}$″; and the length of the ties, center to center of pin holes, is 25′ $5\frac{19}{32}$″ − $\frac{1}{32}$″ = 25′ $5\frac{9}{16}$″. The length of a lower chord member, center to center of pin holes, is 18′ 0″ − $\frac{1}{32}$″ = 17′ $11\frac{31}{32}$″.

COMPRESSION MEMBERS.

POSITIONS OF PINS IN CHORDS AND END POSTS.

1520. The next step in designing the details will be to fix the positions of the pins in the upper chord and end posts. As explained in Arts. **1498** and **1499,** the eccentricity of the pins—that is, the distance of the centers of the pins from the centers of the channels—in a chord member or end post should be the same as the net eccentricity of the member. But it is evident that the pins should have the same relative positions throughout the chord and end posts. In the solutions explained in Art. **1499,** the net eccentricity of the end panel of the upper chord was found to be 1.24 inches; hence, the proper positions of the pins in this member would be at that distance above the centers of the channels. But in solving Example 5 of Art. **1501,** the net eccentricity of the end post was found to be .77 of an inch, at which distance above the centers of the channels are the proper positions of the pins in the end posts.

1521. *In order to develop the full strength of a compression member, it must be so connected that its resultant maximum stress will be distributed uniformly over its section.*

In properly designed bridges, therefore, the net eccentricity should be made the same throughout the entire upper chord and end posts. Had the chords and end posts of the example been properly designed, the weights of the channels in the upper chord would have been increased until the net eccentricity of this member became the same as in the end post. This would increase the sectional area of a member already having an excess of section, to which considerations of economy would offer some objection. The net eccentricity of the end post could have been made the same as the net eccentricity of the end panel of the upper chord without loss of economy, but the net eccentricity of the chord is too great to allow practical and consistent details.

In making general designs of highway bridges, it is frequently the case that the time available is not sufficient for determining sections having exactly the same net eccentricity. Consequently, conditions of eccentricity are often but loosely considered, and in many cases wholly neglected. In other cases, where time is limited, the designer depends upon his judgment to select sections having nearly the same eccentricity. If he is an experienced designer, he may be able to obtain reasonably approximate results; to do this is, at least, better practice than to neglect the eccentricity entirely. In the example, the top chord and end post sections have been proportioned to illustrate a case of this kind.

1522. The designer of details is seldom the maker of the general design. The detailer must take the general design as it comes to him (in which, it is safe to say, the conditions of ideal perfection are seldom realized), and must from it evolve the best, and, at the same time, the most economical, structure possible. He must meet as best he can the conditions as he finds them.

In a case similar to the example, having found that the net eccentricities of the chord and end post are not the same, the best thing to do is to make the eccentricity of the pins in those members a reasonable average between the net eccentricity of the end posts and the connecting or end panels of the upper chord, and agreeing rather more nearly with the eccentricity of the end post. In the example, the net eccentricity of the end posts has been found to be .77 in., and the eccentricity of the end panels of the upper chord 1.24 inches. Hence, in this case, the pins in the top chord and end post are given an eccentricity of $\frac{.77 + 1.24}{2} = 1$ inch. An eccentricity of $\frac{15}{16}$ of an inch would, perhaps, have been as good or even better.

The positions of the pins, with reference to eccentricity, in the intermediate panels of the chord are not of great importance, unless the chord stresses are transmitted through the pins, which is not usually the case. The pins are always given the same eccentricity throughout the upper chord.

GENERAL DIMENSIONS OF THE UPPER CHORD AND END POST.

1523. Sections of the channels used in the chords and end posts are shown in Fig. 79 of Mechanical Drawing. As there shown, the width of the flange in the 10-pound channel (top chord) is $2\frac{1}{16}$ inches, while the width of the flange in the 16-pound channel (end post) is $2\frac{5}{16}$ inches. The latter channel has the wider flange, and as in the built member the flanges of the channels must not project beyond the edges of the cover-plate, the width of this flange will determine the distance between the channels of the end post, back to back. As two channels attach to the cover-plate, the distance back to back of the channels must not exceed $12 - 2 \times 2\frac{5}{16} = 7\frac{3}{8}$ inches. In most cases it is best to allow each edge of the cover-plate to project about $\frac{1}{16}$ of an inch beyond the flanges of the channels; but in the present case, as the channels of the end post, being much heavier than those of the upper chord, have flanges considerably wider than the channels of the upper chord, the flanges of the former channels are placed flush with the edges of the cover-plate. Hence, in the end-post the distance between channels, back to back, is made $7\frac{3}{8}$ inches.

By reference to Fig. 1, of Mechanical Drawing Plate, Title: Highway Bridge: Details I, it is noticed that on the end post the reinforcing pin plates for the hip joint *B* are placed inside the channels, and, in order to grasp the pin, extend beyond the channels of the upper chord. In order that there shall be $\frac{1}{16}$ of an inch clearance upon each side between these pin plates of the end post and the channels of the upper chord, the distance between the latter, back to back, must be $7\frac{3}{8} + 2 \times \frac{1}{16} = 7\frac{1}{2}$ inches. As the pin plates must clear countersunk rivets, it would have been as well or better to have given $\frac{1}{8}$ of an inch clearance on each side, but it would have produced greater bending moment upon the pin. The width, out to out, of the flanges of the channels in the upper chord must not be greater than the width of the cover-plate, i. e., 12 inches. The width, out to out, of the flanges of the channels is found to be $7\frac{1}{2} + 2 \times 2\frac{1}{16} = 11\frac{5}{8}$ inches, which is less than the width of the cover-plate. Sections of the top chord and end post are shown in Figs. 1 and 2 of the Mechanical Drawing plate referred to above.

CLEARANCE.

1524. It is very important that members which are to be connected in the field should have sufficient clearance, so that no difficulty will be experienced in making the connections. In this the designer must be governed largely by his judgment and the conditions of each individual case, but the following general rules indicate good practice, and will serve as a useful general guide:

I. In arranging the several members upon a pin, $\frac{1}{16}$ *of an inch clearance should always be allowed between the connecting parts of adjacent members, if eye-bar heads, forged loops, or perfectly smooth plates of a single thickness.*

II. If the connecting portion of either member consists of several thicknesses of perfectly smooth plates, from $\frac{1}{32}$ *to* $\frac{1}{16}$ *of an inch additional clearance should be allowed for each additional plate.*

III. If there are countersunk rivets in the connecting portion of either member, not less than $\frac{1}{8}$ *of an inch clearance should be allowed, the countersunk heads being also chipped.*

IV. If there are countersunk rivets in the connecting portions of both members, the countersunk heads being chipped, not less than $\frac{3}{16}$ *of an inch clearance should be allowed.*

V. If the connecting portion of either member contains rivets with flattened heads or full heads, $\frac{1}{8}$ *of an inch clearance above the head should be allowed.*

VI. If the heads of countersunk rivets are not chipped, they should be considered as flattened to $\frac{1}{8}$ *of an inch, in estimating the clearance.*

The clearance allowed between the connecting parts of members should always be ample; it should in no case be so scant as to cause squeezing or as to permit the possibility of the members not coming properly together. The preceding rules apply to *each* clearance between the adjacent sides of members.

SIZES OF PINS.

1525. The stresses in the different members connecting upon a pin produce bending moment in the pin, the amount of which may be determined either by computation or by constructing the moment diagrams. According to item (b_1) of Art. **1511,** in determining the bending moment upon a pin the centers of the bearings of the members under stress must be taken as the points of application of the forces. The positions of the centers of the several bearings upon a pin depend to some extent upon their thickness, which in turn depends largely upon the size of the pin; for the required thickness of each bearing can not be determined until the size of the pin is known. The conditions of bearing and bending being thus interdependent, it becomes necessary to assume certain dimensions and make trial calculations. It will be most expedient to assume a size for the pin.

1526. Although the actual requirements are usually different for each pin, it is customary, in order to afford uniformity in the shop work, to make the pins of uniform diameter throughout the lower chord, including the shoe pin. It is evident, therefore, that the diameter of each pin in the chord must be equal to the diameter of the largest pin required in the same. If the truss has an even number of panels, the largest pin required will almost invariably be the pin at the middle joint of the lower chord. If the truss has an odd number of panels, the largest pin will probably be required at the first or second joint from the center of the lower chord. After some experience, this can usually be determined by inspection.

When practical, the diameter of the hip pin is also made the same as the diameters of the lower chord pins.

Item (l_1) of Art. **1511** affords material guidance in assuming the sizes of the pins in the lower chord. This item requires that, in the lower chord, "the diameter of the pin shall not be less than two-thirds the largest dimension of any tension member attached to it."

In the example, the member $c\,c'$ of the lower chord bears the greatest stress; it would be the largest member in the chord, were the chord composed entirely of chord bars, and it may, therefore, be considered as the largest tension member attaching to joint c. As actually designed, the width or largest dimension of the chord bar in this member is $4\frac{1}{2}$ inches, but of this width $\frac{9}{16}$ of an inch is cut out by rivet holes and is not effective section. The width of the effective section is $4\frac{1}{2} - \frac{9}{16} = 3\frac{15}{16}$ inches, or near enough for present purposes, 4 inches; this may be considered the largest dimension of the chord bar. According to the requirements of the specifications, therefore, the diameter of the pin must not be less than $\frac{2}{3} \times 4 = 2.67$ inches $= 2\frac{11}{16}$ inches, nearly.

1527. In a truss having an even number of panels, the maximum bending moment M upon the pin at the center joint of the lower chord is given approximately, but quite closely, by the formula

$$M = \frac{S\,t}{2}, \qquad \textbf{(164.)}$$

in which S is the total live and dead load tensile stress in the adjacent panel of the chord and t the *average* thickness of the heads of the *chord* bars connecting upon the pin plus $\frac{1}{16}$ of an inch. This formula is necessarily applied before the dimensions of the chord bar heads have been determined, but if from the general conditions it is thought that it will be desirable to make the heads somewhat thicker than the body of the bar, allowance should be made for this fact in applying the formula.

The formula may also be applied at the first or second lower chord joint from the center in a truss having an odd number of panels, but the results are not so reliable. In such cases, S should represent the stresses in the adjacent panel *towards the center of the truss.*

Having obtained the maximum bending moment, the size of the pin giving an equal resisting moment may be obtained from Table 40, Art. **1546,** which will be further noticed in that article.

1528. The total stress in the panel $c\,c'$ of the example (see stress sheet) is $48,600 + 20,800 = 69,400$ pounds $= S$.

The thickness of the head of each chord bar in the same panel is $\frac{7}{8}$ of an inch, and, consequently, the value of t is $\frac{7}{8} + \frac{1}{16} = \frac{15}{16}$ inch. Hence, by formula **164,** the maximum bending moment upon the pin in the joint c of the lower chord is, approximately, $\frac{69,400 \times \frac{15}{16}}{2} = 32,530$ inch-pounds. Item (b_1) of Art. **1511** requires that the bending stress upon a pin shall not exceed 18,000 pounds per square inch. From Table 40, it is found that, with a fiber stress of 18,000 pounds per square inch, a bending moment of 32,500 inch-pounds will slightly exceed the resisting moment given by a $2\frac{5}{8}''$ pin.

As the bending moment obtained above is only approximate, it will be well in choosing the diameter of the pin to select a diameter somewhat greater than the diameter thus obtained. Therefore, the diameters of all pins in the lower chord will be assumed to be $2\frac{3}{4}$ inches. With reference to the actual bending moment upon it, this diameter will now be investigated for the pin of the shoe joint.

PROPORTIONING PIN PLATES.

1529. Shearing Upon Pin.—By the first condition of item (b_1) of Art. **1511,** the shearing stress per square inch upon the pin must not exceed 9,000 pounds. The total stress upon the end post is $45,800 + 19,600 = 65,400$ pounds, one-half of which, or 32,700 pounds, is transmitted to the pin through each channel. Hence, the sectional area of the pin, as required to resist the shear, is $\frac{32,700}{9,000} = 3.63$ square inches. The sectional area given by a pin or round bar 2¾ inches in diameter is $.7854 \times 2.75^2 = 5.94$ square inches. The shearing stress is found to be abundantly resisted by this size of pin.

1530. Bearing Upon Pin and Thickness of Pin Plates.—The same item of the specifications requires that the crushing stress (bearing stress) per square inch upon the projected area of the semi-intrados (i. e., the diameter of the pin multiplied by the total thickness of metal bearing upon it, or, in other words, by so much of its length as is covered by the surface of the metal in the member bearing upon it) must not exceed 15,000 pounds, or one and one-half times the stress per square inch allowed upon the member.

At 15,000 pounds per square inch, the allowed bearing upon one lineal inch of the pin is $15,000 \times 2\frac{3}{4} = 41,250$ pounds, requiring for the total thickness of metal bearing upon the pin $\frac{45,800 + 19,600}{41,250} = 1.59$ inches, or a little less than 1⅝ inches.

In finding the value of the allowed bearing stress by taking one and one-half times the allowed stress per square inch, the reduction of the unit stress for length of column (i. e., the negative quantity in the compression formula) is of course, neglected. Neglecting the quantity $-50\frac{l}{r}$ in the compression formula, the live load unit stress allowed upon the end post is 8,750 pounds, and the dead load unit stress is 17,500 pounds [Art. **1410** (*o*)]. Hence, the bearing stresses allowed upon the pin are, for live load stress, $8,750 \times 1\frac{1}{2} = 13,125$ pounds per square inch, and for dead load stress, $17,500 \times 1\frac{1}{2} = 26,250$ pounds per square inch. The bearing stresses allowed upon one lineal inch of the pin are, therefore, $13,125 \times 2\frac{3}{4} = 36,090$ pounds for live load stress, and $26,250 \times 2\frac{3}{4} = 72,190$ pounds for dead load stress. The required total thickness of metal bearing upon the pin is, then, $\frac{45,800}{36,090} + \frac{19,600}{72,190} = 1.54$ inches, or a little less than $1\frac{9}{16}$ inches. Hence, it is found that the thickness of bearing required by the allowed bearing stress of 15,000 pounds per square inch is the greater. Therefore, the required total thickness of bearing is $1\frac{5}{8}$ inches.

It will be well to notice that, as the ratio between the live and dead load stresses is the same throughout the chord and end post, and as the unit stresses allowed upon the end post are less than those allowed upon the chord, it follows that the thickness of bearing required by the allowed bearing stress of 15,000 pounds per square inch will be greater than that required by one and one-half times the allowed stress, if applied to any portion of the chord or end post in this bridge; hence, the latter condition may be neglected and a bearing stress of 15,000 pounds per square inch used. This refers to the bearing stress either upon a rivet or upon a pin.

1531. When, as in the majority of cases, the allowed dead load unit stress is twice the allowed live load unit stress, the preceding operations are very much shortened by the following formulas:

Let d = diameter of pin;
u = allowed live load unit stress;
$2u$ = allowed dead load unit stress;
L = total live load stress upon the member;
D = total dead load stress upon the member.

Then, at 15,000 pounds per square inch, the allowed bearing stress per lineal inch of pin = 15,000 d, and the required thickness t_{15} of metal is given by the formula

$$t_{15} = \frac{L + D}{15,000\,d}. \qquad \textbf{(165.)}$$

The values of 15,000 d are given in Table 40, Art. **1546.** At $1\frac{1}{2}$ $(= \frac{3}{2})$, the allowed unit stress for live load, the allowed bearing per lineal inch of pin $= \frac{3}{2} \times u \times d$, and the required thickness for live load, $t_l = \frac{L}{\frac{3}{2} d u} = \frac{2L}{3du}$. Similarly, the required thickness for dead load stress is $t_d = \frac{D}{\frac{3}{2} \times d \times 2u} = \frac{D}{3du}$. Therefore, the total thickness T in this case is

$$T = i_l + t_a = \frac{2L + D}{3du}. \qquad \textbf{(166.)}$$

In order to know what thickness should be provided for, we must compare t_{ls} with T, and take the larger. But, for this purpose, it is not necessary to compute both values; a general formula may be derived for the difference $t_{ls} - T$. When this difference is positive, t_{ls} is greater than T, and the value of t_{ls}, calculated by formula **165,** should be used; when the difference is negative, the value of T, calculated by formula **166,** should be used. The formula for the difference is found by subtracting formula **166** from formula **165,** which gives, after factoring,

$$t_{ls} - T = \frac{(u - 5{,}000) D - (10{,}000 - u) L}{15{,}000 du}.$$

But, since the denominator does not alter the sign of the fraction, we may neglect it and write

$$\begin{aligned} &\textit{sign of } (t_{ls} - T) = \\ &\textit{sign of } [(u - 5{,}000)\ D - (10{,}000 - u)\ L]. \qquad \textbf{(167.)} \end{aligned}$$

Therefore, formula **165** should be used when the second member of this equation is positive, and formula **166** when negative. Furthermore, by a simple process of factoring we may arrive at some very convenient formulas for the various cases likely to occur. Thus, in the preceding example, and *in all those of the same kind*, we have $u = 8{,}750$. and formula **167** becomes

$$\begin{aligned} &\textit{sign of } (t_{ls} - T) = \textit{sign of } (3{,}750\ D - 1{,}250\ L) = \\ &\quad \textit{sign of } [1{,}250\ (3\ D - L) = \textit{sign of } (3\ D - L)], \end{aligned}$$

whence the following

Rule.—*For the bearing thickness of all posts* [see Art. **1410** (*o*)], *use formula **165**, if three times the dead load stress exceeds the live load stress. In the opposite case use formula **166**.*

1532. For chord segments [see Art. **1410** (*n*)], $u = 10{,}000$, and formula **167** gives

$$\textit{sign of } (t_{16} - T) = \textit{sign of } 5{,}000\, D,$$

which is always positive. Hence,

Rule.—*For the bearing thickness of all chords in compression, use formula **165**.*

For other members to which the same rule applies, see Art. **1399** (*g*).

In all cases, formula **167** may be used to determine what thickness (whether t_{16} or T) should be provided for; but it must be borne in mind that *the preceding rules apply only to members proportioned by Cooper's specifications or by others containing the same requirements.*

1533. Referring to the section of the end post shown in Fig. 1 of Mechanical Drawing Plate, Title: Highway Bridge: Details I, it is found that the thickness of metal in the web of each channel of the end post is $\frac{3}{8}$ of an inch. Hence, the thickness of bearing given by the two channels is $2 \times \frac{3}{8} = \frac{3}{4}$ of an inch, leaving $1\frac{5}{8} - \frac{3}{4} = \frac{7}{8}$ of an inch thickness of bearing to be given by the two reinforcing pin plates; in other words, the two pin plates (one on each channel) are each required to be $\frac{1}{2} \times \frac{7}{8} = \frac{7}{16}$ of an inch in thickness.

It is evident that this thickness, obtained for the pin plates, will apply not only to the shoe connection of the end post, but to the hip connection also, provided the same size pin is used.

1534. Shearing Upon Rivets in Pin Plates.—If the stress is assumed to be distributed uniformly over the bearing surface, the amount of live load stress taken by each pin plate is equal to $\frac{\frac{7}{16} \times 45{,}800}{1\frac{5}{8}} = 12{,}330$ pounds, and the amount of dead load stress is $\frac{\frac{7}{16} \times 19{,}600}{1\frac{5}{8}} = 5{,}280$ pounds, or a total of $12,330 + 5,280 = 17,610$ pounds. This amount of stress must be transferred from the channel to the pin plate by the rivets.

According to item (a_1) of Art. **1511**, the shearing stress per square inch upon rivets must not exceed 9,000 pounds, or three-fourths of the allowed stress upon the member.

The rivets used will be $\frac{5}{8}$ of an inch in diameter, giving for each rivet a sectional area of $(\frac{5}{8})^2 \times .7854 = .3068$ of a square inch. An allowed shearing stress of 9,000 pounds per square inch will give $9,000 \times .3068 = 2,760$ pounds as the amount of shearing stress that may be allowed upon each rivet. Hence, the number of rivets required by this condition is $\frac{17,610}{2,760} = 6.4$, or, as the result can not be fractional, 7 rivets.

Three-fourths of the live load unit stress allowed upon the end post is $\frac{3}{4} \times 8,750 = 6,560$ pounds, and three-fourths of the dead load stress is $\frac{3}{4} \times 17,500 = 13,130$ pounds. The amount of live load shearing stress allowed upon a rivet is $6,560 \times .3068 = 2,010$ pounds, and the amount of dead load stress is $13,130 \times .3068 = 4,030$ pounds. Hence, the number of rivets required by the live load stress is $\frac{12,330}{2,010} = 6.1$, and the number required by the dead load stress is $\frac{5,280}{4,030} = 1.3$; or, a total of $6.1 + 1.3 = 7.4$, or, as the result can not be fractional, 8 rivets.

It may be noticed that, as applied to any portion of the end post of this bridge, the more severe requirement for shear is that the shearing stress per square inch shall not exceed three-fourths of the allowed stress upon the member.

1535. When the allowed dead load unit stress is twice the allowed live load unit stress, we may use the following formulas, in which T is the total bearing thickness ($1\frac{5}{8}''$ in the preceding example), t = thickness of pin plate ($\frac{7}{16}''$ in the example), A = area of rivet, and u, D, and L have the same values as in Art. **1531.**

Taking allowed shearing unit stress at 9,000 pounds, the required number of rivets is

$$n_s = \frac{t\,(D + L)}{9{,}000\,A\,T}. \qquad \textbf{(168.)}$$

Taking allowed shearing unit stress at three-quarters unit stress allowed for members, the number of rivets is

$$N = \frac{2\,t\,(2\,L + D)}{3\,A\,T\,u}. \qquad \textbf{(169.)}$$

By subtracting **169** from **168** it is found that

$$\textit{Sign of } (n_s - N) = \textit{sign of } [(u - 6{,}000)\,D - (12{,}000 - u)\,L] \qquad \textbf{(170.)}$$

If this sign is *plus*, use formula **168**; if *minus*, use **169.**

Thus, in the preceding example, where $u = 8{,}750$, $(u - 6{,}000)\,D - (12{,}000 - u)\,L = 2{,}750\ D - 3{,}250\ L = 250\ (11\ D - 13\ L) = 250\ (11 \times 19{,}600 - 13 \times 45{,}000)$, which, being negative (this is seen at a glance without performing the operations) shows that N is greater than n_s, and that, therefore, formula **169** should be used for the number of rivets.

1536. Bearing Upon Rivets in Pin Plates.—The number of rivets required in the pin plate, when considered with reference to their capacity to resist bearing stress, will now be noticed. The thinnest plate through which the rivet passes is the web of the channel, which is practically $\frac{3}{8}$ of an inch in thickness. As noticed above, the more severe requirement for bearing is that the allowed bearing stress shall not exceed 15,000 pounds per square inch.

At 15,000 pounds per square inch, the bearing stress resisted by a $\frac{5}{8}''$ rivet through $\frac{3}{8}$ of an inch thickness of plate, is $\frac{5}{8} \times \frac{3}{8} \times 15{,}000 = 3{,}520$ pounds. The number of rivets required in each pin plate to resist the bearing is $\frac{17{,}610}{3{,}520} =$ 5 rivets. Hence, at the unit stresses allowed for shearing and bearing, it is found that 8 rivets in each pin plate will fulfil all conditions for resisting the maximum stress in both shearing and bearing.

The number of rivets required in each pin plate, as well as the thickness of the pin plate, is the same for the hip connection of the end post as for the shoe connection. Eight rivets are used in each pin plate at the hip connection of the end post, but at the shoe connection, as six of the rivets are countersunk and four of them also flattened, ten rivets are used. In order to be on the safe side it is well to use one or two more rivets than the calculations require, especially if a number of the rivets are countersunk.

1537. Tables of Shearing and Bearing Values for Rivets.—Operations similar to those explained above, for finding the number of rivets required in a pin plate and for like purposes, may be considerably facilitated by the use of the following tables of the shearing and bearing values of rivets. As allowed by many specifications, the unit value for bearing is double that allowed for shearing, and the tables are arranged accordingly. The values allowed by Cooper's Highway Specifications, however, correspond with Table 37 for bearing and Table 38 for shearing. Other unit values are used for both shearing and bearing, and other tables will be found in various structural hand-books, but the values given in these tables are those commonly used. According to the common practice, the values given in Table 36 are used for wrought iron in railroad bridges; those given in Tables 37 and 38 are used for wrought iron in highway bridges and for medium steel in railroad bridges; while the values given in Table 39 are used for medium steel in highway bridges.

NOTE.—In each table all bearing values above or to the right of the upper zigzag lines are greater than double shear.

Between upper and lower zigzag lines bearing values are less than double shear and greater than single shear.

Below and to the left of lower zigzag lines bearing values are less than single shear.

All values in pounds.

Shearing values at 12,000 pounds per square inch, and bearing values at 24,000 pounds per square inch may be obtained by doubling the values given in Table 36.

TABLE 36.

Diameter of Rivet in Inches.	Area of Rivet in Square Inches.	Shear Values at 6,000 lb. per Square Inch.		Bearing Values for Different Thicknesses of Plate in Inches, at 12,000 lb. per Square Inch.												
		Single Shear.	Double Shear.	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
3/8	.1104	660	1320	1130	1410	1690										
1/2	.1963	1180	2360	1500	1880	2250	2630	3000								
5/8	.3068	1840	3680	1880	2340	2810	3280	3750	4220	4690						
3/4	.4418	2650	5300	2250	2810	3380	3940	4500	5160	5630	6190	6750				
7/8	.6013	3610	7220	2630	3280	3940	4590	5250	5910	6560	7220	7880	8530	9190		
1	.7854	4710	9420	3000	3750	4500	5250	6000	6750	7500	8250	9000	9750	10500	11250	12000

TABLE 37.

Diameter of Rivet in Inches.	Area of Rivet in Square Inches.	Shear Values at 7,500 lb. per Square Inch.		Bearing Values for Different Thicknesses of Plate in Inches, at 15,000 lb. per Square Inch.												
		Single Shear.	Double Shear.	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
3/8	.1104	830	1660	1410	1760	2110										
1/2	.1963	1470	2940	1880	2340	2810	3280	3750								
5/8	.3068	2300	4600	2340	2930	3520	4100	4690	5280	5860						
3/4	.4418	3310	6630	2810	3520	4220	4920	5630	6330	7030	7720	8440				
7/8	.6013	4510	9020	3280	4100	4920	5740	6560	7380	8200	9030	9850	10670	11480		
1	.7854	5890	11780	3750	4690	5630	6560	7500	8440	9380	10310	11250	12190	13130	14060	15000

TABLE 38.

Diameter of Rivet in Inches.	Area of Rivet in Square Inches.	Shear Values at 9,000 lb. per Square Inch.		Bearing Values for Different Thicknesses of Plate in Inches, at 18,000 lb. per Square Inch.												
		Single Shear.	Double Shear.	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
3/8	.1104	990	1990	1690	2110	2530										
1/2	.1963	1770	3530	2250	2810	3380	3940	4500								
5/8	.3068	2760	5520	2810	3520	4220	4920	5630	6330	7030						
3/4	.4418	3980	7950	3380	4220	5060	5910	6750	7590	8440	9280	10130				
7/8	.6013	5410	10820	3940	4920	5910	6890	7880	8860	9840	10830	11810	12800	13780		
1	.7854	7070	14140	4500	5630	6750	7880	9000	10130	11250	12380	13500	14630	15750	16880	18000

TABLE 39.

Diameter of Rivet in Inches	Area of Rivet in Square Inches.	Shear Values at 10,500 lb. per Square Inch.		Bearing Values for Different Thicknesses of Plate in Inches, at 21,000 lb. per Square Inch.												
		Single Shear.	Double Shear.	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16	1
3/8	.1104	1160	2320	1970	2460	2950										
1/2	.1963	2060	4120	2630	3280	3940	4590	5250								
5/8	.3068	3220	6440	3280	4100	4920	5740	6560	7380	8200						
3/4	.4418	4640	9280	3940	4920	5910	6890	7880	8860	9840	10830	11810				
7/8	.6013	6310	12630	4590	5740	6890	8040	9190	10340	11480	12630	13780	14930	16080		
1	.7854	8250	16490	5250	6560	7880	9190	10500	11810	13130	14440	15750	17060	18380	19690	21000

1538. Dimensions of Pin Plates and Arrangement of Rivets.—By reference to the sections of the channels shown in Mechanical Drawing, Fig. 79, it will be noticed that the width of the widest plate that can be used upon the outside of the web of the channel of the end post (16 pounds), without rounding the edges to fit between the flanges of the channel, can not be greater than $8 - 2\left(\frac{5}{8} + \frac{5}{16}\right) = 6\frac{1}{8}$ inches. Fitting the edges would involve additional and expensive shop work, and, therefore, should not be required when not absolutely necessary. Hence, pin plates 6 inches in width will be used upon the end post. The length of the pin plate will depend upon the number of rivets used and the arrangement of the rivet spacing.

By item (e_1) of Art. **1511** the distance between the center of a rivet hole and the outer edge of the plate must not be less than $1\frac{1}{4}$ inches. If the outer lines of rivets in the pin plates be each located $1\frac{1}{4}$ inches from the outer edge of the plate, the distance between these two lines of rivets will be $6 - 2 \times 1\frac{1}{4} = 3\frac{1}{2}$ inches; and if a row of rivets be placed midway between the two outer rows, the distance between two adjacent rows will be $\frac{1}{2} \times 3\frac{1}{2} = 1\frac{3}{4}$ inches. This is between two and one-half and three diameters of the rivets, and hence is a less distance between the rows of rivets than is desirable for chain riveting, but is a sufficient distance between the rows if the rivets are staggered. (Art. **1512,** items II and III.) The rivets will be staggered, and their pitch will be made equal to twice the distance between the adjacent rows, or $3\frac{1}{2}$ inches. The distance from the center of any rivet hole to either the end of the pin plate or the edge of the pin hole should not be less than $1\frac{1}{4}$ inches. The arrangement of the rivets in the pin plate to meet the required condition is clearly shown in Mechanical Drawing Plate, Title: Highway Bridge: Details I, Fig. 1. It will be noticed that with the rivets arranged as there shown, the shortest distance between the centers of any two rivets is the diagonal distance between the centers of the rivets in two adjacent rows; this distance is very nearly $2\frac{1}{2}$ inches, or four diameters. The arrangement of the rivet spacing in the pin plate should usually be deferred until the size of the pin is definitely determined, and should always be worked out fully in pencil. Any change in the diameter of the pin would affect the required thickness of the material bearing upon it; it would, consequently, affect not only the thickness of the pin plate, but also the number of rivets in the same.

MOMENTS ON PINS.

1539. Positions of Bearings Upon the Pin.—The thickness of material necessary to give the proper bearing upon the pin, as required for the standards of the shoe and for the member in the end panel of the lower chord, will now be determined, in order to find the bending moment upon the pin. This is necessary because, according to item (b_1) of the specifications (Art. **1511**), the centers of the bearings of the members must be taken as the points of application of the stresses in determining the bending moment.

The bearings of the shoe standards must be sufficient to resist the total vertical pressure upon them, which will be equal to the reaction due to the total wind pressure against the portal plus that due to the full live and dead loads. By reference to Fig. 277 (b) of Art. **1309**, it will be noticed that the total wind pressure against the portal is equal to $2,700 + 1,350 + 1,350 = 5,400$ pounds. Hence, the *vertical* reaction at the foot of the leeward end post, due to this wind pressure against the portal, is equal to $5,400 \times \frac{18}{19} = 5,116$, or, near enough, 5,120 pounds.

By reference to Arts. **1293** and **1301**, it is found that the reaction due to a full live load is 32,400 pounds, and the reaction due to the dead load is 13,900 pounds. It has been found (Art. **1530**) that for a bearing pressure of 15,000 pounds per square inch the bearing value per lineal inch of a $2\frac{3}{4}''$ pin is 41,250 pounds. Hence, the total thickness of bearing required in the standards of the shoe will be $\frac{32,400 + 13,900 + 5,120}{41,250} = 1.25$ inches, which is a thickness of $\frac{1}{2} \times 1\frac{1}{4} = \frac{5}{8}$ inch for each standard.

1540. The live and dead load stresses in the end panel *a b* of the lower chord are found to be the same as the live and dead load reactions. (This will always be the case for a full uniform load when the panel length and the height of truss at the hip vertical are equal.) Hence, for present purposes, the thickness of bearing of the lower chord upon the shoe pin may be considered the same as that taken for the shoe standards.

The shoe joint may be arranged with the standards of the shoe between the channels of the end post and the lower chord members outside of the channels, but it is a better arrangement to pack the lower chord between the channels, with the shoe standards connecting outside of the end post. As the stress in the end post is resisted by both the stress in the lower chord and by the reaction, applied through the shoe, the bearing of the end post upon the pin should be between the bearing of the chord and the bearing of the shoe, as shown in Mechanical Drawing Plate, Title: Highway Bridge: Details IV.

The distance between the channels of the end post, back to back, being $7\frac{3}{8}$ inches (Art. **1523**), the distance from outside to outside of the pin plates at the shoe joint is $7\frac{3}{8} + 2\left(\frac{3}{8} + \frac{7}{16}\right) = 9$ inches.

Rivets should not be flattened to less than $\frac{1}{4}$ of an inch in height, and are always likely to be about one-sixteenth of an inch higher than marked. The shoe should have a clearance of not less than $\frac{1}{8}$ of an inch on each side of the end post. Hence, the distance between the standards of the shoe is made equal to $9 + 2\left(\frac{1}{4} + \frac{1}{16} + \frac{1}{8}\right) = 9\frac{7}{8}$ inches. The distance from center to center of the pin bearings of the shoe standards is, therefore, $9\frac{7}{8} + \frac{5}{8} = 10\frac{1}{2}$ inches. See Figs. 5 and 9 of Mechanical Drawing Plate, Title: Highway Bridge: Details III.

1541. As the dimensions of the connecting details of the lower chord member *a b* have not yet been determined, the actual distance between the centers of their bearings, and, therefore, their actual positions with reference to the other bearings, are not known. Other conditions (to be hereafter noticed) may require the thickness of metal in the chord bearing upon the pin to be greater than required by the allowed bearing stress. In such cases it is good practice to assume the positions of the bearings to be the same *as determined by the required thickness of bearing*, with liberal allowance for clearance. Assuming the clearance on each side between the chord and end post to be $\frac{1}{8}$ of an inch, the width from outside to outside of the chord connection will be $7\frac{3}{8} - 2 \times \frac{1}{8} = 7\frac{1}{8}$ inches. If each bearing of the chord be assumed to be $\frac{5}{8}$ of an inch in thickness, the same as the shoe standards, the distance between the centers of the chord bearings may be taken at $7\frac{1}{8} - \frac{5}{8} = 6\frac{1}{2}$ inches. The distance between the centers of the bearings of the end post is known to be $7\frac{1}{8} + \frac{5}{8} + \frac{7}{16} = 8\frac{3}{16}$ inches.

1542. Bending Moment Upon the Pin.—The relative positions of the points of application of the forces acting upon the pin (which are the stresses in the connecting members transferred by the pin) have now been determined, and the maximum bending moment upon the pin may be readily obtained. It is convenient to resolve all forces acting upon the pin into their horizontal and vertical components, to determine the bending moments at various points in both the horizontal and vertical plane, and then find the resultants of the bending moments in the two perpendicular planes at corresponding points. The greatest of these resultants will be the maximum bending moment upon the pin.

The forces acting upon the shoe joint, together with the force polygon for the same, are shown in Fig. 313. For each member the live and dead load stresses are combined; the wind stresses are neglected. A mere inspection of the force polygon clearly shows that *2-1*, the vertical component of *3-1*, the stress in the end post, is equal and opposed to the reaction *1-2*; also, that *3-2*, the horizontal component of *3-1*, the stress in the end post, is equal and opposed to *2-3*, the stress in the lower chord. Without performing any operation, it is thus known that the horizontal component of the live and dead load stresses in the end post is equal to 46,300 pounds, or the sum of the live and dead load stresses in the lower chord member with which it connects, and that the vertical component of the live and dead load stresses in the end post, which is equal to the reaction, is also equal to the same amount. The vertical forces acting upon the pin, and the horizontal forces acting upon the same, may each be considered independently of the other, giving very simple problems. The resultant of the two results may then be obtained.

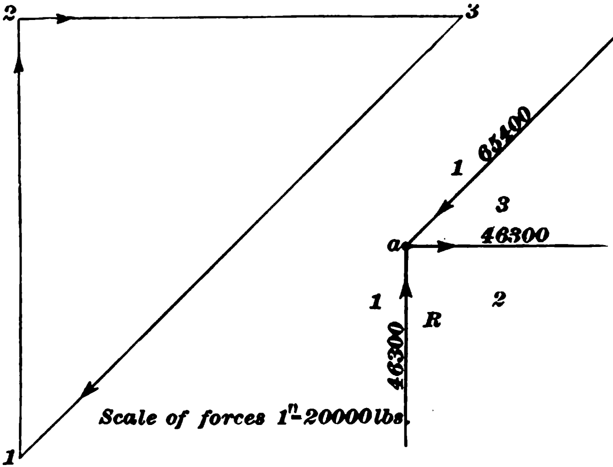


FIG. 313.

1543. In determining the actual bending moment upon a lower chord pin, the effect of the wind stress, though very often neglected, should be considered and provided for in a manner consistent with the method by which its effect upon the chord members connecting upon the pin was estimated in proportioning the material for those members. In other words, the bending moment upon the pin should be determined with regard to the effective sections of the members connecting upon it; a pin having an equal resisting moment would then be as strong as the connecting members. If the designer has not before him the data used in determining the increase of area to provide for the wind load, he may proceed as follows:

From the stress sheet he may take the dead and live load stresses (D and L), divide them by the dead and live load unit stresses, respectively, as given by the specifications, and add the quotients. The result will be the area necessary for live and dead load stresses only. Call this area A. If the area given on the stress sheet is equal to A, this will show that no provision was made for wind stress, and the latter may be neglected in proportioning the pin. But, if the area A_1 given on the stress sheet is different from A, the amount W of wind stress that must be provided for in dimensioning the pin is found by this simple proportion:

$$L + D : A :: L + D + W : A_1;$$

whence, $$W = (L + D)\left(\frac{A_1}{A} - 1\right).$$

In the present case, the live and dead load unit stresses allowed are 10,000 and 20,000 pounds per square inch, respectively. From the stress sheet, the live and dead load stresses upon the lower chord member *a b* are 32,400 and 13,900 pounds, respectively, or a total of $32{,}400 + 13{,}900 = 46{,}300$ pounds. The sectional area required by the live and dead load stresses is $\frac{32{,}400}{10{,}000} + \frac{13{,}900}{20{,}000} = 3.94$ square inches. As this is the area given on the stress sheet, no provision need be made for wind stress.

At the shoe joint, the additional amount of stress assumed to provide for wind stress, when A_1 is greater than A, may be considered to be resisted by the reaction, through the medium of the shoe. At each intermediate joint of the lower chord, if the lateral rods connect upon the floor-beams, and the latter connect upon the pins by means of hangers, this additional stress, assumed to provide for the effect of wind stress, may be considered to come upon the pin through the beam hanger and upon the chord through the pin. In other words, as the wind stresses are resisted by the lateral system, the additional stresses assumed to provide for the effects of the wind stresses upon the lower chord may be considered to come upon the chord through the piece by which the diagonals of the lower lateral system attach to the chord.

This practice is not in all cases thoroughly consistent with accuracy, but the small error will probably be upon the side of safety. It is necessary to make some assumption by which the forces considered to act upon the pin will be in equilibrium.

1544. The distance, as determined above, between the centers of the bearings of each member connecting upon the shoe pin, together with the stress upon the same, is given in tabular form below. Stresses which act upwards or to the right upon the pin are designated by the + sign, and those which act downwards or to the left are designated by the — sign.

Member.	Distance Between Bearings, Inches.	Vertical Stress, Pounds.	Horizontal Stress, Pounds.
Shoe,	$10\frac{1}{2}$	+ 46,300	0,000
End post,	$8\frac{3}{16}$	— 46,300	— 46,300
Lower chord,	$6\frac{1}{2}$	0,000	+ 46,300

It will be noticed that in each column the sum of the stresses designated by the + sign equals the sum of the stresses designated by the — sign.

The stress in each member is delivered upon the pin through two bearings, one-half of each stress through each bearing. The maximum bending moment on the pin might be found by the force and moment diagrams: but in a simple case like this it is easier to find it by direct calculation. Fig. 314 shows one end of the pin with the forces acting upon it. The distance between two consecutive bearing points is easily determined from the distances tabulated above. Thus, distance from shoe bearing to center (middle) of

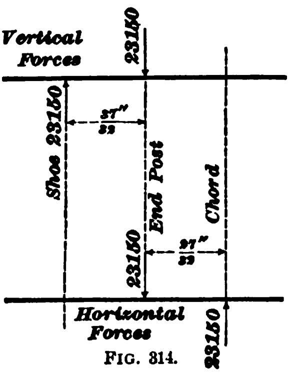


FIG. 314.

pin $= 10\frac{1}{2} \div 2 = 5\frac{1}{4}''$; distance from end post bearing to center of pin $= 8\frac{3}{16} \div 2 = 4\frac{3}{32}''$. Therefore, distance between shoe bearing and end post bearing $= 5\frac{1}{4} - 4\frac{3}{32} = \frac{37}{32}''$. The moment of the vertical forces $= 23{,}150 \times \frac{37}{32} = 26{,}770$ inch-pounds. The moment of the horizontal forces $= 23{,}150 \times \frac{27}{32} = 19{,}530$ inch-pounds.

The resultant bending moment upon the pin is found by constructing a force polygon in the form of a right-angled triangle whose base and altitude represent, respectively, the horizontal and vertical bending moments as obtained. The hypotenuse of the triangle will represent the resultant bending moment upon the pin. This is clearly shown in Fig. 315. Or the resultant moment may be computed from the horizontal and vertical moments in the same manner that the length of the hypotenuse is calculated in a right-angled triangle. It is equal to $\sqrt{26{,}770^2 + 19{,}530^2} =$ 33,100 inch-pounds, nearly.

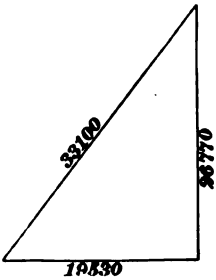


FIG. 315.

1545. The Resisting Moments of Pins.—Having ascertained the maximum bending moment upon a pin, it is necessary to determine next the size of the pin required to resist the moment. The pin acts simply as a solid beam having a circular cross-section.

The allowed bending stress, or stress per square inch upon the extreme fiber of pins (S_4 of formula **73**, Art. **1243**), commonly called the **extreme fiber stress,** is usually taken at 15,000 and 18,000 pounds for wrought iron, and at 20,000, 22,500, and 25,000 pounds for steel, according to the nature of the structure and the requirements of different specifications. For each material the lower unit values are used for railroad, and the higher unit values for highway bridges.

Formula **73** is

$$M = \frac{S I}{c},$$

in which M is the moment of resistance (which must be equal to the bending moment), S is taken to represent the allowed stress in the outermost fiber (i. e., the extreme fiber stress), c is the distance from the neutral axis to the outermost fiber, and I is the moment of inertia of the cross-section of the pin.

From the ninth item of Table of Moments of Inertia, it is known that for a solid circular section $I = \frac{3.1416\,d^4}{64} = .049\,d^4$, and $c = \frac{1}{2}d$, d being the diameter of the section.

By substituting these values of I and c in the preceding formula, we get

$$M = \frac{.049\,S\,d^4}{\frac{1}{2}d} = .098\,S\,d^3. \qquad \textbf{(171.)}$$

$$d = 2.17\sqrt[3]{\frac{M}{S}}. \qquad \textbf{(172.)}$$

The values of M and d given by these formulas are sufficiently exact for all practical purposes. It is, however, much more expedient to obtain the values of M from a table prepared by formula **171.**

1546. Table 40 gives the values of the resisting moments of pins for each eighth of an inch from 1 to 5¾ inches diameter, for fiber stresses of 15,000, 18,000, 20,000, 22,500, and 25,000 pounds. For convenience, the bearing values for 1 inch thickness of plate (= diameter of pin × 1″ × allowed bearing stress per sq. in.) are also given for bearing stresses of 12,000, 15,000, and 18,000 pounds per square inch.

For the resisting moments of pins having diameters expressed in odd sixteenths of an inch, it will be sufficiently correct to use a value 50 pounds less than a mean between the next lower and next higher values. Thus, with a fiber stress of 15,000 pounds per square inch, the resisting moment of a $2\frac{7}{16}''$ pin may be taken at $\frac{19,700 + 23,000}{2} - 50 = 21,300$ inch-pounds.

For intermediate bearing values, a mean between the next lower and next higher values may be taken.

TABLE 40.

BEARING VALUES AND RESISTING MOMENTS OF PINS.

Diameter of Pin in Inches.	Area of Pin in Square Inches.	Bearing Values, in Pounds, for One Inch Thickness of Plate.			Moments, in Inch-Pounds, for Extreme Fiber Stresses of				
		12,000 lb. per Square Inch.	15,000 lb. per Square Inch.	18,000 lb per Square Inch.	15,000 lb. per Square Inch.	18,000 lb. per Square Inch.	20,000 lb. per Square Inch.	22,500 lb. per Square Inch.	25,000 lb. per Square Inch.
1	0.785	12,000	15,000	18,000	1,470	1,770	1,960	2,210	2,450
1⅛	0.994	13,500	16,900	20,300	2,100	2,520	2,800	3.140	3,500
1¼	1.227	15,000	18,800	22,500	2,880	3,450	3,830	4,310	4,790
1⅜	1.485	16,500	20,600	24,800	3,830	4.590	5,100	5.740	6,380
1½	1.767	18,000	22,500	27,000	4,970	5,960	6,630	7.460	8,280
1⅝	2.074	19,500	24,400	29,300	6,320	7,580	8,430	9.480	10,500
1¾	2.405	21,000	26,300	31,500	7,890	9.470	10,500	11,800	13,200
1⅞	2.761	22,500	28,100	33,800	9,710	11,600	12,900	14,600	16,200
2	3.142	24,000	30,000	36,000	11,800	14,100	15,700	17,700	19,600
2⅛	3.547	25,500	31,900	38,300	14,100	17,000	18,800	21,200	23,600
2¼	3.976	27,000	33,800	40,500	16,800	20,100	22,400	25,200	28,000
2⅜	4.430	28,500	35,600	42,800	19,700	23,700	26,300	29,600	32,900
2½	4.909	30,000	37,500	45,000	23,000	27,600	30,700	34.500	38,400
2⅝	5.412	31,500	39,400	47,300	26,600	32,000	35.500	40,000	44,400
2¾	5.940	33,000	41,300	49,500	30,600	36,800	40,800	45,900	51,000
2⅞	6.492	34,500	43,100	51,800	35,000	42,000	46,700	52,500	58,300
3	7.069	36,000	45,000	54,000	39,800	47,700	53,000	59,600	66,300
3⅛	7.670	37,500	46,900	56,300	44,900	53,900	59,900	67,400	74,900
3¼	8.296	39,000	48,800	58,500	50,600	60,700	67,400	75,800	84,300
3⅜	8.946	40,500	50,600	60,800	56,600	67,900	75,500	84,900	94,400
3½	9.621	42,000	52,500	63,000	63,100	75,800	84,200	94,700	105,200
3⅝	10.321	43,500	54,400	65,300	70,100	84,200	93,500	105,200	116,900
3¾	11.045	45,000	56,300	67,500	77.700	93,200	103,500	116,500	129,400
3⅞	11.793	46,500	58,100	69,800	85,700	102,800	114,200	128,500	142,800
4	12.566	48,000	60,000	72,000	94,200	113,100	125,700	141,400	157,100
4⅛	13.364	49,500	61,900	74,300	103,400	124,000	137,800	155,000	172.300
4¼	14.186	51,000	63,800	76,500	113.000	135,700	150,700	169,600	188,400
4⅜	15.033	52,500	65,600	78,800	123,300	148,000	164,400	185,000	205,500
4½	15.904	54,000	67,500	81,000	134,200	161,000	178,900	201.300	223,700
4⅝	16.800	55,500	69,400	83,300	145,700	174,800	194,300	218,500	242,800
4¾	17.721	57,000	71,300	85,500	157,800	189,400	210,400	236,700	263,000
4⅞	18.665	58,500	73,100	87,800	170,600	204,700	227,500	255,900	284,400
5	19.635	60,000	75,000	90,000	184,100	220,900	245,400	276,100	306,800
5⅛	20.629	61,500	76,900	92,300	198,200	237,900	264,300	297,300	330,400
5¼	21.648	63,000	78,800	94,500	213,100	255.700	284,100	319,600	355,200
5⅜	22.691	64,500	80,600	96,800	228,700	274.400	304,900	343,000	381,100
5½	23.758	66,000	82,500	99,000	245,000	294,000	326,700	367,500	408.300
5⅝	24.850	67,500	84,400	101,300	262,100	314,500	349,500	393.100	436,800
5¾	25.967	69,000	86,300	103,500	280,000	335.900	373,300	419,900	466.600
5⅞	27.109	70,500	88,100	105,800	298,600	358,300	398.200	447,900	497,700

In the present structure, according to item (b_1) of the specifications (Art. **1511**), the bending stress must not exceed 18,000 pounds per square inch.

By reference to Table 40, Art. **1546,** it is found that with an extreme fiber stress of 18,000 pounds per square inch, a pin $2\frac{3}{4}$ inches in diameter has a resisting moment of 36,800 inch-pounds.

Hence, this diameter of pin will resist the bending moment upon the shoe pin, which has been found to be 33,100 inch-pounds. It may be noticed that, at 18,000 pounds per square inch, the resisting moment given by a $2\frac{11}{16}''$ pin is practically $\frac{32,000 + 36,800}{2} - 50 = 34,350$ inch-pounds, which would have been sufficient to resist the bending moment.

PIN PLATES FOR THE HIP JOINT OF THE CHORD.

1547. Thickness of Bearings for Chord.—The total maximum stress upon the upper chord is $48,600 + 20,800 = 69,400$ pounds. From Table 40 it is found that, at 15,000 pounds per square inch, the bearing value for one inch thickness of metal upon a $2\frac{3}{4}''$ pin is 41,300 pounds.

The total thickness of bearing surface upon the pin required for the stress in the upper chord is $\frac{69,400}{41,300} = 1.68$ inches, or, practically, $1\frac{11}{16}$ inches. By reference to Fig. 79 of Mechanical Drawing, it is found that the thickness of web in the upper chord channel (10 lb. per ft.) is $\frac{7}{32}$ of an inch, or a total thickness of $\frac{7}{16}$ of an inch for both channels; leaving $1\frac{11}{16} - \frac{7}{16} = 1\frac{1}{4}$ inches thickness of bearing to be given by the two pin plates, or $\frac{5}{8}$ of an inch each.

1548. Pin Plates.—It is desirable that the pin plates should extend beyond and around the pin. Hence, as the pin plates at the upper end of the end post are on the inside of the channels, the pin plates of the chord must be upon the outside. The width from outside to outside of the webs of the channels of the chord is $7\frac{1}{2} + 2 \times \frac{7}{32} = 7\frac{15}{16}$ inches, while the corresponding width from outside to outside of the webs of the channels of the end post is $7\frac{3}{8} + 2 \times \frac{3}{8} = 8\frac{1}{8}$ inches. This will not allow the pin plates directly on the outside of the webs of the channels of the chord to extend beyond the pin, outside of the webs of the channels of the end post. It will, therefore, be necessary to make the pin plates on the chord double, extending only the outer plates beyond and around the pin. Two pin plates, each $\frac{5}{16}$ of an inch thick, instead of one plate $\frac{5}{8}$ of an inch thick, will be placed upon the outside of the web of each channel of each chord, and the outer plate only will extend beyond and around the pin.

As determined above, the total thickness of bearing required for the chord is 1.68 inches, and the thickness given by the pin plates is 1.25 inches. Hence, the amount of live load stress to be transferred to the pin plates by the rivets is $48{,}600 \times \frac{1.25}{1.68} = 36{,}160$ pounds, and the amount of dead load stress to be transferred by the same is $20{,}800 \times \frac{1.25}{1.68} = 15{,}480$ pounds, or a total of $36{,}160 + 15{,}480 = 51{,}640$ pounds.

1549. Rivets Required by Shearing Stress.—Neglecting the reduction for column length, the live and dead load unit stresses allowed upon the upper chord are 10,000 and 20,000 pounds, respectively. Three-quarters of the live load unit stress (see item (a_1) of Art. **1511**) is 7,500 pounds, and from Table 37, Art. **1537,** the value of a $\frac{5}{8}''$ rivet in single shear at 7,500 pounds per square inch is 2,300 pounds, while for dead load stress it will be double this amount, or 4,600 pounds. Hence, the number of rivets required in the pin plates by this condition is $\frac{36{,}160}{2{,}300} + \frac{15{,}480}{4{,}600} = 19.1$, or, as the result can not be fractional, 20 rivets.

At 9,000 pounds per square inch, the value of a $\frac{5}{8}''$ rivet in single shear is 2,760 pounds (Table 38, Art. **1537**), and the total number of rivets required in the pin plates by this condition is $\frac{36,160 + 15,480}{2,760} = 18.7$, or, practically, 19 rivets.

1550. Rivets Required by Bearing Stress.—The thinnest plate through which the rivets pass is the web of the channel, which is slightly less than $\frac{1}{4}''$ in thickness. At 15,000 pounds per square inch, the bearing value of a $\frac{5}{8}''$ rivet in $\frac{1}{4}''$ thickness of plate is 2,340 pounds. (Table 37, Art. **1537.**) Using this value, the total number of rivets required in the pin plates is $\frac{36,160 + 15,480}{2,340} = 22$ rivets. The bearing is found to be the critical condition.

The required number of rivets as found is for the pin plates upon both channels; the number required in the pin plates attaching to each channel is $\frac{22}{2} = 11$ rivets. As the pin plates on each channel are double and the separate plates are of equal thickness, one-half of this number of rivets will be required for each plate. All the rivets necessarily pass through the inner plate, but only half of them, or 6 rivets, are required in the outer plate. The arrangement of the rivets in the pin plates of the chord is very similar to the arrangement in the pin plates of the end post, and will require no special explanation. These pin plates will be noticed again.

MOMENTS ON HIP PIN.

1551. Positions and Intensities of Bearings Upon Pin.—The thickness of the bearing upon the hip pin for each channel of the upper chord, including the pin plates upon it, being assumed equal to $\frac{1}{4} + \frac{5}{16} + \frac{5}{16} = \frac{7}{8}$ inch, the distance from center to center of the two bearings of the upper chord upon the hip pin is equal to $7\frac{1}{2} + \frac{7}{8} = 8\frac{3}{8}$ inches. At the hip joint, the distance between the inner surfaces of the pin plates of the end post is $7\frac{3}{8} - 2 \times \frac{7}{16} = 6\frac{1}{2}$ inches, and the distance from center to center of the two bearing surfaces of the end post upon the hip pin is $6\frac{1}{2} + \frac{7}{16} + \frac{3}{8} = 7\frac{5}{16}$ inches. The main tie bars and hip vertical rods are both packed inside, or between the bearings of the end post. As the main tie bars bear greater stress than the hip vertical rods, the former are packed adjacent to the end post bearings. The thickness of the tie bar is $\frac{13}{16}$ of an inch, but on account of the pin being situated so near to the tops of the channel, thus limiting the diameter of the head of the tie bar, it is very probable that it will be found necessary to thicken the heads of the tie bars, in order to give sufficient metal back of the pin hole. Hence, in obtaining the bending moment, it will be well to consider the tie bar heads to be $\frac{7}{8}$ of an inch thick.

Allowing $\frac{1}{8}''$ clearance on each side, the distance between the outer sides of the heads of the tie bars is $6\frac{1}{2} - 2 \times \frac{1}{8} = 6\frac{1}{4}$ inches, and the distance from center to center of this bearing surface is $6\frac{1}{4} - \frac{7}{8} = 5\frac{3}{8}$ inches. The clear distance between the inner surfaces of the tie-bar heads is $6\frac{1}{4} - 2 \times \frac{7}{8} = 4\frac{1}{2}$ inches.

A clearance of one-sixteenth of an inch is all that is necessary to allow between two eye-bar heads, or between an eye-bar head and a welded loop. (*See Art.* ***1524,*** *I.*)

Hence, the distance from outside to outside of the loops of the hip vertical rods will be $4\frac{1}{2} - 2 \times \frac{1}{16} = 4\frac{3}{8}$ inches, and, as the hip vertical rods are 1 inch square, the distance from center to center of the same is $4\frac{3}{8} - 1 = 3\frac{3}{8}$ inches.

The stress in the upper chord is horizontal, and equal to 69,400 pounds.

The stress upon the hip vertical is vertical, and equal to 20,800 pounds.

The stress in the end post has a horizontal and a vertical component, each of which has been found to be equal to 46,300 pounds. (Art. **1542.**)

It is evident that the greatest bending moment upon the pin will occur with the truss fully loaded, as this condition will give the maximum stress to every member connecting upon the pin except the main tie $B\,c$. The stress upon the main tie with the truss fully loaded, not being the maximum stress upon that member, is not shown upon the stress sheet. But it is evident that, in order that the forces acting upon the hip joint shall be in equilibrium, the horizontal components of the stresses of the main tie and end post must be equal to the horizontal stress in the upper chord, as no other stress having a horizontal component acts upon this joint. Hence, the horizontal component of the stress in the main tie is equal to 69,400 − 46,300 = 23,100 pounds. Similarly, its vertical component must be equal to the vertical component of the stress in the end post minus the stress in the hip vertical and the one-third panel dead load (2,300 pounds) assumed to be supported directly at the hip joint, or 46,300 − (20,800 + 2,300) = 23,200 pounds. As the vertical and horizontal projections of the tie (height of truss and panel length) are equal, the vertical and horizontal components of its stress are found to be practically equal; they are really exactly equal.

The one-third panel load of dead load (2,300 pounds) assumed to be applied directly at the hip joint may be considered to come upon the pin through the bearings of the upper chord.

The distance between the centers of the two bearings of each member connecting upon the pin, together with the horizontal and vertical stress upon the same, as determined above, is given in tabular form below.

Those stresses which act upwards, or to the left, upon the pin are designated by the + sign, and those stresses which act downwards, or to the right, are designated by the − sign.

Member.	Distance Between Bearings, Inches.	Horizontal Stress, Pounds.	Vertical Stress, Pounds.
Upper chord,	$8\frac{3}{8}$	+ 69,400	− 2,300
End post,	$7\frac{5}{16}$	− 46,300	+ 46,300
Main tie,	$5\frac{3}{8}$	− 23,100	− 23,200
Hip vertical,	$3\frac{3}{8}$	0,000	− 20,800

In each column the sum of the stresses designated by the + sign equals the sum of the stresses designated by the − sign.

1552. Bending Moments Upon Pin.—The force diagram and equilibrium polygon for the horizontal forces acting upon the pin are shown at (*a*) in Fig. 316, and at (*b*) in the same figure are shown the force diagram and equilibrium polygon for the vertical forces.

For the horizontal forces the pole distance is 40,000 pounds. The maximum intercept, uniform from *c d*, is, to scale, .74 of an inch. Hence, the maximum horizontal bending moment is 40,000 × .74 = 29,600 inch-pounds.

For the vertical forces the pole distance is 30,000 pounds, and the maximum intercept, uniform from *d* to *e*, is, to scale, 1.04 inches. Hence, the maximum vertical bending moment is 30,000 × 1.04 = 31,200 inch-pounds.

The resultant maximum bending moment is equal to $\sqrt{29,600^2 + 31,200^2} = 43,000$ inch-pounds.

1553. Resisting Moment of Pin.—For a bending stress of 18,000 pounds per square inch, as allowed for wrought iron by item (b_1) of the specifications (Art. **1511**), this amount of bending moment would require a pin 3 inches in diameter. But on account of the pin being located so near the top of the channels, it would be very undesirable to use a pin having a diameter greater than 2¾ inches. Hence, as all the conditions except the bending moment are satisfied with a 2¾″ pin, it will be better, instead of using a larger size of pin, to use a 2¾″ pin of *medium steel.*

Cooper's specifications do not directly state the intensity of bending stress allowed upon pins of medium steel. But, as they allow upon medium steel in the main members of a bridge unit stresses 20 per cent. greater than those allowed upon wrought iron, we may assume the same ratio of increase in the bending stresses to be allowed upon the pins. At 18,000 pounds per square inch, the resisting moment of a 2¾″ pin is 36,800 pounds. (See Table 40, Art. **1546.**)

An increase of 20 per cent. would give a resisting moment of 1.20 × 36,800 = 44,200 inch-pounds, which is sufficient to resist the bending moment of 43,000 pounds, as obtained above.

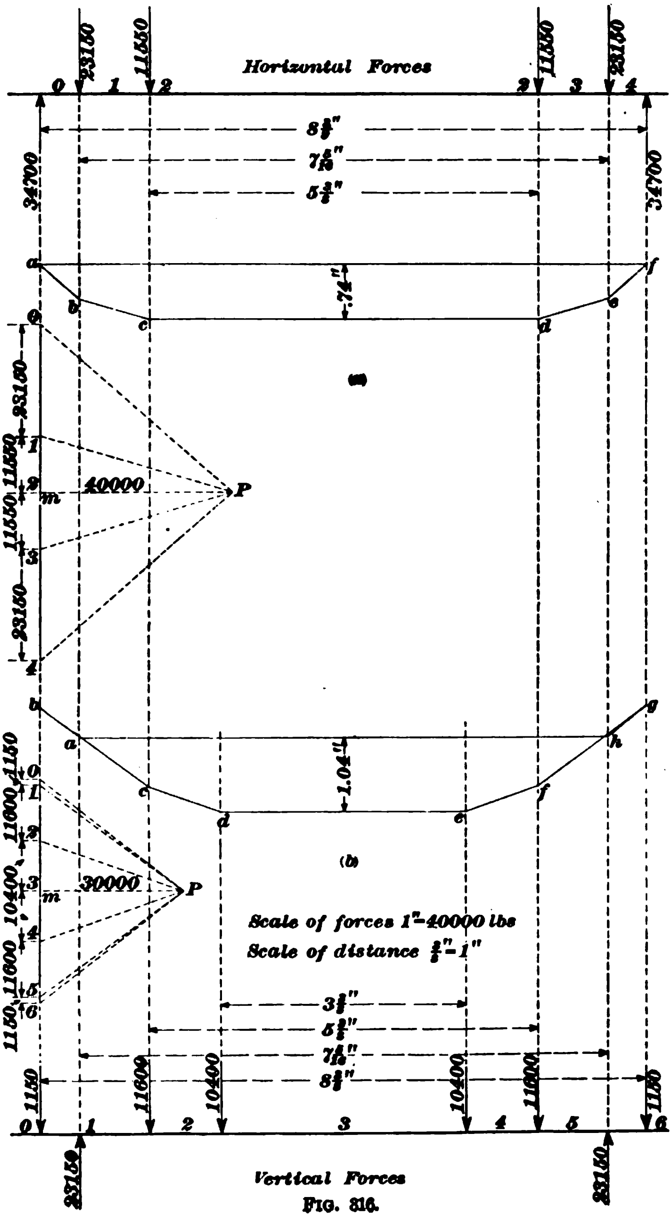

Horizontal Forces
23150
11550
11550
23150
0
1
2
2
3
4
34700
34700
8⅜"
7 5/16"
5⅜"
a
b
c
d
e
f
.74"
(a)
0
1
2
3
4
23150
11550
11550
23150
40000
m
P
b
a
c
d
e
f
h
g
1.04"
(b)
0
1
2
3
4
5
6
1150
11600
10400
10400
11600
1150
30000
m
P
Scale of forces 1"=40000 lbs
Scale of distance ⅜"=1"
3⅝"
5⅜"
7 5/16"
8⅜"
1150
11600
10400
10400
11600
1150
0
1
2
3
4
5
6
23150
23150
Vertical Forces

FIG. 816.

It will be noticed that for this size of pin a bending stress of 22,500 pounds per square inch, which is sometimes used for pins of medium steel, will give a resisting moment of 45,900 inch-pounds.

CONSTRUCTIVE DETAILS OF THE END POST.

1554. The dimensions of the connecting details of the end posts having been determined, the constructive details and rivet spacing for that member, with the exception of the connections for the portal bracing, may now be determined and laid out. These should be drawn out entirely in pencil, as it may afterwards become necessary to modify certain dimensions slightly, in order to provide for the conditions of connecting members. The details of the end posts are shown in Mechanical Drawing Plate, Title: Highway Bridge: Details I, Fig. 1.

1555. The Top View; Positions of Rivet Lines.—The first step with reference to the rivet spacing is to fix the positions of the two lines of rivets connecting the cover-plate to the channels. The distance between these two lines of rivets is governed by the distance a, Fig. 317, from the center of the rivet hole through the flange of the channel to the back of the same, together with the distance, back to back, between the two channels.

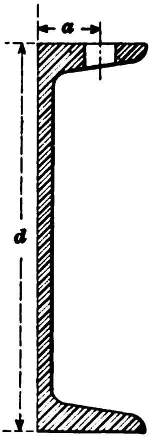


FIG. 317.

As the widths of the flanges of the same size channels vary considerably, not only as rolled in different mills, but also as rolled in different weights in the same mill, it is impossible to fix a general standard for the distance from the back of the channel to the rivet holes through the flange. The following formula, however, gives values for this spacing which are sufficiently close for practice:

$$a = \frac{d}{8} + \frac{1}{4}, \qquad \textbf{(173.)}$$

in which a is the distance from the back of the channel to the center of the rivet hole *for the minimum weight* of channel, and d is the depth of the channel, both in inches. See Fig. 317.

For channels heavier than the minimum weight, but of the same form, except with thicker web (i. e., in which the weight is increased by simply spreading the rolls), the space a should be increased by the amount a_1 as derived by the following formula:

$$a_1 = \frac{.3\,w_1}{d}, \qquad \textbf{(174.)}$$

in which w_1 is the *increase* in pounds of the weight per foot *above the minimum weight*, and d is the depth of channel, as above.

The value given to the space a, Fig. 317, should contain no fraction smaller than one-sixteenth of an inch.

1556. It may here be noticed, incidentally, that from formula **174** may be obtained the thickness of the web or the width of the flange of a channel of any weight, when the thickness of web or the width of flange in the minimum weight of the same size and form of channel is known. In this formula, a_1 *equals the increase in the thickness of web, or width of flange, for each pound per foot increase in weight.*

This formula gives accurate results for wrought iron only.

The values of the spacing given by formula **173** will, in many cases, vary slightly from the standards adopted by various manufacturers. Each large manufacturer of structural material usually has his own standard spacing, suited to the widths of the flanges in the channels manufactured by him.

Perhaps the most notable exception to the correct application of formula **173** occurs in the case of light 8″ channels. As rolled by some mills, the lighter weights of this size of channels (10 pounds per foot) have flanges no wider than the ordinary widths of flanges in 7″ channels. The spacing for the flanges of these lighter weights of 8″ channels will here be obtained by applying formula **173** the same as for 7″ channels. When the weight of the channel does not greatly exceed the minimum weight, formula **174** need not be applied.

Thus, in the example, for the 8″ channels of the upper chord, the space *a* should equal $\frac{1}{8} \times 7 + \frac{1}{4} = 1\frac{1}{8}$ inches. Hence, as the distance between the channels, back to back, is $7\frac{1}{2}$ inches (Art. **1523**), the perpendicular distance between the rows of rivets connecting the cover-plate to the channels in the upper chord is $7\frac{1}{2} + 2 \times 1\frac{1}{8} = 9\frac{3}{4}$ inches. This distance should usually be made the same on the chord and end post. The distance, back to back, between the channels of the end post is $7\frac{3}{8}$ inches. Hence, the distance *a*, from the center of the rivet hole through the flange to the back of the channel, will be made equal to $\frac{9\frac{3}{4} - 7\frac{3}{8}}{2} = 1\frac{3}{16}$ inches. This spacing is shown on the top view of the end post, Mechanical Drawing Plate, Title: Highway Bridge: Details I, Fig. 1. If the channels of the end post were of lighter weight, having narrower flanges, the distance between the backs of the channels, and also between the rivet lines, would be somewhat greater. The positions of the rivet *lines* being thus fixed, the spacing of the rivets is readily laid out.

1557. Spacing of Rivets in Cover-Plate.—By item (d_1) of the specifications the pitch of the rivets must not exceed 6 inches, or sixteen times the thickness of the thinnest outside plate. The cover-plate in the end post is $\frac{3}{8}$ of an inch thick; hence, the pitch of the rivets connecting it to the channels must not exceed $16 \times \frac{3}{8} = 6$ inches. The maximum pitch of the rivets in the cover-plate has been made 5 inches, though, in compliance with the specifications, it could have been 6 inches. Near the ends of the member the pitch of the rivets should always be materially diminished; and, so far as possible, the rivet spacing should be the same at both ends of the member; that is, the rivet spacing on the two halves of the member should be symmetrical with reference to the center (between pin holes) of the member. This facilitates the laying out of the work in the shop. The sum of the rivet spaces between the centers of the pin holes should be exactly equal to the length of the member between those points. Thus, in the end post of the example, the sum of the rivet spaces between centers of pin holes is as follows:

53 spaces at	5 inches	= 22′ 1″
2 spaces at	$4\frac{9}{16}$ inches	= 0′ $9\frac{1}{8}$″
4 spaces at	4 inches	= 1′ 4″
2 spaces at	$3\frac{1}{2}$ inches	= 0′ 7″
2 spaces	3 and $1\frac{1}{4}$ inches	= 0′ $4\frac{1}{4}$″
2 spaces	$2\frac{1}{4}$ and 2 inches	= 0′ $4\frac{1}{4}$″
Distance center to center of pins		= 25′ $5\frac{5}{8}$″

At the lower end of the end post, the rivet spacing between the center of the pin hole and the end of the member should exactly correspond with the length between those two points. At the top of the end post, the rivet spacing must correspond with the holes left vacant for connecting the hip cover-plate, which in this case serves also as a connection for the portal bracing.

The bevel for the hip joint should be carefully determined by laying out in pencil, to a large scale, the general dimensions of a side elevation of a hip joint somewhat similar to the side elevation shown in Mechanical Drawing Plate Title: Highway Bridge: Details IV, Fig. 13; and the exact bevel should be shown on the side elevation by the base and altitude of a right-angled triangle, one of which dimensions should be 12 inches for convenience in laying out in the shop. The exact length of both cover-plate and channels should be given.

1558. The Bottom View; Batten Plates.—For the details of the bottom view, the positions of the batten plates (sometimes also called **tie plates** and **stay plates**) are first approximately determined. They should be as near to the ends of the member as possible, but should give ample clearance for the members connecting upon the pins. The position of the lower batten plate, giving sufficient clearance for the lower chord member, may be found by drawing accurately in pencil a side elevation of the shoe joint similar to that shown in Fig. 17 of the plate just referred to, but omitting unnecessary portions. From this drawing may also be determined the general dimensions of the standards of the shoe; also, how much, if any, of the lower corners of the channels of the end post it will be necessary to cut off, and how far back from the end it will be necessary to cut away a portion of the lower flanges of the channel, in order that the end post will fit into the shoe with sufficient clearance, as shown on the side elevation and bottom view in Fig. 1, same title as above, I.

The position of the upper batten plate on the end post, giving sufficient clearance for the hip vertical rods, may be found by drawing the hip vertical rods in their proper position in the side elevation of the hip joint drawn to determine the bevel. From this drawing may also be obtained the position of the batten plate on the upper chord, giving sufficient clearance for the main tie bars.

1559. In arranging the approximate position of each batten plate, a clearance of about one inch should be given to the member connecting upon the pin. According to item (o_1) of the specifications, each batten plate must have a length equal to $1\frac{1}{2}$ times the width of the member, or, in the present case, $1\frac{1}{2} \times 12 = 18$ inches. The thickness of the batten plates should be not less than $\frac{1}{50}$ of the distance between the rivet lines in the parts which the batten plates connect, or, in the present case, $\frac{9.75}{50} = .195$ of an inch. The thickness of the batten plates, however, may generally be the same as the thickness specified for the lattice bars on the same member, or, in this case, $\frac{1}{4}$ of an inch. According to the first condition of item (d_1) of the specifications, the pitch of the rivets in the batten plates of the example must not exceed $16 \times \frac{1}{4} = 4$ inches. By placing each of the outer rivets in the batten plate at a distance of $1\frac{1}{4}$ inches from the corresponding end of the plate, there will remain

$18 - 2 \times 1\frac{1}{4} = 15\frac{1}{2}$ inches between these rivets, which may be divided into four spaces of $\frac{15.5}{4} = 3\frac{7}{8}$ inches each. The distance between the rivet lines on the lower side of the end post is the same as the corresponding distance on the upper side, or $9\frac{3}{4}$ inches.

1560. It will be well to notice also that item (d_1) of the specifications requires that "the pitch of rivets at the ends of the compression members shall not exceed four diameters of the rivet for a length equal to twice the width of the member." As the batten plate usually comes within this length, the pitch of rivets in it should be considered to be governed by this condition, although this was not done in the example. In any case, however, the following is a safe

Rule.—*In chords and end posts, make the pitch of rivets in the batten plates as nearly as practical the same as in that portion of the cover-plate directly opposite. This pitch, however, must in no case exceed sixteen times the thickness of the batten plate.*

1561. The Lattice Bars.—The portions of the bottom flanges of the channels of the end post situated between the batten plates are to be stayed by diagonal bars, called **lattice bars.** When the diagonal stays or braces of this system do not intersect each other at any point between their ends, as is the case on the chord and end post of the example, the system is sometimes called **lacing,** and the bars are called **lacing bars,** those bars only which cross each other being called lattice bars. In this Course, however, the distinction will not be made, but all such systems of stays or bracings will be designated as lattice, and the bars as lattice bars.

1562. Lattice bars connected by one rivet at each end, as in the present case, should have a width of about three times the diameter of the rivet used, or, in our example, about $3 \times \frac{5}{8} = 1\frac{7}{8}$ inches. Item (o_1) of the specifications (Art. **1511**) requires the size of lattice bars used to connect the flanges of 8″ channels to be 1¾″ × ¼″. The same item specifies that the lattice bars shall be inclined at an angle of not less than 60 degrees with the axis (center line) of the member. The distance between the batten plates may be divided into any number of equal spaces that will give the required pitch. In order that the pitch of the lattice bars may not contain an inconvenient fraction, the spacing may be started from the inner rivets of the batten plates, as shown in Mechanical Drawing Plate, Title: Highway Bridge: Details I, Fig. 1, or it may begin at points on the channels about 2½ inches from those rivets, a distance just sufficient for the lattice bar to clear the batten plate. It is better practice to have the lattice bars connect upon the inner rivets of the batten plates. The positions of the batten plates may be slightly changed to suit the spacing of the lattice bars. It must be remembered that the rivets attaching the lattice bars to one channel are midway between those attaching them to the opposite channel, requiring half spaces at the ends, which may both come upon the same channel and count as a whole space, or one may come upon each channel and thus count as a half space.

1563. The exact length l of the lattice bars, from center to center of rivet holes, should always be given; it may be found by the following formula:

$$l = \sqrt{\left(\frac{p}{2}\right)^2 + w^2}, \qquad \textbf{(175.)}$$

in which p is the pitch of the lattice, i. e., the distance between two adjacent rivets connecting the lattice bars in the same rivet line, and w is the perpendicular distance between the two rivet lines.

If the lattice bars are inclined at an angle of *exactly* 60 degrees,

$$l = p = 1.1547\,w. \qquad \textbf{(176.)}$$

Thus, in the end post of the example, if the inclination of the lattice bars is 60 degrees, the pitch of the lattice is equal to the length of the bars = 1.1547 × 9.75 = 11.2583 inches, or, practically, 11¼ inches. It is found that by taking advantage of the half space of $\frac{11\frac{1}{4}}{2} = 5\frac{5}{8}$ inches, one of which is made to count upon each channel at the opposite ends of the latticed length of the member, the bottom plates can readily be so arranged that a pitch of 11¼ inches can be used.

1564. The distance of the outer rivet in each batten plate from the center of the pin hole should be given. These two distances, together with the rivet spacing on the batten plates and the spacing for the lattice bars, when added together, must give the exact length of the member from center to center of pin holes. On the end post of the example these distances are as follows:

1 space at 12¾ inches = 1′ 0¾″
4 spaces at 3⅞ inches = 1′ 3½″
22 spaces at 11¼ inches = 20′ 7½″
1 space at 5⅝ inches = 0′ 5⅝″
4 spaces at 3⅞ inches = 1′ 3½″
1 space at 8¾ inches = 0′ 8¾″

Distance center to center of pins = 25′ 5⅝″

DETAILS OF THE UPPER CHORD.

1565. The arrangement of the constructive details of the upper chord is so very similar to that for the end post as to require very little special explanation. The thickness of bearing and number of rivets required, determining the sizes of the pin plates upon the chord at the hip, were found in connection with the details of the hip joint. In the cover plate, the pitch of rivets must not exceed 16 × ¼ = 4 inches. In the greater portion of the length of the member the pitch of the rivets is 4 inches, but it is reduced towards the ends of the member. Near each end of the panel the rivet spacing is necessarily accommodated to the details of the connecting members. Between the centers of the pin holes the sum of the rivet spaces exactly equals the panel length.

On top of the cover plates is riveted the bent plate for connecting the portal bracing and also the short pieces of angles forming the lateral connections. The intermediate lateral strut also connects on top of the chord at C, vacant holes for this connection being shown in the cover-plate and cover splice plate.

1566. The **lateral connections** are placed as near to the portal and intermediate struts as practicable, but allowing sufficient clearance. The angles at which the lateral connections or lateral hitches are placed upon the chord are determined by the positions of the points at which the center lines of the lateral rods intersect the center line of the chord. Each point of intersection is also the center, in each direction, between the rivets connecting the short pieces of angles forming the lateral connections, or, in other words, the center of the lateral hitch. Theoretically, these points, where the center lines of the lateral rods intersect the center line of the chord, should be at the intersections of the center lines of the lateral struts with the center line of the chord. It is impossible, however, to locate the lateral hitches at these intersections, but they should be located as near to them as possible.

At the hip joint, the distance from the center of the pin to the center of the lateral hitch is $14\frac{3}{16}$ inches, and at the intermediate joint C, the distance between the center of the pin and the lateral hitch of the end panel is $8\frac{3}{16}$ inches. Hence, as the panel length of the chord is $18'\ 0\frac{3}{8}''$, the longitudinal projection of the lateral rods in the panel $B\ C$, from center to center of connections, i. e., the distance between the centers of the two lateral hitches, is $18'\ 0\frac{3}{8}'' - (14\frac{3}{16}'' + 8\frac{3}{16}'') = 16'\ 2'' = 194$ inches. The lateral projection of the lateral rods, center to center of connections, i. e., the distance from center to center of chords, is $18 + 1 = 19$ feet $=$ 228 inches. In determining the base and altitude of the right-angled triangles which show the angles of the lateral hitches, if the smaller dimension be made 12 inches, by calling the greater dimension x, we have the proportion $194 : 228 :: 12 : x$, whence $x = 14.103$, or slightly more than

$14\frac{3}{32}$ inches. For the convenience of the workmen, one of these dimensions should always be 12 inches.

1567. Splice Plates and Intermediate Joint.—The channels and cover-plate of the upper chord are spliced as near the joint at *C* as is practicable for the details of the connecting members. The ends of the channels and cover-plates are to be planed to give true abutting surfaces. According to item (g_1) of the specifications, the joint must be spliced sufficiently to maintain these parts accurately in contact. A splice plate is placed on top of the cover-plate and on the outside of the web of each channel. Sometimes, in large chords, small angle bar splices are also placed on the lower sides of the lower flanges of the channels, but it was not done in this case. The splice plates on the outsides of the webs of the channels serve also as reinforcing plates for the pin bearings, as do also the plates for the post connection on the insides of the channels, really giving considerably more than the required amount of bearing for the pin.

As the splice plates for the channels and cover-plate of the chord do not bear stress, but simply serve to hold the parts in position, no rule can be given for calculating the number of rivets required. The designer must be guided by his judgment and by such conditions as may arise in each individual case, always endeavoring to make a neat and substantial joint. Sufficient clearance should always be given between the heads of rivets passing through the web of the channel and the heads of rivets passing through the flanges of the same.

The bending moment upon the pin at this joint is produced wholly by the stress upon the counter rods ; it occurs in the plane of these rods, and is found by a single moment diagram. It will not be necessary to explain the process in detail.

1568. Batten Plates and Lattice Bars.—The size of the batten plates, as well as the size of the lattice bars, is the same on the chord as on the end post. The position of the batten plate near the hip joint must be such as to give sufficient clearance for the main tie bars, which is determined by a drawing of the side elevation of the hip joint, as previously stated. The batten plate in the end panel near the intermediate joint *C* is not required to clear any diagonal member, but it should be as near the splice as practicable. It is found that by carefully arranging the positions of the batten plates, the pitch and, consequently, the length of the lattice bars on the chord can be made the same as on the end post. The pitch of the lattice bars throughout the chord and end posts should be made the same where practicable. The rivet spacing on the bottom of the chord between centers of the pin holes should add up exactly equal to the panel length.

All rivet spacing in the center panel of the chord is made as nearly the same as that of the end panels as the conditions will permit. The position of the batten plate, giving sufficient clearance for the counter rods, is determined by drawing a side elevation of the intermediate joint similar to that shown in Mechanical Drawing Plate, Title: Highway Bridge: Details IV, Fig. 14. As this panel of the chord is made perfectly symmetrical with reference to the center of the panel, only one end need be shown.

CONSTRUCTIVE DETAILS OF THE INTERMEDIATE POST.

1569. The Top Connection.—The angles of which the intermediate post is composed connect on the inside of plates that are for that purpose riveted on the inside of the channels of the upper chord. The live and dead load stresses upon the intermediate post are, respectively, 9,700 and 2,300 pounds. Neglecting the reduction for column length, the live and dead load unit stresses allowed upon the post are 8,750 and 17,500 pounds per square inch, respectively. Allowed shearing stresses upon a $\frac{5}{8}''$ rivet, at three-quarters the unit stresses allowed upon the member $=$ $8{,}750 \times \frac{3}{4} \times .3068 = 2{,}010$ pounds for live load stress, and

$17,500 \times \frac{3}{4} \times .3068 = 4,030$ pounds for dead load stress. Hence, the total number of rivets required by this condition for connecting the angles of the intermediate post to the plates riveted upon the inside of the channels of the chord is $\frac{9,700}{2,010} + \frac{2,300}{4,030} = 5.4$, or, practically, 6 rivets.

At 9,000 pounds per square inch, the value of a ⅝″ rivet in single shear, as given by Table 38, Art. **1537,** is 2,760 pounds. Hence, the number of rivets required for the same purpose by this condition is $\frac{9,700 + 2,300}{2,760} = 4.3$, or, practically, 5 rivets.

The thickness of metal in the angles of the intermediate post is ½ of an inch. At 15,000 pounds per square inch, Table 37, Art. **1537,** gives the bearing value of a ⅝″ rivet through ½″ thickness of metal at 2,340 pounds. Hence, the number of rivets required by this condition for bearing stress is $\frac{9,700 + 2,300}{2,340} = 5.1$, or, practically, 6 rivets.

At one and one-half times the unit stresses allowed upon the post, the bearing value for live load stress of a ⅝″ rivet through ½″ thickness of plate is $8,750 \times \frac{3}{4} \times \frac{5}{8} \times \frac{1}{2} = 2,050$ pounds, and for dead load stress, $2,050 \times 2 = 4,100$ pounds. Hence, the number of rivets required by this condition is $\frac{9,700}{2,050} + \frac{2,300}{4,100} = 5.3$, or, practically, 6 rivets.

Therefore, it is found that (5.4) 6 rivets, if machine driven, will meet all the requirements.

But the rivets connecting the angles of the intermediate post to the plates riveted upon the inside of the channels of the chord are necessarily field driven.

According to the latter portion of item (a_1) of the specifications, the specified limits of shearing and bearing stress must, for field riveting, be reduced one-third part; or, what amounts to the same thing, the number of rivets, as required by the limits specified for shearing and bearing, must, for field rivets, be increased one-half. Consequently, the number of field rivets required in the present case is $1\frac{1}{2} \times 5.4 = 8.1$, or, practically, 8 rivets.

1570. The preceding computations may be a little shortened by means of the following formulas, in which $d =$ diameter of rivet, $t =$ thickness of plate, and L, D, u, and A have the same meanings as in formulas **166** and **168.**

Number of rivets required by shearing stress at 9,000 per square inch,

$$n_9 = \frac{L + D}{9{,}000\,A}. \qquad \textbf{(177.)}$$

Number required for bearing stress, at 15,000 per square inch,

$$n_{15} = \frac{L + D}{15{,}000\,d\,t}. \qquad \textbf{(178.)}$$

Number required for shearing, at $\frac{3}{4}$ unit stress on member,

$$n_s = \frac{2\,(2\,L + D)}{3\,A\,u}. \qquad \textbf{(179.)}$$

Number required for bearing, at $\frac{3}{2}$ unit stress on member.

$$n_b = \frac{2\,L + D}{3\,d\,t\,u} = \frac{2\,(2\,L + D)}{2 \times 3\,d\,t\,u}. \qquad \textbf{(180.)}$$

The values of 9,000 A are given by Table 38, Art. **1537,** and those of 15,000 $d\,t$ by Table 37, same article.

Instead of computing all the values, we proceed as follows: Since n_9 and n_{15} have the same numerator, that will be greater which has the smaller denominator; but, as 9,000 A and 15,000 $d\,t$ have the common factor 3,000, we only have to compare 3 A and 5 $d\,t$. In the present case, $3\,A = 3 \times .3068 = .9204$, and $5\,d\,t = 5 \times \frac{5}{8} \times \frac{1}{4} = \frac{25}{8} \times \frac{1}{4} = .7 +$. It is not necessary to continue the latter operation, as it is seen at once that the result will be smaller than 3 A. Therefore, n_{15} is greater than n_9, and formula **178** should be used ($15{,}000\,d\,t = 2{,}340$, as given by Table 37). The result is 5.1, as found before.

In a similar manner, to find whether **179** or **180** should be used, we compare $3\,A\,u$ with $6\,d\,t\,u$, or, since $3\,u$ is common, we compare A and $2\,d\,t$. Now, $A = .3068$, **and** $2\,d\,t = 2 \times \frac{5}{8} \times \frac{1}{4} = \frac{5}{16} = .31 +$, which is greater than A. Therefore, n_s is greater than n_f, and formula **179** should be used. This gives

$$n_s = \frac{2\,(19{,}400 + 2{,}300)}{3 \times .3068 \times 8.750} = \frac{2{,}170}{3 \times .1534 \times 875} = 5.4,$$

as found before.

In all computations where the same rules are used, it is convenient to construct a general formula, as this saves time and often suggests very short methods for finding the required results. The preceding formulas are derived by very simple arithmetical reasoning, and it would be a good exercise for the student to try to derive them himself.

1571. The width of the post, perpendicular to lattice bars, is $3\frac{1}{2} + \frac{1}{2} + 3\frac{1}{2} = 7\frac{1}{2}$ inches. Plates $7\frac{1}{2}'' \times \frac{5}{16}''$ are used for the connection. As the distance from back to back of the channels in the upper chord is $7\frac{1}{2}$ inches, the clear width between these connecting plates is $7\frac{1}{2} - 2 \times \frac{5}{16} = 6\frac{7}{8}$ inches. The post is given no clearance between these plates, but its width, parallel to lattice bars, is made the same as the clear distance between the plates, i. e., $6\frac{7}{8}$ inches. As the upper ends of the angles of the post extend but $\frac{1}{8}$ of an inch above the lower edges of the channels of the chord, the post can probably be placed in position without clearance, but it would usually be better to give a clearance of $\frac{1}{16}$ of an inch on each side, making the width of the post, in this case, $6\frac{3}{4}$ inches.

1572. Spacing for Rivet Lines in Angles.—The lattice bars will be $\frac{1}{4}$ of an inch thick, and, in order that two bars can connect upon each rivet between the angles, the latter must be $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ inch apart. Batten plates $\frac{1}{4}''$ thick are used at the ends, requiring also filler plates $\frac{1}{4}''$ thick between the angles, except at the two rivets where the lattice bars connect upon the batten plates.

As was stated in Mechanical Drawing, Art. **56,** the position of the rivet line in an angle bar is always fixed by the distance from the back or corner of the angle. For this distance the values given by the following formula agree with most of the adopted standards:

$$a = \frac{d}{2} + c, \qquad (181.)$$

in which a is the distance from the back or corner of the angle to the center of the rivet hole, d is the nominal length of the leg (see Fig. 318), and c is a constant, having the following values:

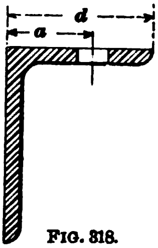


FIG. 318.

For values of d of $1\frac{7}{8}''$ or less, $c = \frac{1}{16}''$.
For values of d of from $2''$ to $2\frac{3}{4}''$, $c = \frac{1}{8}''$.
For values of d of from $3''$ to $5''$, $c = \frac{1}{4}''$.
For values of d above $5''$, $c = \frac{1}{2}''$.

These values of c, when substituted in formula **181,** will give the values of a that agree with what is probably the most common practice. But, as in the case of channels, the practice is not uniform, though it is more nearly so. For instance, it is not uncommon to give c a value of $\frac{1}{8}$ of an inch for values of d of from $1\frac{1}{2}$ to $2\frac{1}{2}$ inches, inclusive.

In the intermediate post, the connected leg of the angle is 2 inches wide $= d$. Hence, by formula **181,** the distance from the back of the angle to the rivet line is $\frac{2}{2} + \frac{1}{8} = 1\frac{1}{8}$ inches. This fixes the distance between rivet lines at $6\frac{7}{8} - 2 \times 1\frac{1}{8} = 4\frac{5}{8}$ inches.

1573. The Bottom Connections.—The rivets connecting the pin plates at the lower end of the post with the free legs of the angles are staggered with those connecting the angles with the batten plates. As the stress upon the post requires the same number of rivets to connect it at the bottom as at the top, it is evident that more rivets are used to connect the pin plates at the lower end of the post than are required by the stresses.

The pin plates should be thick enough to give the required bearing upon the lower chord pin. Applying rule given in Art. **1531**, we see that 3 × 2,300 is less than 9,700. Therefore, the required thickness is given by formula **166**, as follows:

$$T = \frac{2 \times 9{,}700 + 2{,}300}{3 \times 2.75 \times 8{,}750} = .30, \text{ or say } \tfrac{5}{16}''.$$

This would require each pin plate to be only $\frac{5}{32}''$, but such thickness would be too small for strut resistance.

1574. Forked Ends of Compression Members.—Item (q_1) of the specifications (Art. **1511**) requires that "Where the ends of compression members are forked to connect to the pins, the aggregate compressive strength of these forked ends must equal the compressive strength of the body of the members."

The pin plates form the forked ends of the post. In order that the angles forming the post may clear the ends of all other members connecting upon the pin, the pin plates must project beyond the ends of the angles a distance depending upon the dimensions of the other connecting members. The aggregate sectional area of the pin plates should not be less than that of the angles forming the post. These angles weigh 4.4 pounds per foot, giving an aggregate sectional area of $4.4 \times 4 \times \frac{3}{10} = 5.28$ square inches. The post is $7\frac{1}{2}$ inches wide, and pin plates of the same width and $\frac{3}{8}$ of an inch thick will give an aggregate sectional area of $7\frac{1}{2} \times \frac{3}{8} \times 2 = 5.63$ square inches.

1575. The strut resistance of these pin plates will now be investigated. The length of the strut formed by a pin plate may be taken as the distance between its bearing upon the pin and the first rivet connecting it to the angles, if the batten plate supports or stays the angles at or very near their ends. It will be on the side of safety, in such a case, to take the distance between the center of the pin hole and the center of the first rivet as the strut length of the pin plates. In the present case, the distance is 6 inches.

According to formula **140,** Art. **1433,** the least radius of gyration of each pin plate is equal to $.289 \times \frac{3}{8} = .1084$ of an inch. Hence, the live load unit stress, as allowed by formula **125,** Art. **1410,** is $8,750 - 50 \times \frac{6}{.1084} = 5,980$ pounds. The dead load unit stress is $5,980 \times 2 = 11,960$ pounds. The area necessary to resist the stresses upon the post is, therefore, $\frac{9,700}{5,980} + \frac{2,300}{11,960} = 1.81$ square inches. As found above, the sectional area given by the two pin plates is 5.63 square inches. It is, therefore, found that the strut resistance of these pin plates is ample, when computed upon the same basis as the resistance of the post as a whole.

Considered as struts, the two pin plates will bear $5.63 \times 5,980 = 33,670$ pounds of live load stress. The rivets connecting the pin plates to the angles of the post should have nearly the same resistance as the pin plates considered as struts. At 9,000 pounds per square inch, the value of a $\frac{5}{8}''$ rivet in single shear is 2,760 pounds. (Table 38, Art. **1537.**) The metal in the angles forming the post is $\frac{1}{4}$ of an inch thick. At 15,000 pounds per square inch, the bearing value of a $\frac{5}{8}''$ rivet through $\frac{1}{4}$ of an inch thickness of metal is 2,340 pounds. (Table 37.) The latter is the critical condition. Hence, in order that the rivets connecting the pin plates to the angles shall have the same resistance as the pin plates considered as struts, the number of rivets required for this purpose is $\frac{33,670}{2,340} = 14.4$ rivets. Twelve rivets are used for this connection, which is nearly twice the number required by the live and dead load stresses in the post, as found for the connection of the top of the post. The pin plates at the bottom of the intermediate post do not strictly comply with all requirements as indicated in item (q_1) of the specifications, but they may be considered practically to fulfil the really important requirement of that item; namely, that the aggregate compressive strength of the forked end shall equal the compressive strength of the body of the member.

CONSTRUCTIVE DETAILS OF THE PORTAL BRACING.

1576. Conditions Governing the Arrangement of Lattice Bars.—The portal is shown in Mechanical Drawing Plate, Title: Highway Bridge: Details II, Fig. 1. The design of the details of this member consists principally in determining the proper arrangement and dimensions of its connections, and also the arrangement of the lattice bars, and their connections with the flange angles. These are for the most part questions of judgment rather than of calculation. Certain conditions, however, which are susceptible of calculation, must be fulfilled. The conditions governing the arrangement of the lattice bars will first be noticed.

The lattice bars must be connected to and support the flange angles at such frequent intervals that the strut resistance of the flange in the plane of the lattice bars will be at least as great as its resistance in a plane perpendicular to the lattice bars, as determined in proportioning the material for the flanges. In order that this may be the case, the distance l_0 between the rivets connecting the lattice bars to the flanges, in inches, must never be greater than given by the formula

$$l_0 = \frac{l r_0}{r}, \qquad (182.)$$

in which l is the length of the portal flange (usually taken center to center of chords), r is the radius of gyration of same about an axis parallel to plane of the lattice bars, and r_0 is the radius of gyration of the same about an axis perpendicular to plane of the lattice bars, all in inches.

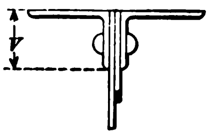


FIG. 319.

The value of r is given in formula **113**, Art. **1408**.

For two angles with the shorter legs connected back to back, but separated by lattice bars (see Fig. 319), the value of the radius of gyration about an axis parallel to longer flanges of angles, i. e., the value of r_0, is given by the formula

$$r_0 = \frac{28}{100} V. \qquad (183.)$$

For any ordinary section, the value of r_0, as given by formula **183,** will never be less than one-fourth the value of r, as given by formula **113,** and it will seldom have a value so relatively small. Hence, the greatest allowed length of l_0, as limited by this condition, may be found by substituting $\frac{1}{4}r$ for r_0 in formula **182,** as follows:

$$l_0 = \frac{l \times \frac{1}{4}r}{r} = \frac{l}{4}. \qquad \textbf{(184.)}$$

In the present case, the value of r is $\frac{24}{100} \times 7.5 = 1.8$ inches, and the value of r_0 is $\frac{28}{100} \times 2.5 = .7$ of an inch. The value of l is 19 feet = 228 inches. Hence, the value of l_0 must not exceed $\frac{.7}{1.8} \times 228 = 88\frac{2}{3}$ inches. As determined by formula **184,** the value of l_0 must not exceed $\frac{228}{4} = 57$ inches, which is less than the value just obtained, and very materially on the side of safety.

In compliance with other conditions, however, the pitch of the rivets connecting the lattice bars to the flange angles can never be made as great as either of the preceding values of l_0; it is commonly made about 12 inches. Hence, with flanges composed of angles connected by lattice bars, this condition is never liable to be violated, and may be neglected. But there also exists a much more severe condition.

1577. The pitch of the rivets along each lattice bar, that is, those connecting the bar with the lattice bars which cross it at right angles, or nearly so, and to the flange angles, must not exceed a certain multiple of the thickness of the bar. This is a much more critical condition than the preceding. In order that a lattice bar shall have a reasonable amount of resistance as a

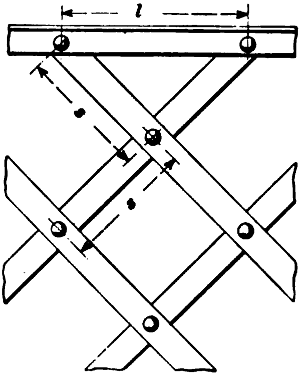


FIG. 320.

strut, its length between connections should not exceed about 125 times its least radius of gyration. Formula **140,** Art. **1433,** gives the value of the least radius of gyration of a rectangular bar = .289 t, t being the thickness; 125 times this = 36 t. Hence, the pitch S of the rivets connecting the bars (see Fig. 320) should not exceed the value given by the formula

$$S = 36\,t. \qquad \textbf{(185.)}$$

In the example, the lattice bars are $\frac{1}{4}$ of an inch in thickness; consequently, the value of S should not exceed 36 × $\frac{1}{4}$ = 9 inches. If the lattice bars are placed at an angle of 45 degrees with the axis of the member, i. e., at right angles to each other, the maximum value of l_0 is given by the simple formula

$$l_0 = \sqrt{(36t)^2 + (36t)^2} = 36t\sqrt{2} = 51t. \qquad \textbf{(186.)}$$

In the example, $t = \frac{1}{4}$, and the value that l_0 must not exceed = 51 × $\frac{1}{4}$ = 12.75 inches. This is, then, the governing condition.

1578. In general, if a and b are the sides, and h the hypotenuse of a right-angled triangle, it is well known that

$$h = \sqrt{a^2 + b^2}.$$

If $a = b$, which is always the case when one of the acute angles = 45°, we have,

$$h = \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2} \times a = 1.4142a. \qquad \textbf{(187.)}$$

Conversely, if h is known, the value of a is

$$a = \frac{h}{1.4142} = .7071h. \qquad \textbf{(188.)}$$

1579. The Practical Arrangement of the Lattice Bars.—In the example the clear width of the roadway is 18 feet = 216 inches. At each side (end) of the portal the lattice bars are attached to a somewhat larger vertical bar. By trial it is found that by allowing on each side $1\frac{1}{16}$ inches between the outer line of rivets attaching the lattice bars and the inner edge of the upper chord, the space remaining between the two outer lines of rivets attaching the lattice bars ($= 216 - 2 \times 1\frac{11}{16} = 212\frac{5}{8}''$) may be divided into 18 spaces of $\frac{212.625}{18} = 11.8125$, or $11\frac{13}{16}$ inches each.

The vertical pitch of the lattice bars is made the same as the horizontal pitch, or $11\frac{13}{16}$ inches. The legs of the flange angles to which the lattice bars attach are $2\frac{1}{2}''$ long, and by formula **181,** Art. **1572,** the distance from the back of the angles to the rivet line is $\frac{2\frac{1}{2}}{2} + \frac{1}{8} = 1\frac{3}{8}$ inches. Hence, the depth of the portal, from out to out of flange angles, is $2 \times 11\frac{13}{16} + 2 \times 1\frac{3}{8} = 26\frac{3}{8}$ inches. The space, or pitch, S, along the lattice bars, as given by formula **188,** is $.7071 \times 11.8125 = 8.3526$, or $8\frac{23}{64}$ inches. A single space is usually expressed to the nearest sixteenth of an inch; but, in case the same space is several times repeated, the error thus accumulating may become considerable; hence, in the present case, the pitch of the rivets in the lattice bars is expressed to the nearest sixty-fourth of an inch. The latticing of the brackets is simply a continuation of the latticing of the main or horizontal portion of the portal.

1580. The Portal Connections.—The upper flange of the portal is to be riveted to the bent plate shown on top of the chord in Mechanical Drawing Plate, Title: Highway Bridge: Details I, Fig. 2. It is also to be connected to the chord and end post by means of the hip cover-plate, which is shown attached to the flange. The details of these connections are somewhat complicated, but the dimensions of the connecting pieces may be determined by accurately drawing them in position. The number of rivets required to connect the upper flange may readily be determined from the stress. The plates used for the connection are $\frac{1}{2}''$ thick. By reference to Table 37, Art. **1537,** it is found that, at 15,000 pounds per square inch, the bearing value of a $\frac{5}{8}''$

rivet through a $\frac{1}{4}''$ plate is 2,340 pounds, while Table 38 of the same article gives the value of a $\frac{5}{8}''$ rivet in single shear, at 9,000 pounds per square inch, to be 2,760 pounds. Hence, the bearing is the critical condition. The external forces acting upon the portal are shown in Fig. 278, Art. **1311.** By taking moments about the point where the flange of the bracket connects to the end post, the stress at the point where the upper flange connects to the end post is found to be $\frac{2,700 \times 21 + 4,050 \times 5}{5} = 15,390$ pounds. Hence, the number of rivets required for the connection of the upper flange of the portal is $\frac{15,390}{2,340} = 6.6$ rivets. As five of these rivets are to be field driven, eight rivets are used; three shop rivets connect the flange to the hip cover-plate, and five field rivets connect it to the bent plate on top of the chord, counting as $3 + \frac{2}{3} \times 5 = 6.3$ rivets.

The lower horizontal flange of the portal and the lattice bars of the bracket attach by means of a 5″ × ¼″ plate to an angle bar shown riveted on the web of the end-post channel, while the flange angles of the bracket are to be riveted directly to the web of the channel. It will be noticed that the 5″ × ¼″ connecting plate has ⅛″ clearance from the web of the channel. It would be impossible to determine accurately the number of rivets required for this connection, but by comparison with the number of rivets required for the upper flange, it is seen that a sufficient number is used. It is necessary to cut away portions of the connecting legs of the angles of the bracket flange, in order to connect between the flanges of the channels of the end post. All dimensions of the connecting parts, as well as the holes and rivet spacing, must be accurately and plainly shown and marked. The dimensions of the connecting parts may be determined and the rivet holes located by carefully drawing them in place to a sufficiently large scale. This need be only a pencil sketch, but it must be accurate. It is always well to check the results by calculations.

CONSTRUCTIVE DETAILS OF THE LATERAL STRUTS AND KNEE BRACES.

1581. The Lateral Strut.—The details of this member are shown in Mechanical Drawing Plate, Title: Highway Bridge: Details II, Fig. 2; they are exceedingly simple. The strut is to be connected to each chord by means of rivets, for which vacant holes are shown through the horizontal legs of the angles. The number of rivets for this purpose may be calculated from the initial stress assumed upon the strut. The thickness of metal in the angles of the strut is $\frac{3}{8}$ of an inch, while in the chord the rivets pass through both the cover-plate and the cover splice plate, or $\frac{1}{2}$ inch thickness of metal. At 15,000 pounds per square inch, the bearing value of a $\frac{5}{8}''$ rivet through $\frac{3}{8}$ of an inch thickness of metal is found to be 3,520 pounds (Table 37, Art. **1537**), while at 9,000 pounds per square inch the value of a $\frac{5}{8}''$ rivet in single-shear is 2,760 pounds. (Table 38.) The latter is the critical condition. In Art. **1425,** the resultant stress assumed upon the lateral strut was found to be 11,400 pounds. Hence, the number of rivets required to connect the end of the lateral strut is $\frac{11,400}{2,760} = 4.1$ rivets. As these rivets are field driven, there will be required $1\frac{1}{2} \times 4.1 = 6.2$; 6 are used.

At each end of the strut the two middle holes connect at the center of the cover-plate of the chord, while two holes connect at each line of rivets connecting the cover-plate to the channels; hence, these holes are spaced $\frac{9\frac{3}{4}}{2} = 4\frac{7}{8}$ inches apart. The distance from the two middle holes at one end of the strut to the corresponding holes at the opposite end is 19 feet, or the same as the distance from center to center of chords. The longer or 5″ legs of the angles are riveted together, back to back. By formula **181,** Art. **1572,** the rivet line is fixed at $\frac{5}{2} + \frac{1}{4} = 2\frac{3}{4}$ inches from the corners of the angles. The pitch of the rivets connecting the two angles together will not, within any reasonable limits, be so great as to weaken the member as a strut; hence, within such limits, these rivets may be given any convenient pitch. A good rule for such cases is as follows:

Rule.—*In angles connected closely together, back to back, make the pitch of the rivets, based upon the thickness of the angles, double that allowed by item* (d_1) *of the specifications* (Art. **1511**).

Thus, in the example, the pitch of the rivets connecting the vertical legs of the angles composing the lateral strut is $16 \times \frac{3}{8} \times 2 = 12$ inches.

1582. The Knee Brace.—The form of the section of the knee brace, Fig. 3 of plate referred to in the preceding article, is similar to that of the lateral strut. The brace is usually inclined at an angle of 45 degrees. The horizontal and vertical projections of the inclined portion of the brace are usually made about 3 feet. In the present case, each is made $3'\ 0\frac{3}{8}''$, or 36.375 inches; hence, by formula **187,** the length from bend to bend is $1.4142 \times 36.375 = 51.4415$, or, very closely, $4'\ 3\frac{7}{16}''$.

The lower end of the brace attaches to the angles of the intermediate post. As the post is $6\frac{7}{8}$ inches wide, from out to out of angles, and the inner rivet holes through the end of the brace to connect it with the lateral strut are $1\frac{9}{16}''$ from the bend, it follows that *on the lateral strut* the distance from the middle pair of rivet holes at the end of the strut (at center of chord) to the nearest pair of holes connecting the knee brace is $\frac{6\frac{7}{8}}{2} + 36\frac{3}{8} + 1\frac{9}{16} = 41\frac{3}{8}$ inches. Deducting one space of $4\frac{7}{8}$ inches at the end of the strut, we get $41\frac{3}{8} - 4\frac{7}{8} = 36\frac{1}{2}''$, or $3'\ 0\frac{1}{2}''$, as marked upon the strut. The positions of the holes in the angles of the intermediate post for the connection of the knee brace are determined in a similar manner.

The $3''$ legs of the angles are riveted together, back to back. The distance from the backs of the angles to the rivet line, as fixed by formula **181,** is $\frac{3}{2} + \frac{1}{4} = 1\frac{3}{4}$ inches. According to the rule given above, the pitch of the rivets is made double that allowed by item (d_1) of the specifications. The vacant holes in the knee brace are placed rather nearer to the outer edges of the angles than is usual, better to accommodate the holes in the angles of the post and strut.

1583. The Shoe Strut.—This member, which is shown in Mechanical Drawing Plate, Title: Highway Bridge: Details III, Fig. 4, is a lateral strut, and will next be noticed. It can not be completely detailed until the dimensions of the shoe are determined, but as the details for this member are so simple as to require very little special explanation, they will here be briefly noticed.

The angles are riveted together with the shorter legs back to back, and the plate between them. The spacing of the rivets is determined according to the rule given above for the lateral strut, which it resembles in form. The distance from the corners of the angles to the rivet line, as fixed by formula **181,** is $\frac{2\frac{1}{2}}{2} + \frac{1}{8} = 1\frac{3}{8}$ inches.

From Art. **1434,** it is known that the shoe strut is proportioned to resist a resultant stress of 7,260 pounds. At 9,000 pounds per square inch the value of a ⅝″ rivet in single shear is 2,760 pounds (Table 38, Art. **1537**), while at 15,000 pounds per square inch the bearing value of a ⅝″ rivet through a ¼″ plate is 2,340 pounds (Table 37). Hence, the latter is found to be the critical condition, and the number of rivets required for connecting the end of the shoe strut, if machine driven, is $\frac{7,260}{2,340} = 3.1$, or, if field driven, $1\frac{1}{2} \times 3.1 = 4.7$ rivets; 4 are used.

When the dimensions of the shoe have been determined, the position of the lateral pin in the shoe and the length of the shoe strut may be found by drawing a plan of the shoe and its lateral connections, similar to that shown in Mechanical Drawing Plate, Title: Highway Bridge: Details III, Fig. 17. The lateral strut must be short enough to clear the loop of the lateral rod.

CONSTRUCTIVE DETAILS OF THE FLOOR-BEAM.

1584. Positions of Rivet Lines.—In detailing the floor-beam, Mechanical Drawing Plate, Title: Highway Bridge: Details II, Fig. 4, it will be well first to fix the positions of the rivet lines for the rivets connecting the flange angles to the web-plate, and then determine the required spacing of the rivets along the same. The depth of the web-plate is 24 inches, but (see Art. **1319**) the depth of a floor-beam, out to out of flange angles, may be made from $\frac{1}{4}$ to $\frac{1}{2}$ of an inch deeper than a web plate. In the present case the depth of the beam, out to out of flange angles, is made $\frac{1}{2}''$ deeper than the web-plate, or $24\frac{1}{2}$ inches. The flange angles are $4'' \times 3''$ angles with the $3''$ legs connected to the web. As fixed by formula **181**, the rivet lines are $\frac{3}{2} + \frac{1}{4} = 1\frac{3}{4}$ inches from the backs of the free legs of the angles, making the distance betwen the rivet lines of the upper and lower flanges $24\frac{1}{2} - 2 \times 1\frac{3}{4} = 21$ inches.

1585. Bending Moments and Increments of Flange Stress.—The bending stress in the floor-beam is zero at the supports and increases to maximum at the center of the beam, while the shear is maximum at the supports and diminishes to zero at the center. This is true of any simple beam supporting a uniform load, and it is true of a simple beam supporting any load or system of loads, with the exception that the maximum bending stress and zero shear may occur at some point other than the center of the beam. The stress, which at the point of support is wholly a shearing stress, gradually changes towards the center of the beam, till at the point of maximum bending moment it becomes wholly a bending stress. The change, however, is not uniform, but more rapid near the supports. Under the assumption that the bending stress is borne entirely by the flanges, and the shearing stress is borne wholly by the web-plate, it is evident that between any two points the rivets connecting the flange angles to the web-plate must be of sufficient strength to transfer to the web-plate the difference between the bending stresses at those two points. When the load rests upon the upper flange, the rivets connecting it to the web-plate must also be of sufficient strength to transmit the pressure of the load to the web as shear, although this is done largely by the stiffeners, when the latter are used.

The bending moment M_0 at any point along the loaded portion of the beam is given by formula **98**, Art. **1318.** By applying this formula at the center of the beam and at certain intervals (say at each foot) along its length towards the end, and dividing each result by the effective depth of the beam, the several quotients will be the flange stresses at the respective points along the half length of the beam. The stress increment, or difference between the flange stresses at any two consecutive points, must be taken by the rivets connecting the angles to the web-plate in that portion of the beam included between the two points.

1586. The Pitch of Rivets in the Flange.—If the stress increment be divided by the critical resisting value of one rivet, the quotient will be the number of rivets required between the two points. Although this method is quite commonly employed, the same result may be more easily obtained by the following method:

Assuming the shear to be resisted entirely by the web-plate, and the bending stress to be resisted wholly by the flange angles, the pitch p of the rivets connecting the flange angles to the web-plate is given by the following formula:

$$p = \frac{r\,h}{S}, \qquad \textbf{(189.)}$$

in which r is the least, or critical, value of one rivet, h is the distance between the rivet lines in the two flanges, and S is the vertical shear in the beam at the point under consideration.

The rivets connecting the flange angles to the web-plate are in double shear. From Table 38, Art. **1537,** it is found that at 9,000 pounds per square inch, the shearing value of a $\frac{5}{8}$-inch rivet in double shear is 5,520 pounds. At 15,000 pounds per square inch, the bearing value of a $\frac{5}{8}$-inch rivet through $\frac{3}{8}$ of an inch thickness of plate, as given by Table 37, is 3,520 pounds. The bearing is thus found to be the critical condition, and 3,520 pounds is the value to be given to r in the preceding formula. As stated above, the distance h between the rivet lines in the upper and lower flanges is 21 inches.

S varies in intensity at different points along the beam, being, as previously stated, maximum at the supports and zero at the center.

As stated in Art. **1227,**

The vertical shear equals the reaction of the left-hand support minus all the loads (or all that portion of the uniform load) on the beam to the left of the point considered.

Formula **189** will be applied to the floor-beam at the support and at consecutive points one foot apart along the length of the beam. From Art. **1316,** the total amount of load upon the beam is 41,200 pounds.

But the designer of details must obtain his information from the stress sheet. According to the latter, the maximum flange stress is 53,700 pounds. The depth of the web-plate is 24 inches, and, consequently, the effective depth of the beam is $24.5 - 1.5 = 23$ inches. Hence, by applying formula **97,** Art. **1318,** and dividing by d, the effective depth of the beam, we have $53{,}700 = \frac{M}{d} = \frac{W(18+2)\,12}{8 \times 23}$, from which $W = 41{,}170$, or, practically, 41,200 pounds.

As this is a uniform load, the reaction of each support is $\frac{41{,}200}{2} = 20{,}600$ pounds, and the load upon each lineal foot of the beam is $\frac{41{,}200}{18} = 2{,}289$, or, practically, 2,290 pounds. No load is carried upon the half foot of beam adjacent to each support.

The computations for determining the required pitch of the rivets at consecutive points one foot apart along the beam are tabulated below. The results in column (*e*) will be referred to further along:

(*a*) Distance of Point from Support.	(*b*) Amount of Load at Left of Point.	(*c*) Shear at Point, $S = 20,600 - (b)$	(*d*) Required Pitch for Rivets, $p = \frac{r\,h}{S}$	(*e*) Final Pitch, $a'\,b_1$, $a'\,c_1$, etc.
Feet.	Pounds.	Pounds.	Inches.	Inches.
0	0,000	20,600	3.59	3.59
1	1,140	19,460	3.80	3.73
2	3,430	17,170	4.31	4.20
3	5,720	14,880	4.97	4.81
4	8,010	12,590	5.87	5.61
5	10,300	10,300	7.18	6.72
6	12,590	8,010		
7	14,880	5,720		
8	17,170	3,430		
9	19,460	1,140		

As the pitch of the rivets must not exceed 6 inches, the computations are completed only to a point 5 feet from the support, where the computed pitch becomes 7.18 inches. It will be sufficiently accurate to apply each computed pitch to that portion of the beam extending 6 inches in each direction from the point considered.

1587. Effect of Vertical Load; Final Pitch.—As the beam is loaded along its upper flange, the rivets in this flange must, as previously stated, bear also the pressure due to the load upon the beam, exclusive of the weight of the beam itself. From Art. **1315,** it may be found that the estimated load supported by the floor-beam, exclusive of its own weight, is $32,400 + 7,540 = 39,940$ pounds, or $\frac{39,940}{18 \times 12} =$ 185 pounds per lineal *inch* of the loaded portion. But it is usually necessary for the designer of details to get all his information from the stress sheet. As previously determined, the total estimated load upon the loaded portion of the beam is 2,290 pounds per lineal foot. From this is to be deducted the weight per foot of the beam itself. The weight per foot of flanges is $4 \times 8.4 = 33.6$ pounds, and the weight per foot of the web is $24 \times \frac{3}{8} \times \frac{10}{3} = 30$ pounds, making $33.6 + 30 = 63.6$ for flanges and web. If 10 per cent. be added for weight of details, it will make a total of $63.6 \times 1.10 = 70$ pounds (nearly) per foot. Hence, the load per foot upon the upper flange of the beam is $2,290 - 70 = 2,220$ pounds, making the load upon each lineal inch of the flange equal to $\frac{2,220}{12} = 185$ pounds, as found before.

This load produces vertical pressure or stress upon the rivets connecting the upper flange angles to the web-plate at all points along the loaded portion of the beam, while the increments of the flange stress act upon the same rivets as horizontal stresses. The stresses borne by the rivets are the resultants of these vertical and horizontal stresses. The pitch, as given in column (*d*) of the preceding table, provides for the horizontal stresses only, and must be reduced so as to provide for the vertical stresses also. The pitch thus reduced may be called the **final pitch,** and can be determined graphically more easily than by calculation.

1588. If *i* is the vertical load upon each lineal *inch* of the loaded flange, and *p* and *r* represent the same values as

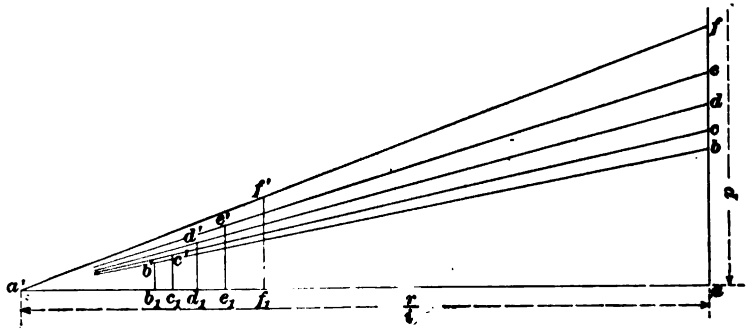


FIG. 321.

in formula **189**, the graphical construction giving the final pitch is as follows:

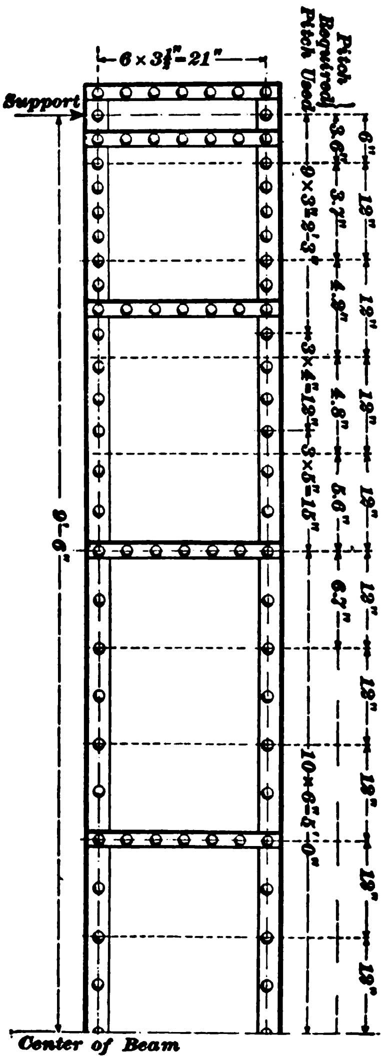


FIG. 322.

Construct a right-angled triangle, as a' a b, Fig. 321, whose base a' a is by any convenient scale made equal to the quantity $\frac{r}{i}$, considered as inches, and whose altitude a b is by the same scale made equal to the pitch p in inches, as obtained by formula **189**. *From a' lay off a' b' on the hypotenuse equal to $a\ b = p$, and by a vertical line, as $b'\ b_1$, project a' b' upon the base of the triangle. The projected line $a'\ b_1$ thus obtained will be the pitch necessary to resist the resultant of the horizontal and vertical stresses.*

All the necessary constructions for the various points along a beam are readily combined in one figure. In Fig. 321 are combined the constructions for determining the final pitch in the six items (except the first) of the calculations tabulated above. The triangles have a common base, which is equal to $\frac{r}{i} = \frac{3,520}{185} = 19$ inches. The altitudes of the several triangles are equal to the pitch required by the respective stress increments, as given in column (*d*) of the table in Art. **1586** (except first item). The final pitches, i. e., the results $a'\ b_1$, $a'\ c_1$, etc., are given in column (*e*) of the same table. As there is no vertical load upon the upper flange at the support, the pitch at that point is given the same in column (*e*) as in column (*d*). For that point, the pitch as given in column (*d*) is the final pitch.

The rivet spacing along the flange, giving the pitch of rivets as required and used, is clearly shown in Fig. 322. Near the end of the beam the pitch is made somewhat less than the required pitch, on account of the connections for the lateral rods. The rivet spacing in the lower flange should usually be made the same as in the upper flange.

1589. Stiffeners.—In each stiffener the pitch of the rivets should not exceed that required in the adjacent portion of the flange. For practical considerations, however, it is desirable that when possible the pitch of rivets should be uniform in all stiffeners on the same beam. Single stiffeners are sometimes used on the floor-beams of highway bridges, being placed alternately on either side of the beam. The practice, however, is not to be commended.

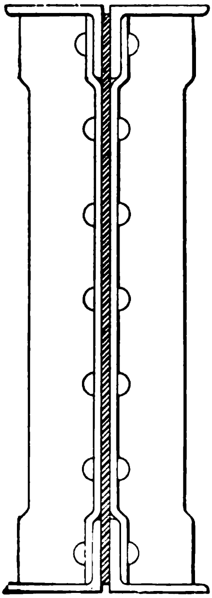


FIG. 323.

If the beam is supported upon its bottom flange, as in the present case, it should have two pairs of stiffeners at the end, in order to transmit properly the shear upon the bearing. In case of heavy beams and plate girders, that portion of the web above the bearings should be also reinforced by additional plates. Were this necessary in the example it could readily be accomplished by using a $7\frac{1}{2}'' \times \frac{5}{8}'' \times 18\frac{1}{2}''$ plate on each side of the web-plate, extending as a filler plate under the two end stiffeners. As stated in Art. **1448**, stiffeners are not necessary in this case, but are used simply as an illustration. They should always be used at the ends of floor-beams.

Stiffeners are sometimes used without fillers, in which case the stiffener angles must be offset or crimped to fit over the legs of the flange angles, as in the end view of the beam shown in Fig. 323. It is not economy to do this, however, except in heavy beams. In light beams the cost of offsetting the angles will usually be greater than the cost of the material for fillers.

1590. Positions of Beam Hangers.—The positions of the holes for the beam hangers through the horizontal legs of the flange angles are fixed longitudinally of the beam by the length of the beam from center to center of chords. Their distance apart across the flange, as well as the diameters of the holes, are determined by the dimensions of the hanger [Mechanical Drawing Plate, Title: Highway Bridge: Details IV, Fig. 7]. The end of the beam must project a sufficient distance beyond the hanger holes to afford a bearing for the end of the intermediate post, this being necessary to hold the beam securely in position.

1591. The Connections for Lateral Rods.—The details of the plate and angle lugs for lateral connections are largely matters of arrangement, but sufficient bearing for the lateral pins must be provided. As the four floor-beams are made alike, the bearings for the lateral pins must be sufficient to resist the greatest stress upon any lateral rod attaching to the beam. The rod in the end panel of the lower lateral system is 1⅛ inches in diameter, and bears a stress of 14,900 pounds. In order to admit this size of rod the clear distance between the angle lug and horizontal leg of the angle of the top flange should not be less than 1¼ inches. As the rivets attaching the plate for the lateral connection to the flange angles must be countersunk on the under side, requiring ample clearance, the top of the angle lug is placed at the center of the flange rivets, giving a clearance between the bearings for the lateral pin of $1\frac{3}{4} - \frac{3}{8} = 1\frac{3}{8}$ inches. The holes for the lateral pins are so near to the edges of the flange angles that the resistance of the latter should not be counted. Hence, neglecting the bearing of the flange angle, the distance between the centers of the bearings for the lateral pins is $1\frac{3}{4} + \frac{3}{8} = 2\frac{1}{8}$ inches.

By applying formula **106,** Art. **1371,** the bending moment upon the pin is found to be $\frac{14,900 \times 2\frac{1}{8}}{4} = 7,920$ inch-pounds. From Table 40, Art. **1546,** it is found that, with a fiber stress of 18,000 pounds per square inch, a pin $1\frac{3}{4}$ inches in diameter will give the required resisting moment. From the same table it is found that, at 15,000 pounds per square inch, the bearing value of a $1\frac{3}{4}''$ pin through 1 inch thickness of metal is 26,300 pounds. Hence, the total thickness of bearing required for the lateral pin is $\frac{14,900}{26,300} = .57$ of an inch. Two bearings, each $\frac{5}{16}$ of an inch thick, giving a total bearing of $2 \times \frac{5}{16} = \frac{5}{8}$ (.63) of an inch, will give the required bearing surface. As, however, the vertical leg of the angle lug is subjected to a certain amount of bending stress, the metal in the angle is given a thickness of $\frac{3}{8}$ of an inch; the plate on top of the flange is given the same thickness.

At 9,000 pounds per square inch, the shearing value of a $\frac{5}{8}''$ rivet is 2,760 pounds (Table 38, Art. **1537**), which is less than its bearing value through a $\frac{3}{8}$ inch plate at 15,000 pounds per square inch (= 3,520 lb., Table 37). Consequently, as the pin plate is subjected to one-half the stress upon the lateral rod, the number of rivets required to connect the plate to the flange is $\frac{1}{2} \times \frac{14,900}{2,760} = 2.7$, or, practically, 3 rivets. Four rivets are used.

1592. As it is not permissible to use rivets in tension, the angle lugs are attached by means of bolts. The sectional area of the $1\frac{1}{8}''$ lateral rod is .99 of a square inch, and it will be practically correct to make the aggregate sectional area of the two bolts somewhat more than one-half of this amount. After deducting the section cut away by the threads, the sectional area given by two $\frac{3}{4}''$ bolts is about .6 of a square inch.

The lateral connections on the floor-beam should be placed as near to the beam hangers as other necessary conditions will permit. After the dimensions and positions of the lower chord connections are determined, the position of the lateral connections on the floor-beam may be fixed by drawing plans of the lower chord joints similar to those shown in Figs. 15 and 16 of the plate just referred to.

TENSION MEMBERS.

EYE-BAR HEADS.

1593. General Requirements and Manufacture.—In pin-connected trusses the main tension members are usually composed of eye-bars. The relative dimensions of eye-bar heads will now be considered. Item (j_1) of the specifications requires that "The heads of eye-bars shall be so proportioned and made that the bars will preferably break in the body of the original bar, rather than at any part of the head or neck."

The heads of wrought-iron eye-bars are commonly formed by piling the proper amount of scrap iron, carefully selected and suitably arranged, upon the end of the bar, and then, after heating to the required temperature, forging the head into the desired shape in a die under a steam hammer or hydraulic pressure. Other similar processes are occasionally employed. If the welding is perfect, the head becomes thoroughly identified with the bar; but this is seldom the case. Nevertheless, wrought-iron eye-bars have been very commonly produced by the process indicated above which, when tested to destruction, would break in the body of the bar, rather than in the head or neck.

Eye-bars are now quite generally made of medium steel, a material which has been found excellent for this purpose. Steel eye-bars are usually formed by first upsetting the bar sufficiently to form the head, then forging the latter into the required form in a die under steam or hydraulic pressure. The bars thus obtained are very satisfactory. Steel eye-bars have, however, been successfully made by piling carefully selected scrap and forging, in much the same manner as for wrought-iron eye-bars; but great care is required in the arrangement of the scrap and in the forging, and a suitable quality of steel is required for both the bar and the scrap. Whatever the process of manufacture, the bars must be subsequently reheated and annealed before boring the pin holes; this should never be omitted.

1594. Dimensions of Wrought-Iron Eye-Bar Heads.—The relative dimensions of eye-bar heads to fulfil the conditions of item (j_1), quoted above, are determined by experiment. A form of head which has been very extensively used for wrought-iron eye-bars is shown in Fig. 324.

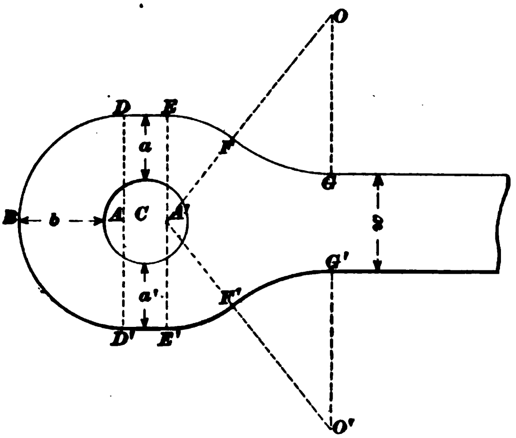


FIG. 324.

If the thickness of the head is the same as that of the original bar, the dimensions of the head may be expressed in terms of w, the width of the original bar. The dimensions a and a' (see Fig. 324) are each given by the formula

$$a = a' = \tfrac{2}{3} w. \qquad \textbf{(190.)}$$

$CA = CA'$ = radius of pin hole $= r$. $E D$ and $E' D'$ are straight lines parallel to the center line of the bar. $D B D'$ is a semicircle of radius $A D = r + a$, with the position of the center A so taken on the center line of the bar that the dimension b will have the value given by the formula

$$b = \tfrac{7}{8} w. \qquad \textbf{(191.)}$$

This makes $C A = r - (A D - b) = r - (r + \frac{2}{3} w - \frac{7}{8} w) = \frac{5}{24} w$. $E F$ and $E' F'$ are circular arcs having their center at A' and radii $A' E$ and $A' E'$ equal to $A D$. $A' C = A C$. The arcs $F G$ and $F' G'$ should have equal radii $O F$ and $O' F'$, which should be of sufficient length to gradually connect the head with the main body of the bar; the radius $O F$ of the neck is often made approximately equal to one and one-half times the radius $A' F$.

If the heads are made thicker than the body of the bar, the letters a, a', b, and w in the two preceding formulas must be considered to represent *sectional areas* instead of dimensions, the sectional areas to be taken on the respective lines as shown in Fig. 324.

1595. At the present time, however, eye-bar heads are generally made circular, as shown in Fig. 325. The center of the pin hole is at the center of the head. The dimensions for this form of head being simpler, the dies are less expensive than for the form of head shown in Fig. 324. According to the most approved practice, the *net sectional area* A through the pin hole (i. e., area A = areas $a + a' = t (D - d)$, t being the thickness) should not be materially less than given by the following formulas, in which B represents the *sectional area* of the body of the bar:

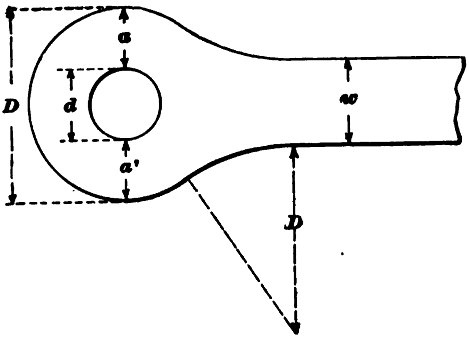


FIG. 325.

When the diameter d of the pin hole does not exceed $\frac{2}{3}$ *of the width w of the bar:*

$$A = 1.6\,B. \qquad \textbf{(192.)}$$

When the diameter d of the pin hole materially exceeds the width w of the bar:

$$A = 1.8\,B. \qquad \textbf{(193.)}$$

If the thickness of the head is the same as the thickness of the bar, $a + a' = D - d$ may be substituted for A, and w for B. For wrought-iron eye-bars with circular heads, as used in highway bridges, the net sectional area A across the head is often made equal to one and one-half times the sectional area of the bar. This practice has, in most cases, been found to give satisfactory results.

1596. Dimensions of Steel Eye-Bar Heads.—Steel eye-bar heads, when properly made and annealed, possess somewhat greater relative efficiency than those of wrought iron, and, consequently, do not require so large a percentage of additional material. The heads are generally made circular in form, as in Fig. 325, and the thickness of the head is always made the same as that of the original bar. (See Art. **1512,** VII.) If, for steel eye-bars, the *net dimension* across the head be represented by $A, = a + a' = D - d$, then this net dimension, as compared with the width w of the bar, is given by the following formulas:

When the diameter d of the pin hole does not exceed ¾ of the width w of the bar:

$$A = 1.33\,w. \qquad \textbf{(194.)}$$

When the diameter d of the pin hole exceeds the width w of the bar:

$$A = 1.4\,w. \qquad \textbf{(195.)}$$

1597. The diameters D of eye-bar heads are usually expressed nominally in multiples of a half inch; but, as actually manufactured, heads whose diameters are multiples of an inch are quite commonly made $\frac{1}{8}$ of an inch less than their nominal diameters, in order to admit of being packed inside of channels of the same nominal size.

1598. On account of the practical conditions of manufacture and of the liability of thin eye-bar heads to buckle behind the pin hole when under strain, the thickness of the head should not be less than given by the following formula:

$$t = \frac{D + w + 8}{32}. \qquad \textbf{(196.)}$$

For highway bridges the minimum limit of thickness allowed for eye-bar heads should never be less than ⅝ of an inch. For railroad bridges the minimum limit is ¾ of an inch, and this limit might properly be applied to all bridges.

CONSTRUCTIVE DETAILS OF THE LOWER CHORD MEMBERS.

1599. The End Panel *a b*; Bearing on Shoe Pin.—The details of the member in the end panel of the lower chord are shown in Mechanical Drawing Plate, Title: Highway Bridge: Details III, Fig. 1. The end *a* of this member connects upon the shoe pin by means of pin plates riveted to the angles composing the member. These pin plates must be thick enough to afford sufficient bearing upon the pin, and, as they practically fulfil the office of eye-bar heads, they must also provide sufficient metal, at the section through the pin hole and beyond the pin, to comply with the conditions of formulas **190** and **191**.

According to item (b_1) of the specifications, as this member is not composed of forged eye-bars, its bearing stress upon the pin must not exceed 15,000 pounds per square inch, or one and one-half times the unit stress allowed upon it. As one and one-half times the allowed unit stresses would be $1\frac{1}{2} \times 10{,}000 = 15{,}000$ pounds, and $1\frac{1}{2} \times 20{,}000 = 30{,}000$ pounds per square inch for live and dead load stresses, respectively, it is readily seen that an allowed bearing stress of 15,000 pounds per square inch for all stresses is the more severe requirement. The stress sheet shows the live and dead load stresses for this member to be 32,400 pounds and 13,900 pounds, respectively. At 15,000 pounds per square inch, Table 40, Art. **1546,** gives 41,300 pounds as the bearing value of a 2¾-inch pin through 1 inch thickness of plate. Hence, the total thickness of bearing for the pin plates required by the stresses upon the member is $\frac{46{,}300}{41{,}300} = 1.121$, or about 1⅛ inches. Two pin plates, each ⅝ of an inch thick, would give ample bearing surface upon the pin. But at the shoe joint, other considerations will require a greater thickness for the pin plates upon the chord.

1600. Section of Pin Plate Across the Pin Hole.—From the stress sheet it is found that the net sectional area given by the four angles composing this member is 4.02 square inches. By applying formula **190,** at the section on the lines *a* and *a'* (Fig. 324) through the pin hole, and using *areas* instead of *dimensions* for the values of *w* and *a*, the net sectional area required at the pin hole is found to be $2 \times \frac{2}{3} \times 4.02 = 5.36$ square inches. Hence, as the pin plate is made 6½ inches wide, the total thickness of metal required for the pin plates by this condition is $\frac{5.36}{6.5 - 2.75} =$ 1.43 inches, which is practically ¾ of an inch for each plate. But this will probably be found not to be the critical condition.

1601. Critical Section of Pin Plate.—As the space between the shoe pin and the cover-plate of the end post is limited, it will be necessary to either cut away the cover-plate sufficiently to allow the pin plates to project through it, or to plane off the corner of each pin plate sufficiently to clear the cover-plate. As the former would have a rather unworkmanlike appearance, the latter expedient will be employed. It will be necessary to cut away a large piece of the metal of the pin plate; consequently, the amount of metal remaining between the pin and the edge of the plate thus formed, i. e., on the line *c*, Fig. 326, will probably be found to be the critical condition in complying with the requirements, and no advantage will be obtained by making the pin plate wider than the member itself, or $3 + \frac{1}{2} + 3 = 6\frac{1}{2}$ inches.

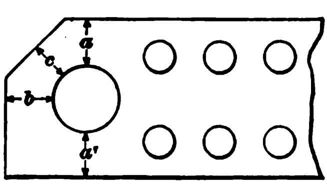


FIG. 326.

From Mechanical Drawing Plate, Title: Highway Bridge: Details I, Fig. 1, it is known that the distance between the center of the pin and the lower side of the cover-plate of the end post is 3 inches. Consequently, by allowing $\frac{1}{16}$ of an inch as the smallest permissible clearance, the net width *c* (see Fig. 326) of that portion of the pin plate between the edge of the pin hole and the lower side of the cover-plate of the end post can not be more than $3 - \frac{1}{2} \times 2\frac{3}{4} - \frac{1}{16} = 1\frac{9}{16} = 1.56$ inches. As given by formula **190**, the sectional area required in the vertical section above or below the pin hole, i. e., the section on the line *a* or *a'*, Fig. 326, is $\frac{2}{3} \times 4.02 = 2.68$ square inches, while the sectional area required on the horizontal section behind the pin, i. e., on the line *b*, is $\frac{7}{8} \times 4.02 = 3.52$ square inches. (Formula **191.**)

1602. In the case of a pin plate having the corner cut away, as shown in Fig. 326, in order that the section on the line *c*, making an angle of 45 degrees with the center line of the member, shall give practically the same resistance as the sections on the lines *a* and *b*, as derived by formulas **190** and **191**, respectively, the sectional area on the line *c* must have the value given by the formula

$$c = \frac{25}{32}\,w, \qquad \textbf{(197.)}$$

in which *w* represents the sectional area of the member. In the example, therefore, the sectional area required on the line *c* is $\frac{25}{32} \times 4.02 = 3.14$ square inches.

Hence, the total thickness of the pin plates required by this condition is $\frac{3.14}{1.56} = 2.01$ inches. This is, therefore, the governing condition. Two pin plates each 1 inch in thickness might have been used; instead, however, four pin plates each $\frac{1}{2}$ inch thick (two double plates) are used.

As each pin plate is of uniform thickness, the dimensions *b* and *c* will be proportional to the area of the sections on those lines. The dimensions *c* and both areas being known, we have the proportion 3.14 : 3.52 :: 1.56 : *b*, from which $b = \frac{3.52 \times 1.56}{3.14} = 1\ 75$ inches. Hence, the required distance from the center of the pin to the end of the pin plate is $1.75 + 1.38 = 3.13$ inches, or say $3\frac{1}{8}$ inches.

1603. Rivets for Connecting Pin Plates.—The number of rivets required to connect the pin plate to the angles composing the main portion of the member is determined by the most severe condition of the requirements for the shearing and bearing. The unit stresses allowed upon the member are 10,000 and 20,000 pounds for live and dead load stresses, respectively.

As all the determinant conditions (the values of A, d, and t) are the same as in Art. **1570,** formulas **178** and **179** should be used, and the results compared. As $d = \frac{5}{8}''$, and $t = \frac{1}{4}''$, Table 37, Art. **1537,** gives 15,000 $d\ t = 2{,}340$. Formula **178** gives

$$n_{lb} = \frac{32{,}400 + 13{,}900}{2{,}340} = 19.8.$$

Formula **179** gives $n_s = \dfrac{2(2 \times 32{,}400 + 13{,}900)}{3 \times .3068 \times 10{,}000} = 17.1.$

The required number of rivets is, therefore, 19.8, that is, 20.

Twenty rivets are used to connect the angles composing the member to the pin plates at each end. Sixteen rivets (four in each angle) connect the free vertical legs of the four angles directly to the pin plates, and four rivets (two in each pair of angles) connect the horizontal legs of the angles to four short pieces of angles attaching to the pin plates. It will be noticed that each of the four rivets last mentioned has a double bearing value and is also in double shear, which gives practically a total value of 24 rivets in the connection of the pin plates, thus making ample allowance for countersunk rivets. The details of this connection, though fulfilling all requirements, could be somewhat improved.

1604. Pin Plates at Joint *b*.—For connecting the pin plates at the opposite end *b* of the member, the number of rivets required is the same. It was found above that two pin plates each $\frac{5}{8}$ of an inch thick, or a total thickness of $1\frac{1}{4}$ inches, will give sufficient bearing for this member upon a $2\frac{3}{4}$-inch pin. With a total thickness of $1\frac{1}{4}$ inches, the required width of the pin plates, as given by applying formula **190,** Art. **1594,** is $\frac{2 \times \frac{2}{3} \times 4.02}{1.25} + 2.75 = 7.04$ inches. Two pin plates 7″ × $\frac{5}{8}$″ will be used. As determined by formula **191,** the pin plates must project beyond the edge of the pin a distance of $\frac{\frac{7}{8} \times 4.02}{1\frac{1}{4}} = 2\frac{13}{16}$ inches (very closely), or a distance of $2\frac{13}{16} + 1\frac{3}{8} = 4\frac{3}{16}$ inches beyond the center of the pin. It will be well to make this dimension practically $4\frac{1}{4}$ inches; for convenience in shortening the distance between pin centers $\frac{1}{32}$ of an inch, in compliance with the rule given in Art. **1519,** this dimension is increased to $4\frac{9}{32}$ inches at one end.

1605. Metal Behind Pin; Another Condition.—Pin plates for tension members should not only comply with the conditions required by formulas **190** and **191** for eyebar heads, but, besides, the dimension b, Fig. 326, should never be less than given by the following formulas:

For wrought iron:

$$b = \frac{9\,d\,t}{8\,T}. \qquad \textbf{(198.)}$$

For medium steel:

$$b = \frac{d\,t}{T}. \qquad \textbf{(199.)}$$

In both of these formulas d is the diameter of the pin hole, t is the thickness of pin plate required by the bearing stress, and T is the actual thickness of the pin plate.

It was found above that the total thickness t for the pin plates at each end of this member, as required by the bearing stress, is 1.12 inches. At the end a of the member, the total thickness T is 2 inches. Hence, at this end of the member the dimension b, or clear distance from the edge of the pin hole to the end of the member, must not be less than $\frac{9 \times 2.75 \times 1.12}{8 \times 2} = 1.73$ inches; or, to express it otherwise, the distance from the center of the pin hole to the end of the pin plate, as required by this condition, must not be less than $1.73 + \frac{2.75}{2} = 3.11$ inches. At the end *b* of the member the dimension *b* should not be less than $\frac{9 \times 2.75 \times 1.12}{8 \times 1.25} = 2.77$ inches, making the distance from the center of the pin hole to the end of the pin plate, as required by this condition, equal to $2.77 + 1.38 = 4.15$ inches. As the distance adopted is greater than this, it is found that the condition required by formula **198** has been complied with.

It will not be necessary to take special notice of the arrangement of the lattice bars for this member; it involves no unusual or difficult conditions, and will be readily understood without explanation.

1606. The Second Panel *b c*.—The dimensions of the pin plates for the member in the second panel *b c* of the lower chord, Mechanical Drawing Plate, Title: Highway Bridge: Details III, Fig. 2, are determined in substantially the same manner as explained above for the lower chord member *a b*; it will be unnecessary to explain the process in detail. It will be noticed that the connections of the pin plates on this member are more simple than upon the member *a b*, Fig. 1 of the same plate. Simplicity of detail is always desirable and should be obtained whenever possible. The manner of connecting the pin plates shown in Fig. 2 (same plate) is far preferable to that shown in Fig. 1. In order that the relative widths of the members would be such as to connect properly upon the pins, the arrangements of the pin plates as shown for the joint *a* of the member *a b* became necessary.

It will be noticed that in the preceding computations the dimensions of the pin plates have been proportioned according to the requirements for elongated eye-bar heads. This method, although not very generally employed, is given here as a safe and reliable practice for the pin plates of tension truss members.

1607. Stiffened Eye-Bars; Panel *c c'*.—The member in the middle panel of the lower chord is shown in Fig. 3

of the plate just referred to. It is composed of eye-bars of the usual form, except that, in order to illustrate another form of stiffened chord, the bars are shown connected by bent lattice bars. The heads are of the form shown in Fig. 325, and, as the diameter of the pin does not exceed three-quarters of the width of the bar, the diameters of the heads are determined by formula **192.** By reference to items (b_1) and (l_1) of the specifications, it is found that for forged eye-bars no definite limit is placed upon the intensity of bearing stress, other conditions being usually such that the permissible intensity of bearing stress will not be likely to be exceeded. In the present case, however, the thickness of the eye-bar heads has been increased to ⅞ of an inch, in order that the bearing stress shall not exceed 15,000 pounds per square inch. According to the stress sheet the net sectional area of the member is 6.40 square inches. Hence, by formula **192,** the net sectional area of the head across the pin hole, i. e., on the lines *a* and *a'*, Fig. 325, must be for each of the two bars $\frac{1.6 \times 6.40}{2} = 5.12$ square inches. With a thickness of ⅞ of an inch, the diameter D required for each head is $\frac{5.12}{\frac{7}{8}} + 2.75 = 8.60$ inches. Heads nominally 9 inches in diameter, but having an actual diameter of 8⅞ inches, are used.

When the eye-bars are not connected by lattice bars, it is necessary to designate only the size of the bar, the diameter and thickness of each head, the size of each pin, the distance from center to center of pin holes, and the number of bars required. In many bridge offices the practice is to simply make a list of the eye-bars required for each structure, giving the above items; in such cases the eye-bars are not shown on drawings, except when required to be latticed.

In some bridge works it is the practice to state the diameter of the *pins* to be used, as is shown in Mechanical Drawing Plates, Titles: Highway Bridge: Details I to IV, while in other shops it is the practice to state the exact diameters required for the *pin holes.*

It is never necessary to give the radius for the necks of eye-bars. As they are always tension members, the distance from center to center of pin holes must be shortened $\frac{1}{32}$ of an inch, in compliance with the rule given in Art. **1519.**

The latticing of chord bars is merely a matter of arrangement, needing no special explanation.

CONSTRUCTIVE DETAILS OF THE TENSION WEB MEMBERS OF THE TRUSS.

1608. The Main Tie Bars *B c*.—The dimensions of the main tie bar are shown in Mechanical Drawing Plate, Title: Highway Bridge: Details IV, Fig. 1. At the hip joint, the distance between the center of the pin and the cover-plate of the chord, or of the end post, is 3 inches. Hence, it is evident that the radius of the eye-bar head connecting upon this pin must be slightly less than this. The nominal diameter of the head will be 6 inches; the actual diameter, $5\frac{7}{8}$ inches. The diameter of the pin hole will be somewhat in excess of the width of the bar, but not greatly so; consequently, the net sectional area A across the head may be taken at a mean between the values given by formulas **192** and **193,** that is, as equal to $\frac{1.6 + 1.8}{2} = 1.7\,B$.

The sectional area of one bar is $2 \times \frac{13}{16} = 1.63$ square inches, and, as the diameter of the head is fixed at $5\frac{7}{8}$ $(= 5.875)$ inches, the thickness of the head must be $\frac{1.63 \times 1.7}{5.88 - 2.75} = .885$, or, practically, $\frac{7}{8}$ of an inch. Heads $5\frac{7}{8}'' \times \frac{7}{8}''$ will be used. As in the present case it is essential that the diameter of the head should under no condition exceed $5\frac{7}{8}$ inches, the exact diameter of the head is marked, instead of its nominal diameter. As the pins at each end of the bar are of the same size, both heads will be made alike. The length of the tie bar, from center to center of pin holes, is $25'\ 5\frac{9}{16}''$. (Art. **1519.**)

1609. The Hip Vertical *B b*.—A hip vertical rod is shown in Fig. 3 of the plate just mentioned; it is 1 inch square. Each hip vertical member consists of two rods like this. Hip vertical members sometimes consist of ordinary eye-bars, however. In the present case the loop, by which each end of the rod connects upon the pin, is formed by bending the end of the rod into the desired form and welding it back upon the bar, making the rod as nearly of the desired length as possible. The length of such bars is always given between the inner edges of the extremities of the two loops, as shown; it is commonly called the length back to back, and is designated on drawings by the letters *B to B* or *b to b*.

Rule.—*The length back to back of a tension rod having loop ends is the length from center to center of pins plus one-half the diameter of each pin, minus* $\frac{1}{32}$ *of an inch if the rod is not adjustable.*

Thus, in the present case, the length *b to b* is equal to $18'\ 0'' + \dfrac{2\frac{3}{4}'' + 2\frac{3}{4}''}{2} - \frac{1}{32}'' = 18'\ 2\frac{23}{32}''$. The exact length *b to b* is finally obtained in reaming the inner sides of the loops to fit the pins.

Rule.—*The inside length of a bent loop is always made equal to 2½ diameters of the pin upon which it connects; or, in other words, the distance from the center of the pin to the inner angle of the weld is made equal to 2 diameters of the pin.*

In the present case, the length of each loop is $2\frac{1}{2} \times 2\frac{3}{4} = 6\frac{7}{8}$ inches.

1610. The Counter Tie *C c'*.—Counter ties usually consist of square or round bars, though frequently of flat bars. The counter shown in Fig. 2 (title as above, IV) is a square bar and connects upon the pins by means of bent loops in substantially the same manner as the hip vertical. The form and relative dimensions of the loop are also the same as described for the hip vertical. The two ends of the rod connect upon different sizes of pins, and the dimensions of each loop depend upon the size of the pin upon which it connects; the loop upon the lower end *c'* of the rod, connecting upon the 2¾″ pin, is flattened to ¾ of an inch in thickness in order to pack upon the pin within the required space.

Counters are made adjustable, usually by means of turnbuckles, as shown. Sometimes, however, only one of the diagonals in the center panel of a truss is made adjustable. This counter tie will be further noticed in the following articles.

UPSET SCREW ENDS.

1611. The end of rods upon which threads are formed are enlarged by upsetting, so that after the threads are cut the remaining portion of the end will be as strong as the unaltered portion of the rod. As the process of upsetting somewhat reduces the strength of the metal, bars in which the diameter at the root of the thread is the same as that of the bar itself, invariably break in the screw end, when tested to destruction, without developing the full strength of the bar. It, therefore, becomes necessary to make up for this loss of strength by upsetting the ends sufficiently so that the sectional area at the root of the thread will be somewhat in excess of the sectional area of the bar.

According to item (m_1) of the specifications (Art. **1511**), "all rods and hangers with screw ends shall be upset at the ends, so that the diameter at the bottom of the threads shall be $\frac{1}{16}$ inch larger than any part of the body of the bar."

According to Thacher's specifications, "the area at root of thread in the upset ends of rods shall be greater than the area of the rod at least 17 per cent."

According to Lewis's specifications, "the area at base of thread and at all parts of upset ends will be 15 per cent. in excess of the area of the bar."

The common practice, which varies slightly in different shops, gives an excess of effective sectional area of the screw end over the area of the bar of from about 17 to 50 per cent., the diameter of the upset end always being a multiple of ⅛ of an inch.

The following table, which gives the standard proportions for upset screw ends as used by the Keystone Bridge Company, is fairly representative of the best practice for upset ends:

TABLE 41.

UPSET SCREW ENDS FOR ROUND AND SQUARE BARS.

Diam. of Round or Side of Square Original Bar.	Round Bars.				Square Bars.			
	Diameter of Upset Screw End.	Diameter of Screw at Root of Thread.	Number of Threads per Inch.	Excess of Effective Area of Screw End over Bar.	Diameter of Upset Screw End.	Diameter of Screw at Root of Thread.	Number of Threads per Inch.	Excess of Effective Area of Screw End over Bar.
In.	Inches.	Inches.		Per cent.	Inches.	Inches.		Per cent.
½	¾	.62	10	54	¾	.62	10	21
⅝	⅞	.73	9	37	1	.84	8	41
¾	1	.84	8	25	1⅛	.94	7	23
⅞	1¼	1.07	7	48	1⅜	1.16	6	38
1	1⅜	1.16	6	35	1½	1.28	6	29
1⅛	1½	1.28	6	30	1⅝	1.39	5½	20
1¼	1⅝	1.39	5½	23	1⅞	1.62	5	31
1⅜	1¾	1.49	5	18	2	1.71	4½	22
1½	2	1.71	4½	30	2⅛	1.84	4½	18
1⅝	2⅛	1.84	4½	28	2⅜	2.09	4½	30
1¾	2¼	1.96	4½	26	2½	2.18	4	21
1⅞	2⅜	2.09	4½	24	2⅝	2.30	4	18
2	2½	2.18	4	18	2⅞	2.55	4	28
2⅛	2⅝	2.30	4	17	3	2.63	3½	20
2¼	2⅞	2.55	4	28	3⅛	2.75	3½	18
2⅜	3	2.63	3½	23	3⅜	3.00	3½	26
2½	3⅛	2.75	3½	21	3½	3.10	3¼	21

The diameter of the round bar or side of the square bar being known, the corresponding diameter of the upset end may be obtained from the table. Upset screw ends are sometimes required on flat bars. In such a case it is necessary to find first the side of a square bar having the same sectional area as the flat bar, and then to find from the table the diameter of the upset end as required for the square bar. The length of the upset end may be determined by formula **200**, which is given in connection with the dimensions of turnbuckles.

TURNBUCKLES.

1612. Having obtained the required diameter of the upset screw end, the dimensions of the turnbuckles are next to be determined. The threaded ends of the two portions of the rod screw into the ends of the turnbuckle, or, in other words, the turnbuckle screws upon the ends of the rods by means of a right-handed screw at one end and a left-handed screw at the other end, so that by turning it in one direction both ends are screwed up, while by turning it in the opposite direction both ends are unscrewed. As the finished turnbuckles commonly used are to be obtained in the market in sizes suitable for the different diameters of thread ends, the various dimensions of turnbuckles are not commonly given on the drawings, it being usually sufficient to designate the dimensions of the thread ends and state the sizes of turnbuckles to be used. But, in order that the proportions of this detail may be clearly understood, the relative dimensions of a common form of turnbuckle will be given.

The forms and dimensions of turnbuckles vary somewhat as manufactured by different firms, but the form shown in Fig. 327 may be considered as fairly representative. The letters by which the various dimensions are designated are shown in

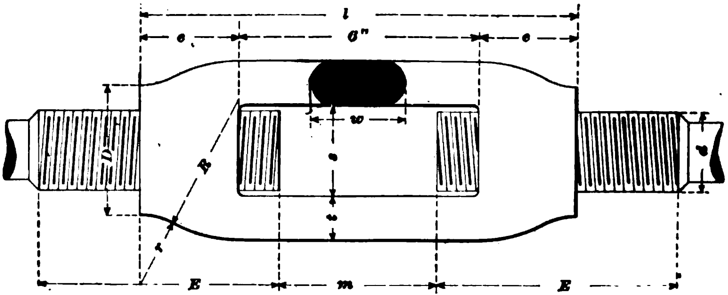


FIG. 327.

the figure. E is the length of the upset end, of which d is the diameter; m is the assumed distance between the two ends when the turnbuckle is screwed up in position upon them. All other dimensions relate to the turnbuckle.

The several dimensions for this form of turnbuckle, and also the length of the upset ends, may be expressed in terms of d as follows:

$$E = 2d + 1\tfrac{1}{2}''. \qquad \textbf{(200.)}$$

$$l = 2d + 7''. \qquad \textbf{(201.)}$$

$$e = d + \tfrac{1}{2}''. \qquad \textbf{(202.)}$$

$$D = 1\tfrac{1}{2}d + \tfrac{1}{4}''. \qquad \textbf{(203.)}$$

$$s = d + \tfrac{1}{4}''. \qquad \textbf{(204.)}$$

$$t = \frac{d}{2} + \tfrac{1}{8}''. \qquad \textbf{(205.)}$$

$$w = d + \frac{d - 1\tfrac{1}{4}''}{2}. \qquad \textbf{(206.)}$$

$$R = 1.417\,e. \qquad \textbf{(207.)}$$

$$r = .708\,e. \qquad \textbf{(208.)}$$

$$m = 4''. \qquad \textbf{(209.)}$$

The value of E, formula **200,** is usually made a multiple of ½ inch, although there is no objection to giving this dimension the exact value derived from the formula, which will always be a multiple of ¼ inch. This dimension is commonly given values varying from $2\,d + 2''$ to $2\,d + 1''$; a value of $2\,d + 1\tfrac{1}{2}''$ is to be preferred. The minimum value which may be given to E is 4 inches. Formula **200** and the preceding remarks apply not only to turnbuckles, but to all ordinary upset screw ends.

In the expression for the value of w, formula **206,** when the fraction $\frac{d - 1\frac{1}{4}''}{2}$ becomes negative it is simply given a value of zero.

The distance m between the ends of the screws is variously taken as 3, 4, and even 5 inches; 4 inches is a good value. A value of 3 inches is, however, not uncommonly assumed.

It may be noticed that values of E and m, as given by formulas **200** and **209**, will give practically the same relative positions and conditions of the screw ends as a value of $2\,d + 2''$ for E, used with a value of 3 inches for m. Slight variations in these dimensions are not of consequence, and any consistent rule will be found satisfactory.

1613. The Counter Tie Continued.—Turnbuckles upon counters are usually placed at a distance of about five feet from the lower end. As determined in Art. **1519,** the length of the tension diagonal, center to center of *pin holes*, is $25'\,5\frac{9}{16}''$; hence (see Art. **1609**), the length of the counter from back to back of loops will be $25'\,5\frac{9}{16}'' + \frac{1\frac{3}{4}'' + 2\frac{3}{4}''}{2} = 25'\,7\frac{13}{16}''$. As an error of less than $\frac{1}{16}$ of an inch is of no consequence in an adjustable member, the allowance of $\frac{1}{32}$ of an inch for clearance of pin holes may be wholly neglected. If the portion of the counter below the turnbuckle is made $5'\,0''$ long, back to end, then, with a distance of 3 inches between the ends, the length of that portion of the bar above the turnbuckle will be $25'\,7\frac{13}{16}'' - 5'\,3'' = 20'\,4\frac{13}{16}''$, end to back, or, with an allowance of 4 inches between the screw ends, the length of the larger portion of the bar will be $20'\,3\frac{13}{16}''$ from back to end.

CONSTRUCTIVE DETAILS OF THE LATERAL RODS. DIMENSIONS OF NUTS.

1614. The Lower Lateral Rods; Intermediate Panels.—The rods in the three intermediate panels of the lower lateral system will be first considered. The dimensions for each of these rods are shown in Mechanical Drawing Plate, Title: Highway Bridge: Details IV, Fig. 6. By reference to the detailed drawing of the floor-beam (same title, II), it will be found that the distance between the centers of the lateral pin holes at opposite ends of the floor-beam is $19'\,0'' - 2 \times 8\frac{3}{4}''$, or $2 \times (8'\,9\frac{1}{4}'') = 17'\,6\frac{1}{2}'' = 210\frac{1}{2}$ inches. This distance is the lateral projection, or reach of the lateral rods, in each of these three panels. In the same panels the longitudinal projection, or reach of each lateral rod, is equal to the panel length minus the distance between the centers of the two lateral pin holes in the same end of the beam, or $18' \; 0'' - 0' \; 5\frac{3}{4}'' = 17' \; 6\frac{1}{4}'' = 210\frac{1}{4}$ inches. Hence, the length of the lateral rod, from center to center of pin holes, is equal to $\sqrt{210.5^2 + 210.25^2} = 297.515''$, or $24' \; 9\frac{33}{64}''$. As the rod is adjustable, it will be sufficiently accurate to call this length $24' \; 9\frac{1}{2}''$.

It will be noticed that the lateral projection of the rod ($210\frac{1}{2}$ inches) and its longitudinal projection ($210\frac{1}{4}$ inches) happen to have very nearly the same values; the length from center to center may therefore be determined with sufficient accuracy for an adjustable rod by applying formula **184,** Art. **1576,** using a mean between the two projections. The mean of the two projections is $\frac{210.5 + 210.25}{2} = 210.375$ inches, from which formula **184** gives $1.4142 \times 210.375 = 297.512$ inches, or, practically, $24' \; 9\frac{1}{2}''$, as the length from center to center of pins.

As proportioned in Art. **1591,** all lateral pins are $1\frac{3}{4}$ inches in diameter. The length of this lateral rod from back to back of loops is, therefore, $24' \; 9\frac{1}{2}'' + \frac{1\frac{3}{4}'' + 1\frac{3}{4}''}{2} = 24' \; 11\frac{1}{4}''$. If 3 inches be allowed between the screw ends, the length of the longer piece of the rod will be $24' \; 11\frac{1}{4}'' - 5' \; 3'' = 19' \; 8\frac{1}{4}''$ from back to end; or, if 4 inches be allowed between the screw ends, the length of the longer piece will be $19' \; 7\frac{1}{4}''$ from back to end. The lengths of the loops are found in the manner explained for the hip verticals, and the diameter and length of the upset ends are obtained as explained in the preceding articles.

1615. The Lower Lateral Rods; End Panels.—The dimensions for each lower lateral rod in the end panels are shown in Mechanical Drawing Plate, Title: Highway Bridge: Details IV, Fig. 5. One end of this lateral rod connects upon the lateral pin in the shoe, and, therefore, its length can not be determined until the position of the lateral pin in the shoe has been fixed. However, as it is desirable to notice here the length of this lateral rod, it will be assumed that the lateral pin holes in the shoes have been previously located as shown in Figs. 5 and 9 of Plate III, same title. By reference to those figures, it is found that the center of each lateral pin is laterally opposite the center of the shoe pin and at a distance of $10\frac{3}{16}$ inches from the center of the chord. From Fig. 4 of Plate II, title as above, it is found that the lateral projection of this rod is $19'\ 0'' - 10\frac{3}{16}'' - 8\frac{3}{4}'' = 17'\ 5\frac{1}{16}'' = 209\frac{1}{16}$ inches, and that its longitudinal projection is $18'\ 0'' - 2\frac{7}{8}'' = 17'\ 9\frac{1}{8}'' = 213\frac{1}{8}$ inches. Hence, the length of the rod from center to center of pin holes is equal to $\sqrt{209.063^2 + 213.125^2} = 298.546$ inches $= 24'\ 10\frac{35}{64}''$, or, near enough for an adjustable rod, $24'\ 10\frac{1}{2}''$. The length of rod from back to back of loops is $24'\ 10\frac{1}{2}'' + \frac{1\frac{3}{4}'' + 1\frac{3}{4}''}{2} = 25'\ 0\frac{1}{4}''$. If the shorter piece is $5'\ 0''$ from back to end, and the distance between the screw ends assumed to be 3 inches, the length of the longer piece will be $25'\ 0\frac{1}{4}'' - 5'\ 3'' = 19'\ 9\frac{1}{4}''$ from end to back; but if 4 inches be allowed between the screw ends, the length of the longer piece will be $19'\ 8\frac{1}{4}''$ from back to end. The relative dimensions of the loops, screw ends, and turnbuckles have been fully explained.

1616. Dimensions of Nuts.—As nuts are used upon the ends of the upper lateral rods, before calculating the lengths of the rods it will be expedient to notice briefly the relative dimensions of nuts. The standard screw threads recommended by the Franklin Institute have been quite generally adopted throughout the United States.

When this thread is used, the diameter d_o of the bolt at root of thread, or the approximate diameter of rough hole in nut before cutting the thread, is given by the formula

$$d_o = d - 1.3\,p, \qquad \textbf{(210.)}$$

in which d is the original diameter of the bolt and p is the pitch of the thread. (The pitch of the thread is equal to one divided by the number of threads per inch.)

The standard proportions for finished nuts recommended by the Franklin Institute are as follows:

Thickness t of nut:

$$t = d - \tfrac{1}{16}''. \qquad \textbf{(211.)}$$

Short diameter S of square or hexagonal nut:

$$S = 1\tfrac{1}{8}\,d + \tfrac{1}{16}''. \qquad \textbf{(212.)}$$

Diagonal L_s of square nut:

$$L_s = 1.414\,S. \qquad \textbf{(213.)}$$

Long diameter L_h of hexagonal nut:

$$L_h = 1.155\,S. \qquad \textbf{(214.)}$$

In all these formulas d is the diameter of the bolt.

In rough nuts, the dimensions t and S are each $\tfrac{1}{16}$ of an inch greater than in finished nuts.

These proportions for nuts have not been very generally adopted, on account of the uncommon sizes of bar (not usually rolled by the mills) required to make the nut. Consequently, other standards are used for the dimensions of nuts. In bridge construction the standard commonly used for nuts is what is known as the **manufacturers' standard.** Table 42 gives sizes and weights of hexagon nuts by this standard.

1617. The Upper Lateral Rods; Center Panel.—Referring to Mechanical Drawing Plate, Title: Highway Bridge: Details I, Fig. 2, it will be noticed that, in the center panel of the upper chord, the center of the lateral hitch (with reference to the rivet holes) is at the center of the chord, laterally, and at a distance of $8\tfrac{3}{16}$ inches from the center of the pin. Hence, the lateral projection of the lateral rod, between centers of the lateral hitches, is equal to the width of the bridge from center to center of chords, or 19 feet = 228 inches. The longitudinal projection between the same points, or the distance between the two lateral hitches on the same chord member, is equal to $18'\ 0\tfrac{3}{8}'' - 2 \times 8\tfrac{3}{16}'' = 16'\ 8'' = 200$ inches, and the length of the diagonal between the same points is $\sqrt{228^2 + 200^2} =$

TABLE 42.

SIZES AND WEIGHTS OF HEXAGON NUTS.

MANUFACTURERS' STANDARD.

Size of Bolt.	Size of Rough Hole.	Thickness of Nut.	Short Diameter.	Long Diameter.	Weight of 100 Nuts.
Inches.	Inches.	Inches.	Inches.	Inches.	Pounds.
1/2	7/16	1/2	7/8	1.01	7.1
1/2	7/16	1/2	1	1.15	9.8
5/8	9/16	5/8	1 1/8	1.30	14.7
5/8	9/16	5/8	1 1/4	1.44	19.1
5/8	9/16	3/4	1 1/4	1.44	22.9
3/4	21/32	3/4	1 3/8	1.59	27.2
3/4	21/32	7/8	1 1/2	1.73	39.
7/8	25/32	7/8	1 5/8	1.88	44.
7/8	25/32	1	1 5/8	1.88	50.
1	7/8	1	1 3/4	2.02	57.
1	7/8	1 1/8	1 3/4	2.02	64.
1 1/8	15/16	1 1/4	2	2.31	96.
1 1/4	1 1/16	1 3/8	2 1/4	2.60	134.
1 3/8	1 3/16	1 1/2	2 1/2	2.89	180.
1 1/2	1 5/16	1 5/8	2 3/4	3.18	235.
1 5/8	1 7/16	1 3/4	3	3.46	300.
1 3/4	1 9/16	1 7/8	3 1/4	3.75	370.
1 7/8	1 11/16	2	3 1/2	4.04	460.
2	1 13/16	2	3 1/2	4.04	450.
2 1/4	2	2 1/4	3 3/4	4.33	560.
2 1/2	2 1/4	2 1/2	4 1/4	4.91	810.
2 3/4	2 7/16	2 3/4	4 1/2	5.20	980.
3	2 11/16	3	4 3/4	5.48	1150.

303.289 inches = 25′ 3$\frac{9}{32}$″, or, near enough for an adjustable rod, 25′ 3¼″.

From Fig. 4 of the same plate the distance from the center (between rivet holes) of the lateral hitch and the outside of the 3½-inch leg of the 3½″ × 5″ angle is 2⅝ inches, and, consequently, the length of the lateral rod from outside to outside of the angles of the hitch is 25′ 3¼″ + 2 × 2⅝″ = 25′ 8½″.

From Table 41, Art. **1611,** the diameter of the upset end, for a round rod 1 inch in diameter, is 1⅜ inches, and from Table 42, Art. **1616,** it is known that, for a bolt, or the screw end of a rod, having a diameter of 1⅜ inches, the thickness of the nut is 1½ inches. With regard to the amount which the screw end of the rod should project beyond the nut, no definite rule can be given. Some engineers consider a projection of one inch at each end to be ample, but for small rods the projection given by the following rule is very commendable:

Rule.—*Make the projecting end of the rod equal to the thickness of the nut.*

This rule is here followed. Hence, the total length of the lateral rod in the center panel is 25′ 8½″ + 4 × 1½″ = 26′ 2½″, as marked in Plate IV, title as above, Fig. 4.

1618. The Upper Lateral Rods ; End Panels.—By referring again to Plate III, title as above, Fig. 2, it is found that, in the end panels, the lateral projection of the lateral rod, between centers of the lateral hitches, is the same as in the center panel, or 19 feet = 228 inches, and that its longitudinal projection, between the same points, is equal to 18′ 0⅜″ − (14$\frac{3}{16}$″ + 8$\frac{3}{16}$″) = 16′ 2″ = 194 inches. Hence, the diagonal distance between the centers of the lateral hitches is equal to $\sqrt{228^2 + 194^2}$ = 299.366 inches = 24′ 11⅜″, nearly, and the total length of each lateral rod in the end panel of the upper lateral system is 24′ 11⅜″ + 2 × (2⅝″ + 1½″ + 1½″) = 25′ 10⅝″, as marked upon the drawing. The length of the thread ends will be the same as given in connection with the dimensions of turnbuckles. A length of 4¼ inches, as given by formula **200,** would be sufficient for each threaded end.

CONSTRUCTIVE DETAILS OF THE BEAM HANGER.

1619. The Hanger Rods.—The beam hanger is shown in Plate, Title : Highway Bridge : Details IV, Fig. 7. On account of its form, this member is also called a **stirrup.** The size of the beam hanger is sometimes determined in making the general design, but it is often left to be proportioned as a detail. The total load upon each floor-beam is 41,200 pounds (Art. **1586**) ; hence, the load upon the hanger at each end is $\frac{41,200}{2} = 20,600$ pounds. In the present case, the hangers are composed of bar iron with forged (upset screw) ends, and the unit stress allowed upon them is 9,000 pounds per square inch. [See Art. **1399** (*c*).] The sectional area required for each hanger is, therefore, $\frac{20,600}{9,000} = 2.29$ square inches.

Referring to Fig. 7 of the same plate it will be noticed that the section is given by two bars, or, rather, by the two parts or branches of the same bar, one of which, when in position, will be on each side of the beam. Consequently, the sectional area of the bar forming the beam hanger must not be less than $\frac{2.29}{2} = 1.15$ square inches. As the bar is bent slightly from a vertical position, in that portion of the hanger above the top of the floor-beam, its sectional area should be somewhat more than this. A bar 1⅛ inches square will give a sectional area of 1.27 square inches. This is the size of rod most commonly used for beam hangers in highway bridges, and would ordinarily be considered sufficient for the present case. As, however, the bar is weakened by bending, a bar 1¼ inches square has been used.

1620. From Table 41, Art. **1611,** it is found that the diameter of the upset screw end required upon a bar 1¼ inches square is 1⅞ inches. These hangers fit upon 2¾″ pins. Hence, in order that the screw ends of the hanger may pass over and easily clear the pin, the distance between the centers of the two screw ends of the hanger should be about one-eighth of an inch more than the diameter of the pin plus the nominal diameter of one screw end, or $2\frac{3}{4} + 1\frac{7}{8} + \frac{1}{8} = 4\frac{3}{4}$ inches, as marked in the figure. This determines the distance apart of the two branches of the hanger, which should be parallel from the bottom of the hanger to the top of the floor-beam. The clear distance between the square bars in the parallel portion of the hanger is $4\frac{3}{4} - 1\frac{1}{4} = 3\frac{1}{2}$ inches, as marked.

1621. The length of the hanger should be such that it may be screwed up so that it will hold the beam firmly against the pin plates of the intermediate post. Fig. 3 of Plate, Title: Highway Bridge: Details I, shows that the pin plates of the intermediate post extend $4\frac{3}{4}$ inches below the center of the pin ; whence it follows that the distance from the upper part of the parallel portion of the hanger (at the top of the floor-beam) to the inner side of the bend should be equal to this projecting length of the pin plates plus one-half the diameter of the pin, or $4\frac{3}{4} + 1\frac{3}{8} = 6\frac{1}{8}$ inches, as marked. This distance should always be determined and marked to the *inner side* of the bend, *not* to the *center* of the pin. On actual shop drawings, the radius of the bend should not be marked, but the diameter of the pin should be given.

The top of each upset should be about $\frac{1}{2}$ or $\frac{3}{4}$ of an inch above the bottom of the beam ; in the present case it is made $\frac{1}{2}$ inch above the bottom of the beam. The amount which each end of the hanger extends below the bottom of the beam is determined by the thickness of the nuts and of the hanger plate, together with the amount which each thread end should project beyond the nut.

1622. **The Hanger Nuts.**—Each end of the hanger should always be provided with two nuts, the principal, or **standard, nut** and a small nut called a **check nut.** The check nut should always be placed *above* the standard nut. Hexagon nuts are generally used for hangers. The size of the principal nut is obtained from the standard sizes given in Table 42, Art. **1616,** by reference to which the thickness of the nut for $1\frac{7}{8}$ inches diameter of bolt or thread end is found to be 2 inches. The thickness of the check nut is usually $\frac{3}{4}$ of an inch. Each end of the hanger should project not less than 1 inch beyond the nut. In the present case the hanger plate is $\frac{3}{4}$ of an inch thick. The length of the thread end will be as follows:

Above bottom of beam,	$\frac{1}{2}''$
Through hanger plate,	$\frac{3}{4}''$
Through check nut,	$\frac{3}{4}''$
Through standard nut,	2″
Projection,	1″
Total,	5″

The length from the end of the hanger to the inside of bend will, therefore, be $5'' + 24'' + 6\frac{1}{8}'' = 2'\ 11\frac{1}{8}''$, as marked. It is not necessary to give the exact dimensions of nuts on working drawings. The practice in different shops varies somewhat, but in most cases it is sufficient to mark the drawing thus:

$1\frac{7}{8}''$ *Thread Ends.*
Standard Nuts.
$\frac{3}{4}''$ *Check Nuts.*

Sometimes special forms of locknuts are used for hangers, in which case the kind of nut required should be stated on the drawings.

1623. Hanger Plates.—The width of the hanger plate should be somewhat greater than the long diameter of the nuts used. Single hanger plates are generally made 5, $5\frac{1}{2}$, or 6 inches in width. In determining the thickness of a hanger plate, the latter may be treated as a beam simply supported at the ends and carrying a uniform load. By considering the resistance of the flange angles of the beam, as supported by the stiffeners, and the support given to the hanger plate by the nuts, it will be seen that this assumption is well upon the side of safety.

Under this assumption, the bending moment is $M = \frac{W l}{8}$ (formula **94,** Art. **1316**), and the moment of resistance, $R = \frac{S I}{c}$ (formula **73,** Art. **1243**). For a rectangular section, $I = \frac{b d^3}{12}$, and $c = \frac{1}{2} d$ (Table of Moments of Inertia). Hence, for this form of section, $\frac{S I}{c} = S \frac{\frac{1}{12} b d^3}{\frac{1}{2} d} = \frac{S b d^2}{6}$, or

$$R = \frac{S b d^2}{6}, \qquad \textbf{(215.)}$$

in which S is the fiber stress ($= S_4$, when R is the ultimate resisting moment), b is the breadth, and d the depth of section. This formula expresses the moment of resistance of any rectangular section.

As the moment of resistance must equal the bending moment, $M = R$, or, in this case, $\frac{W l}{8} = \frac{S b d^2}{6}$, from which

$$d = \sqrt{\frac{3 W l}{4 S b}}. \qquad \textbf{(216.)}$$

In our example, $W = 20,600$ pounds, $l = 4\frac{3}{4}$ inches, b will be taken equal to 6 inches, and S ($= S_4$) may be taken at 18,000 pounds, the same as for pins. Therefore, the thickness of the hanger plate should be,

$$d = \sqrt{\frac{3 \times 20,600 \times 4\frac{3}{4}}{4 \times 18,000 \times 6}} = .82 \text{ of an inch.}$$

A plate $\frac{3}{4}$ of an inch thick has been used. By noticing the manner in which the lower flange of the floor-beam is supported by the stiffeners near the beam hanger (Plate, Title: Highway Bridge: Details II, Fig. 4), it will be seen that, as a considerable portion of the load upon the beam is delivered directly upon the hanger nuts, the hanger plate is relieved of much of its stress as a beam; hence, in this case, a thickness of $\frac{3}{4}$ of an inch is probably ample.

If the flanges of the beam are not supported by stiffeners reasonably near to the position of the hanger, the hanger plate should be considered as a beam carrying a single load concentrated at the center, in which case the quantity under the radical sign will be just twice as great as in formula **216.** But, in such a case, the thickness of the flange angles to some extent helps to make up the thickness required for the hanger plate. The thickness, as obtained by formula **216,** will generally be somewhat greater than the values commonly used.

CONCLUDING REMARKS.

1624. All calculations necessary for proportioning, according to the specifications used, the constructive details of the members of the bridge shown in the five plates of Mechanical Drawing, to which we have referred, have now been explained. Various small details, however, such as pins, shoes, rollers, bed-plates, etc., still remain to be noticed. These, as well as certain features relating to the main members, will be considered in subsequent articles.

Conditions and requirements for the details varying from those here noticed are commonly met with, and are required by other specifications. It should also be noticed that, besides the connecting details explained in preceding pages, many other forms of connections are often used. This statement applies especially to the connections for the lateral rods and struts, the portal bracing, and the floor-beam. It is not within the limits of this Course to compare and discuss the various forms of practical details. A really comprehensive knowledge of these can only be acquired by practical experience.

The details that have been explained are not in all cases the best possible for the purposes intended; but it has been the aim to illustrate thoroughly the application of the underlying principles governing the proportioning of details, rather than to discuss the various forms of the latter.

The principles explained in the preceding pages apply to the calculations and design of details in general; and if the student has thoroughly mastered them, he will experience no serious difficulty in proportioning the details and connections for the members of a bridge according to any specifications, and in conformity with the requirements of approved practice.

It is absolutely impossible to formulate any system of rules that will apply to all cases and conditions. The practical designer is constantly meeting new conditions, and in many things he must be guided by his judgment and experience.

DETAILS, BILLS, AND ESTIMATES.

SHOES, ROLLERS, AND BED-PLATES.

1625. Shoes.—For the structure shown in Mechanical Drawing Plates, Titles: Highway Bridge: Details I, II, III, IV, and Highway Bridge: General Drawing, the shoes are shown in Figs. 5 and 9 of Plate, Title: Highway Bridge: Details III. In a shoe, the upright plates which support the pin are called **standards** or **ribs.** In the present case, the thickness of each of the two standards is ⅝ of an inch, and the clear distance between them is 9⅞ inches. (Arts. **1539** and **1540.**)

The lower or horizontal plate forming the bottom of the shoe is called the **shoe plate** or **sole plate.** Before determining the required thickness of this plate, it is necessary to ascertain its required length and breadth.

In the present case, the width from out to out of standards is $9\frac{7}{8} + 2 \times \frac{5}{8} = 11\frac{1}{8}$ inches. In order that it may be connected to the standards by 3″ × 3″ angles, the shoe plate must, as a rule, extend somewhat more than 3 inches beyond each standard, which gives $11\frac{1}{8} + 2 \times 3 = 17\frac{1}{8}$ inches for the least permissible width of the plate. The width will be made 17½ inches. The length of the shoe plate in the roller shoe depends upon the number of rollers required. Consequently, before proceeding with the dimensions of the shoe, it will be expedient to notice the dimensions of the rollers.

1626. Rollers.—The rollers are shown in Fig. 7 of the last plate referred to. As may be seen in the figure, they are secured together by two bars, called **side bars.** The turned ends of the rollers are made to run in holes drilled in

For notice of copyright, see page immediately following the title page.

the side bars much the same as the ends of pulleys or shafting run in journals. The ends of the side bars are generally connected by round bars, the ends of which are riveted down upon the side bars, attaching them to each other. Such an assemblage of rollers is called a **nest of rollers.**

1627. The design of the rollers is governed by item (t_1) of the specifications (Art. **1511**). According to this specification, the diameter d of the roller shall not be less than 2 inches, and the pressure per lineal inch of the roller shall not exceed $500\sqrt{d}$.

If the diameter of the rollers be assumed to be 2 inches, the pressure per lineal inch must not exceed $500\sqrt{2} = 707$ pounds. The vertical pressure upon the shoe is equal to the reaction from live and dead load (46,300 pounds) plus the pressure at the foot of the leeward end post, due to the wind force against the upper chord, which is equal to $150 \times 18 \times \frac{5-1}{2} \times \frac{18}{19} = 5{,}100$ pounds. The total vertical load that can come upon a shoe is, therefore, $46{,}300 + 5{,}100 = 51{,}400$ pounds, requiring the aggregate length of the rollers available for bearing to be $\frac{51{,}400}{707} = 72.7$ inches.

The constructive details generally require the length of the rollers to be about 1 inch shorter than the width of the shoe plate, or, in the present case, $17\frac{1}{2} - 1 = 16\frac{1}{2}$ inches long, approximately. Fig. 7 of Plate, Title: Highway Bridge: Details III, shows that the actual length of the roller, from shoulder to shoulder, is $16\frac{5}{8}$ inches. It will also be noticed that grooves are turned in the centers of the rollers, in order that they may pass over small bars riveted on the bottom of the shoe plate and top of the bed-plate (Figs. 5 and 8, same plate). The office of these bars is to act as a track or guide, over which the grooves in the rollers may run, holding the rollers and also the shoe truly in position.

In some cases these bars on the shoe plates and bed-plate are solid ribs formed by planing down the bearing portions of the plates. In the present case the bars riveted to the shoe plate and bed-plate are $1\frac{1}{4}$ inches wide, and the grooves $1\frac{3}{8}$ inches wide. As the grooves are made to clear the bars, the rollers have no bearing in this portion of the length, and, therefore, the effective or bearing length of each roller must be considered to be shortened $1\frac{3}{8}$ inches by this detail, making the length of each roller, effective in bearing, equal to $16\frac{3}{8} - 1\frac{3}{8} = 15$ inches. The number of rollers required is, therefore, equal to $\frac{72.7}{15} = 4.8$, or, practically, 5 rollers.

Not less than $\frac{1}{8}$ of an inch clearance must be allowed between each two adjacent rollers, and a clearance of $\frac{1}{4}$ of an inch may be used when obtainable. In the present case, if $\frac{1}{4}$ of an inch clearance is used, the distance from out to out of the rollers will be $5 \times 2 + 4 \times \frac{1}{4} = 11$ inches.

1628. Coefficient of Expansion.—According to item (u_1) of the specifications (Art. **1511**), "variations in temperature, to the extent of 150 degrees (Fahrenheit) must be provided for."

For a variation in temperature of 150 degrees (Fahrenheit), the amount of expansion or contraction of wrought iron is given approximately by the formula

$$e = \frac{l}{1,000}, \qquad \textbf{(217.)}$$

in which e is the total amount of expansion, and l the length, in inches, of the piece of iron (the span of the bridge, in this case) considered.

This is usually expressed as a coefficient, and is called the **coefficient of expansion.** Thus, for a variation of 150 degrees, the coefficient of expansion for wrought iron is .001. For structural steels it is substantially the same.

In the example, then, the amount of expansion (or contraction) to be provided for at the roller shoe is equal to $\frac{90 \times 12}{1,000} = 1.08$, or, nearly, $1\frac{1}{8}$ inches. By adding this amount to the width out to out of rollers, it is found that the

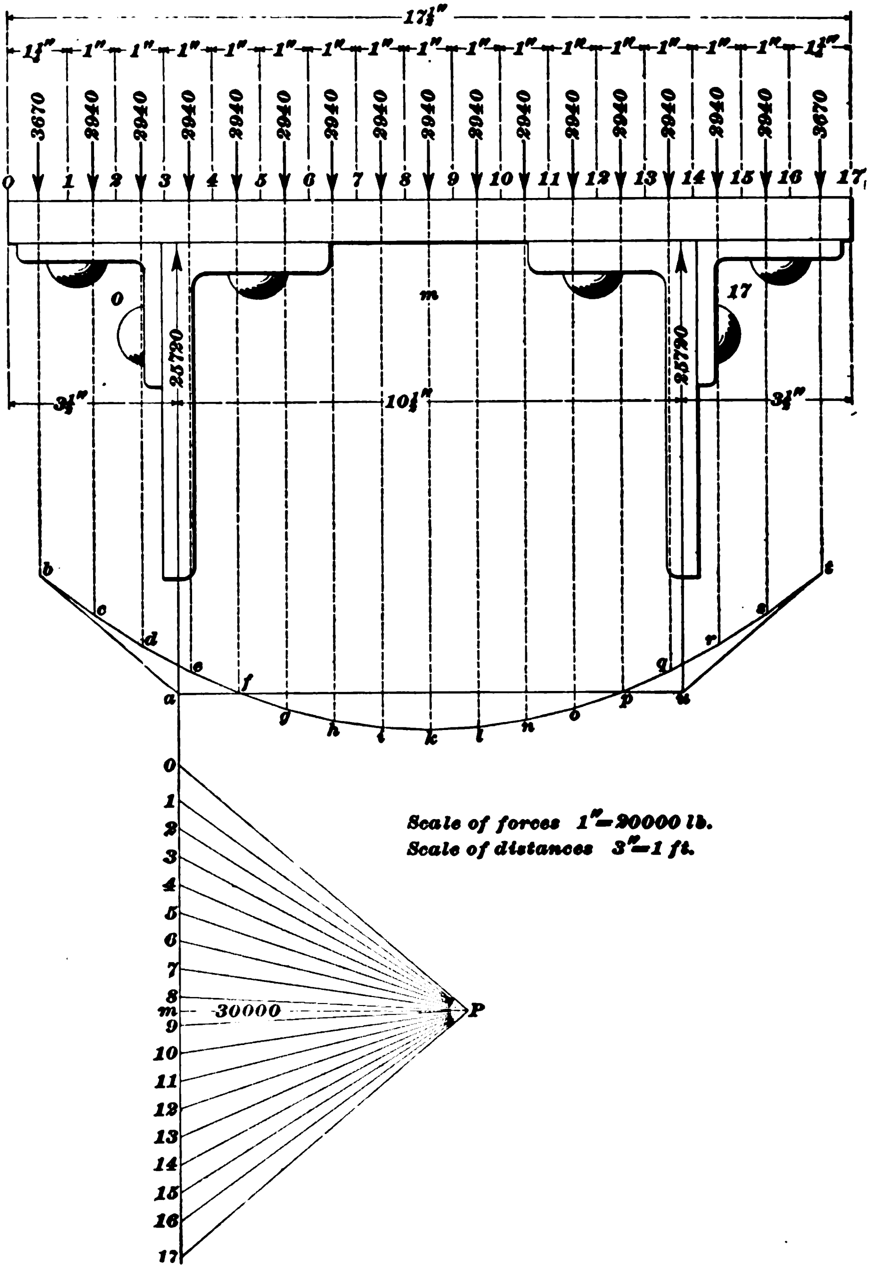

17½"
1¼"
1"
1¼"
3670
2940
25790
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
m
3¼"
10½"
a
b
c
d
e
f
g
h
i
k
l
n
o
p
q
r
s
t
u
30000
P
Scale of forces 1" = 20000 lb.
Scale of distances 3" = 1 ft.

FIG. 328.

required length of the shoe plate is 11 + 1⅛ = 12⅛ inches. As but a small portion of the width of each roller bears upon the plate, the distance out to out of the *bearings* of the roller will be somewhat less than 11 inches, and a length of 12 inches could have been used for the shoe plate. A length, however, of 13 inches is used.

1629. Bending Moment on Shoe Plate.—It will be sufficiently correct to consider the pressure upon the bottom of the shoe to be uniformly distributed over its bottom surface. This will be practically correct for the anchored shoe, and for the roller shoe the error will be slight. The load upon the shoe is delivered upon the shoe plate through the standards. If, for convenience, we imagine the shoe to be inverted, we shall have, in the case of a shoe with two standards, a uniformly loaded beam resting upon two supports and overhanging each support.

As previously determined, the maximum total load upon a shoe is 51,400 pounds. The load upon each standard or, considering the standards as supports, the reaction of each support is $\frac{51,400}{2} = 25,700$ pounds. The distance between the standards, from center to center, is 9⅞ + ⅝ = 10½ inches.

In Fig. 328 the shoe is shown inverted, acting as a uniformly loaded beam upon two supports, extending beyond each support. The uniform load upon the beam is represented as divided into sections, and the weight of each section is considered to be applied at its center of gravity. Each section is 1 inch long, except the two end sections, which are each 1¼ inches long. Consequently, the weight upon each intermediate section is equal to $\frac{51,400}{17.5} = 2,937$, or, practically, 2,940 pounds, and the weight upon each end section is 1¼ × 2,937 = 3,671, or, practically, 3,670 pounds, as indicated in the figure. (See also Art. **1171**, Rule.)

The same figure shows the force diagram and moment diagram for this beam. In the force diagram, the pole distance is made equal to 30,000 pounds. It will be noticed that the bending moment in this beam is negative at the supports and positive in the middle portion. (It is evident that such a beam could project far enough beyond the supports so that the bending moment at the center also would be negative.)

Either the positive or the negative bending moment may be the maximum bending moment, and, consequently, the maximum intercept of each character must be measured. The maximum negative intercept occurs at either support and measures — .60 of an inch, nearly; hence, the maximum negative bending moment will be — .60 × 30,000 = — 18,000 inch-pounds.

The maximum positive intercept occurs at the center *k*, and measures .76 of an inch. This shows that the absolutely maximum bending moment occurs at this point. Its value is .76 × 30,000 = 22,800 inch-pounds.

On account of the uniform load being assumed to be concentrated at the centers of the various sections, this bending moment is slightly in excess of the true one, but it is sufficiently correct for all practical purposes.

1630. The exact bending moment upon a shoe of this form may be easily calculated. If a beam is uniformly loaded over its entire length *b*, Fig. 329, and the amount *p* which it projects beyond one support is equal to the amount *p'* which it projects beyond the opposite support, the bending moment *M* at the center will be given by the formula

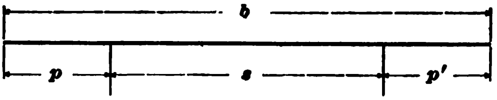


FIG. 329.

$$M = \frac{W}{4}\left(s - \frac{b}{2}\right) = \frac{W}{4}\left(\frac{s}{2} - p\right), \qquad (218.)$$

in which *W* is the total load upon the beam, and *s* and *b* have the values indicated in the figure. This bending moment will be positive or negative, according as the result is positive or negative.

This formula assumes a uniform load over the entire breadth b of the shoe. When the rollers are grooved in the center, as in Plate, Title: Highway Bridge: Details III, Fig. 7, they will support no load upon the groove, and the bending moment at the center of the shoe, as given by formula **218,** will be a little too large when positive, and too small when negative. It would be possible to state a formula for such cases, but the bending moment for each particular case can be readily computed by applying the principles of moments.

The negative bending moment $-M$ at either support will be given by the formula

$$-M = \frac{Wp^2}{2b}, \qquad \textbf{(219.)}$$

in which p may represent either p or p' of Fig. 329, for this formula applies whether p and p' be equal or unequal.

In our case the bending moment at the center of the shoe is, by formula **218,** $\frac{51,400}{4} \times \left(10.5 - \frac{17.5}{2}\right) = 22,488$ inch-pounds. The bending moment at either support is (formula **219**) $-\frac{51,400 \times 3.5^2}{2 \times 17.5} = -17,990$ inch-pounds.

1631. Thickness of Shoe Plate.—We have seen (Art. **1623**) that the moment of resistance of a beam of rectangular section is $R = \frac{Sbd^2}{6}$; whence,

$$d = \sqrt{\frac{6R}{Sb}}. \qquad \textbf{(220.)}$$

In the present case the length of the shoe plate, corresponding to the total width of the beam, is 13 inches. But, as shown in Fig. 5 of the plate just referred to, the section at the center of the shoe plate of the expansion shoe (the position of maximum bending moment) is reduced by three rivet holes.

By considering the diameter of each rivet hole to be $\frac{1}{8}$ of an inch greater than the diameter of the undriven rivet, the width to be deducted for the three rivet holes is $3 \times \frac{3}{4} = 2\frac{1}{4}$ inches, making the net length of the shoe plate, acting as the effective width of the beam, equal to $13 - 2\frac{1}{4} = 10\frac{3}{4}$ inches $= b$. As the load is somewhat distributed, and the shoe plate materially supported and strengthened by the horizontal legs of the angles forming the standards, the value of S may be taken at 18,000 pounds, the same as for pins. By substituting the values of R, S, and b in formula **220**, we get, for the thickness of the shoe plate, $d = \sqrt{\frac{6 \times 22{,}800}{18{,}000 \times 10.75}} = .84$, or, nearly, $\frac{7}{8}$ of an inch.

There are no rivet holes to be deducted from the section at the center of the shoe plate of the anchored shoe, and, consequently, the value of b for this shoe is 13 inches, and $d = \sqrt{\frac{6 \times 22{,}800}{18{,}000 \times 13}} = .76$, or, practically, $\frac{3}{4}$ of an inch. A thickness of $\frac{7}{8}$ of an inch, however, is used for the shoe plates of both the roller and the anchored shoes. It is to be noticed that, for the roller shoe, the rollers and bed-plate may be considered to act somewhat as beams, thus assisting the shoe plate sufficiently to make up for the section cut out by the rivet holes. This being the case, a thickness of $\frac{3}{4}''$ would probably be sufficient for the shoe plates of both anchored and roller shoes.

The standards or ribs of the shoe must be given such form as will clear the upper flanges of the channels of the end post. For bridges of moderate span the form of the standards is commonly made symmetrical with reference to a vertical line through the center of the pin hole. In many shoes the horizontal legs of the angles forming the standards are turned outwards, and the smaller angles (in the present case, $3'' \times 3'' \times \frac{3}{8}''$ angles), connecting the vertical legs of the larger angles to the shoe plate, are omitted. Sometimes, also, each standard consists simply of a vertical plate, connected to the shoe plate by a single angle. The standards shown in Plate, Title: Highway Bridge; Details III, Figs. 5 and 9, are very substantial. The two shoes there shown are alike, with the exception of the small guide bar on the bottom and the connection for the shoe strut on the roller shoe, and the holes for anchor bolts in the anchor shoe.

1632. Another Form of Shoe.—Although the form of shoe shown in the plate just referred to is commonly used for bridges of moderate span, other forms are often used. A form of shoe similar to that shown in Fig. 330 is used for long and heavy spans. A side elevation and rear elevation of the shoe are shown, and also a side elevation of the lower end of the end post, with a section on the center line of the same.

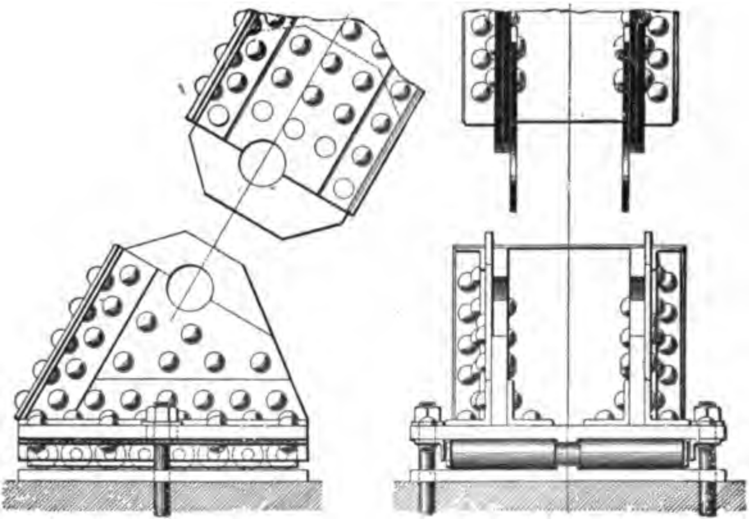


FIG. 330.

In this shoe the bearings of the end post upon the pin are so nearly opposite those of the shoe standards as to produce but a comparatively small amount of bending moment in the pin. The standards of the shoe are supported laterally by a plate corresponding to the cover-plate of the end post. The rollers are held in position by angles attached to the *shoe* plate, thus allowing the accumulations of dirt on the bed-plate to be cleaned out between the rollers. The anchorage is given by anchor bolts passing through slotted holes in the shoe plate.

1633. The Bed-Plate.—The bed-plate is shown in Plate, Title: Highway Bridge: Details III, Fig. 8. Item (v_1) of the specifications (Art. **1511**) requires all bed-plates to be of such dimensions that the greatest pressure upon the masonry shall not exceed 250 pounds per square inch. This condition, together with the dimension of the shoe and rollers, determines the size of the bed-plate. In the present case, as determined for the shoe, the total load upon the bed-plate is 51,400 pounds, and, consequently, the bed-plate must have a bearing surface upon the masonry of $\frac{51,400}{250} = 205.6$ square inches. The bed-plate must be as long as the shoe plate and should generally be somewhat longer; in the present case it is made 14 inches long. Hence, the width of the bed-plate necessary to give the required bearing upon the masonry is $\frac{205.6}{14} = 14\frac{3}{4}$ inches, nearly.

Item (u_1) of the specifications requires that "while the roller ends of all trusses must be free to move longitudinally under changes of temperature, they shall be anchored against lifting or moving sideways;" consequently, at the sides of the bed-plate, outside of the rollers and shoe, there must be upright pieces, the tops of which extend inwards over the edges of the shoe plate, or some equivalent arrangement, to prevent the shoe from lateral movement or lifting. As these side pieces can be most advantageously formed of pieces of angles or **Z** bars (also written zee bars), of which the horizontal legs attaching to the bed-plate must be turned outwards, the bed-plate will usually be required to be about 6 inches wider than the shoe plate. In the present case the bed-plate is made 24 inches wide. Its bearing surface upon the masonry is, therefore, greatly in excess of the bearing surface required.

1634. By reference to Figs. 7 and 8 of the same plate (III), it will be seen that each end of the bed-plate projects $\frac{24 - 16\frac{3}{8}}{2} = 3\frac{13}{16}$, or, near enough for present purposes, 3.8 inches beyond each end of the rollers. The pressure upon the bottom surface of these projecting ends produces a bending moment in the bed-plate at the ends of the rollers, whose value is $\frac{51{,}400 \times 3.8^2}{2 \times 24} = 15{,}463$ inch-pounds (formula **219**). The thickness of the bed-plate must, therefore (formula **220**), be $\sqrt{\frac{6 \times 15{,}463}{18{,}000 \times 14}} = .61$ of an inch, or say $\frac{5}{8}$ of an inch.

This assumes the pressure upon the bottom of the bed-plate to be uniformly distributed over its surface. But, as determined above, the limit of the pressure allowed upon the masonry requires a width of only $14\frac{3}{4}$ inches for the bed-plate, which is less than the bearing length of the rollers. Hence, the entire pressure upon the bottom of the bed-plate may be considered to be applied upon that portion of its width between the ends of the rollers, assuming no pressure upon the projecting ends, in which case there will be no bending moment.

As shown in the figure, a piece of 3″ × 4″ × 3″ × $\frac{5}{8}$″ **Z** bar is riveted upon one side of the bed-plate in such manner that the upper horizontal leg of the bar will extend inwards over the edge of the shoe plate. This leg of the bar is cut off sufficiently to clear the heads of the rivets in the shoe plate. On the opposite side of the bed-plate, a 3″ × 3″ × $\frac{3}{8}$″ angle is used with a small piece of 3″ × 2″ angle riveted to one end in such manner that one leg of the latter will extend over the edge of the shoe plate. The piece of 3″ × 2″ angle can not be riveted to the opposite end of the 3″ × 3″ angle because it would so interfere with the connection of the shoe strut as to prevent the shoe being placed in position upon the bed-plate. Consequently, at one end of the 3″ × 3″ angle, a small bent anchor plate is made to serve the purpose of anchorage against lifting. This plate connects upon the anchor bolt and is detachable; it is shown between Figs. 8 and 9, on Plate, Title: Highway Bridge: Details III.

1635. Anchor Bolts.—Item (v_1) of the specifications requires that for trusses the anchor bolts shall not be less than 1¼ inches in diameter. It is the general practice to use two anchor bolts to each anchored shoe and to each bed-plate, placing each bolt at the center of the side or flange, corresponding to the position of the anchor bolt in the outer (upper) side of the anchor shoe, Plate, Title: Highway Bridge: Details III, Fig. 9.

In the present case, however, it was found more convenient to use three 1-inch anchor bolts in each anchored shoe and in each bed-plate.

In fixing the sizes and positions of the holes through the shoes and bed-plates for the anchor bolts, liberal clearance should be allowed in the holes, and their positions should be such that the nuts will sufficiently clear the upright pieces. By reference to Table 42, Art. **1616,** it is found that, for a bolt 1 inch in diameter, the long diameter of a hexagon nut is 2.02 inches. Hence, the center of the bolt hole should be at least 1⅛ inches from the nearest vertical piece in the shoe or bed-plate. In the bed-plate (Fig. 8, same plate), the center of each hole for the anchor bolt is $1\frac{7}{8} - \frac{5}{8} = 1\frac{1}{4}$ inches from the nearest side of the vertical leg of the **Z** bar, and $1\frac{7}{8} - \frac{3}{8} - \frac{3}{8} = 1\frac{1}{8}$ inches from the vertical portion of the bent anchor plate. In the shoe, the centers of the holes for the anchor bolts are at a distance of $1\frac{3}{4} - \frac{3}{8} = 1\frac{3}{8}$ inches from the vertical leg of the 3″ × 3″ angle.

The anchor bolt is shown in Fig. 6, same plate. The thread end is the same as that of an ordinary bolt, but at the opposite end, instead of having a head, the end of the bolt is split to receive a small wedge, as shown. The anchor bolt is set in the following manner: A hole being drilled in the masonry, the wedge is started into the split in the bolt sufficiently to hold it in position, then the bolt is placed in the hole and driven down over the wedge, thus forcing open the split and spreading the lower end of the bolt. The bolt being placed in its proper position, the hole is filled with melted sulphur or lead, or with neat Portland cement mortar, which, when it has hardened or set, firmly holds the bolt in place. The form of anchor bolt shown in Fig. 6 (same plate as above) is called a **wedge bolt.** It is the best form of anchor bolt, though not the cheapest.

In Fig. 331 are shown three other forms of anchor bolts commonly used for bridges. The form shown at *a* is called a **rag bolt,** or **swedged bolt,** and the same names are sometimes applied rather indiscriminately to the form shown at *b*, though this may be more properly called a **barbed bolt.** The form shown at *c* is simply a threaded bolt. These forms of anchor bolts are generally set into the masonry about 6 inches. If it is necessary for an anchor bolt to take direct tensile stress, as at the bottom of a high tower, it is usually made long with a head at the lower end carrying a large washer or plate, and is built into the masonry a sufficient depth so that the weight of the masonry above the head and plate will resist the tension upon the bolt.

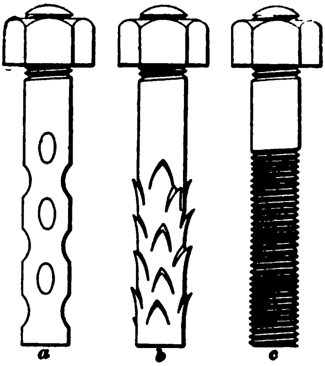


FIG. 331.

CHORD PINS, PIN NUTS, COTTER PINS, AND PIN WASHERS.

1636. The Chord Pins.—The chord pins are shown in Plate, Title: Highway Bridge: Details IV, Figs. 9, 10, and 11. The diameters required for the pins have been determined in previous articles, and only the required lengths of the pins and the dimensions of the thread ends and nuts remain to be determined.

1637. Grip of Pins.—In a chord pin, the length G, Fig. 332, is called the **grip** of the pin. The grip of a chord pin may be determined by the following rules:

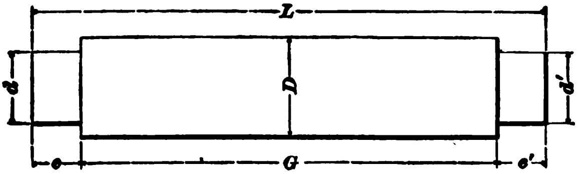


FIG. 332.

Rule I.—*If the outer dimensions of the member through which the pin passes are fixed, the grip will be equal to the net width of the member plus a constant c.*

II.—*If the pin passes through chord bars only, or chord bars packed outside of a riveted member, the grip will be equal to the net width plus $\frac{1}{16}$ of an inch for each chord bar, and plus $\frac{1}{8}$ of an inch for each riveted member; but the total amount added should never be less than the constant c.*

For the value of the constant c, the practice varies. The value of this constant, as determined by any one of the three methods indicated below, will be found satisfactory.

(*1*) *For all chord pins:*

$$c = \tfrac{1}{2}''. \qquad \textbf{(221.)}$$

(*2*) *For chord pins having diameters*

Less than 3 inches: $c = \frac{1}{4}''$. **(222.)**

From 3 to 6 inches: $c = \frac{1}{2}''$. **(223.)**

Greater than 6 inches: $c = \frac{3}{4}''$. **(224.)**

(*3*) *For chord pins of any diameter D:*

$$c = \frac{D}{16} + \tfrac{1}{4}''. \qquad \textbf{(225.)}$$

In using the last formula, the value of c may be taken to the nearest sixteenth of an inch.

In practice, however, when the grip of the pin, as estimated by any one of the preceding formulas in connection with either of the above rules, contains a fraction smaller than one-eighth of an inch, the fraction should be called one-eighth.

The amount added to the net width of the member or members, to obtain the grip of the pin, is to insure the finished portion of the pin passing entirely through the member, making allowance for probable variations in width. A perfect bearing is thus obtained for the pin, which is an essential condition. The practice indicated by formula **221** is the most simple, but that indicated by formula **225** is the most satisfactory. In determining the grip of the chord pin shown on Plate, Title: Highway Bridge: Details IV, formula **221** was used.

Thus, the distance out to out of the standards of the shoe (Plate, Title: Highway Bridge: Details III, Fig. 5) is $9\frac{7}{8} + 2 \times \frac{5}{8} = 11\frac{1}{8}$ inches. In compliance with rule **I**, using formula **221** for the value of c, the grip of the shoe pin is made equal to $11\frac{1}{8} + \frac{1}{2} = 11\frac{5}{8}$ inches. (Plate, Title: Highway Bridge: Details IV, Fig. 9.) The diameter of the pin is $2\frac{3}{4}$ inches. Hence, if the value of c were taken according to formula **222,** the grip of the pin would be $11\frac{1}{8} + \frac{1}{4} = 11\frac{3}{8}$ inches. If formula **225** were used, the value of c would be $\frac{2.75}{16} + \frac{1}{4} = \frac{7}{16}$ of an inch, and the estimated grip of the pin would be $11\frac{1}{8} + \frac{7}{16} = 11\frac{9}{16}$ inches, while the practical grip of the pin would be $11\frac{5}{8}$ inches.

1638. Screw Ends and Pilot Nuts.—As shown in Fig. 332, the screw ends of chord pins are turned down smaller than the main portion of the finished pin, in order that they may pass through the pin holes easily and without injury to the threads. This, of itself, would require the screw ends to be but slightly smaller than the body of the pin, but it has been found desirable to make each screw end of a pin enough smaller than the body, or finished portion, to permit a dome-shaped nut, finished to the same diameter as the pin, to be screwed upon the end. This nut serves to facilitate the process of entering and driving the pin through the various members which it connects, and also to protect the threads of the pin during the process. Such a nut is known as a **pilot nut** or **pin pilot.** A pilot nut is shown in Fig. 333, from which its form will be readily understood.

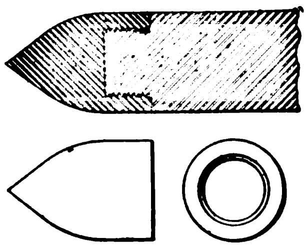


FIG. 333.

For the dimensions of the screw ends of chord pins, as relating to the finished sizes of the pins, the practice varies considerably. As a rule, each bridge office adopts and uses such dimensions of screw ends as satisfactorily fulfil the requirements of its shop practice, and thus forms a standard of its own. The dimensions of screw ends given by the following formulas, however, will be found satisfactory; they probably represent the average practice.

Diameter of screw end:

$$d = \frac{3}{4} D. \qquad \textbf{(226.)}$$

Length of screw end:

$$e = \frac{d}{8} + 1''. \qquad \textbf{(227.)}$$

In these formulas D is the diameter of the finished pin; d is the diameter of the screw end to the nearest eighth of an inch, and e is the length of the screw end to the nearest sixteenth of an inch. (See Fig. 332.)

Thus, for the shoe pin (Plate, Title: Highway Bridge: Details IV, Fig. 9), the diameter d of the screw end is $\frac{3}{4} \times 2\frac{3}{4} = \frac{33}{16}$, or, practically, 2 inches. The length e of the same is $\frac{2}{8} + 1 = 1\frac{1}{4}$ inches.

1639. The Diameters of Finished Pins.—It will be well to notice here the practice with regard to the diameters of finished pins, which varies in different shops. Round bars of diameters greater than $1\frac{9}{16}$ inches are not commonly rolled in sixteenths. As rolled, the diameters of round iron greater than $1\frac{9}{16}$ inches are multiples of $\frac{1}{8}$ of an inch.

In finishing the pin, it is the practice of some shops to turn off $\frac{1}{8}$ of an inch, i. e., to make the diameter of the finished pin $\frac{1}{8}$ of an inch less than the diameter of the rolled bar, while in other shops the practice is to turn off only $\frac{1}{16}$ of an inch in finishing the pin. The former practice gives excellent results, but is not economical. The latter practice is economical, but it does not in every case produce a well-finished pin.

It is evident that by the former practice the diameter of the finished pin, if greater than $1\frac{7}{16}$ inches, will always be a multiple of $\frac{1}{8}$ of an inch, while by the latter practice the diameter of the finished pin, if greater than $1\frac{1}{2}$ inches, will always be in odd sixteenths of an inch.

Most bridge offices adopt a standard minimum size of pins, and use no pins for any purpose having diameters less than the minimum size adopted; $1\frac{3}{4}$ inches is a standard minimum size often used for the diameters of pins, and will be adopted in this Course.

1640. The Pin Nuts.—As now quite generally made, pin nuts have recesses on the inner sides to admit the end or shoulder of the finished portion of the pin. This form of pin nut, known as a **recessed nut,** is shown in Fig. 334. Before recessed nuts came into use, washers fitting over the ends of the pins were used in connection with flat nuts; they are still used, but not extensively. The first recessed nut which appeared on the market was a pressed wrought-iron nut, known as the **Lomas nut.** This was a patented nut; it was somewhat similar in form to the nut shown in Fig. 334, and was at one time widely used. The name *Lomas nut* is sometimes indiscriminately applied to any recessed nut. A malleable iron recessed nut, which seems to be satisfactory, is now much used. Its

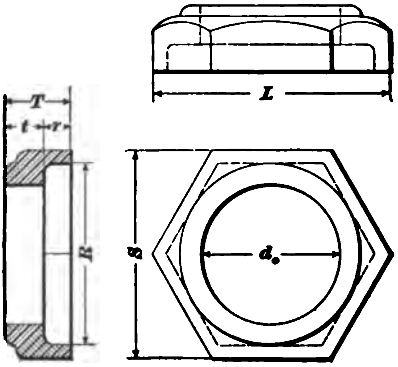


FIG. 334.

general form is shown in Fig. 334. It is unnecessary to give here the exact dimensions of this nut, which vary somewhat irregularly in the different sizes, but for convenience the following approximate dimensions may be given with reference to D, the diameter of the finished pin. See the figure.

Diameter d_0 of rough hole for screw:

$$^{*}d_0 = \frac{3}{4}D - \frac{3''}{16}. \qquad \textbf{(228.)}$$

Diameter R of recess:

$$R = D + \frac{1''}{8}. \qquad \textbf{(229.)}$$

Short diameter S of nut:

$$S = 1\tfrac{1}{16}D + \frac{3''}{8}. \qquad \textbf{(230.)}$$

Long diameter L of nut:

$$L = 1.155\,S. \qquad \textbf{(231.)}$$

Length t of thread:

$$t = \frac{3}{32}D + \frac{1''}{2}. \qquad \textbf{(232.)}$$

Depth r of recess:

$$r = \frac{D}{16} + \frac{5''}{16}. \qquad \textbf{(233.)}$$

Total thickness T of nut:

$$T = t + r = \frac{5}{32}D + \frac{13''}{16}. \qquad \textbf{(234.)}$$

In applying formulas **226** to **234,** inclusive, the diameter D of the pin should contain no fraction smaller than ¼ inch; if the actual diameter of the pin contains a smaller fraction, the full quarter inch next above should be used.

Values from these formulas will be obtained to the nearest sixteenth of an inch, except formulas **226** and **228,** for which the values will be obtained to the nearest eighth of an inch. It will be noticed that the length of the thread, as given by formula **232,** will be in all cases $\frac{1}{2}$ inch less than the length of the screw end of the pin, as given by formula **227;** which may be expressed in the form $t = \frac{d}{8} + \frac{1}{2}''$.

---

* This is correct for 7 threads to the inch, and may be considered near enough for 8 and 6 threads to the inch, which are the sizes of threads commonly used for pin nuts.

It is not necessary to give the dimensions of pin nuts on shop drawings (or shop lists); it is usually sufficient to give the required dimensions of the pins and state the kind of nut required.

1641. Cotter Pins.—Pins of the form shown in Plate, Title: Highway Bridge: Details IV, Fig. 12, are called **cotter pins.** They are often used for lateral pins and for certain other purposes, but are not commonly used when the diameter of the pin is greater than about 3 inches. In general form, a cotter pin is somewhat similar to a bolt. It usually has a round (cylindrical) head, and at its opposite end, instead of having threads cut, it has a hole drilled to receive a small split pin formed by bending a piece of half round iron, so the flat sides will be in contact.

The split pin so formed is sometimes called a **key,** but it is more commonly known as a **cotter.** The end of a cotter pin is tapered slightly in order that it may be more easily inserted into the pin hole.

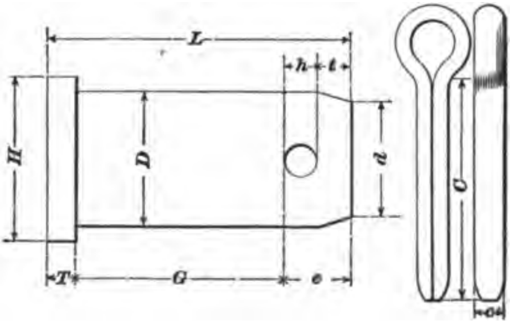


FIG. 335.

A cotter pin is shown in Fig. 335. The relative dimensions of cotter pins given by the following formulas will be in accordance with good practice, each result being taken to nearest sixteenth of an inch. See the figure.

Thickness T of head:

$$T = \frac{D}{8} + \frac{1}{8}''. \qquad \textbf{(235.)}$$

Diameter H of head:

$$H = D + T. \qquad \textbf{(236.)}$$

Length C of cotter:

$$C = 1\tfrac{1}{2}D + \frac{1}{4}''. \qquad \textbf{(237.)}$$

Diameter or width c of cotter:

$$c = T. \qquad \textbf{(238.)}$$

Diameter h of cotter hole:

$$h = c + \frac{1}{16}''. \qquad \textbf{(239.)}$$

Diameter d of end:

$$d = \frac{7}{8}D. \qquad \textbf{(240.)}$$

Length e of end:

$$e = \frac{3}{8}D + \frac{1}{4}''. \qquad \textbf{(241.)}$$

Length t of taper:

$$t = \frac{1}{2}e. \qquad \textbf{(242.)}$$

Total length L of pin:

$$L = G + T + e. \qquad \textbf{(243.)}$$

In these formulas D is the diameter of the finished pin.

1642. The grip of a cotter pin is determined substantially the same as for a chord pin, but formula **222** should generally be used for the value of the constant.

The grip of a cotter pin should always be given from the under side of the head to the inner (or nearest) edge of the cotter hole, as shown at G in Fig. 335.

The preceding formulas represent practical and economical dimensions for cotter pins, but as the diameter and grip are the only really important dimensions of a cotter pin, slight variations in other dimensions are not of consequence.

EXAMPLE.—For a cotter pin having a diameter of $1\frac{3}{4}$ inches, and a net grip of $2\frac{1}{2}$ inches, what are the remaining dimensions, as given by the preceding formulas?

SOLUTION.—Thickness of head, $T = \frac{1\frac{3}{4}}{8} + \frac{1}{8} = \frac{3}{8}''$.

Diameter of head, $H = 1\frac{3}{4} + \frac{3}{8} = 2\frac{1}{8}''$.
Length of cotter, $C = 1\frac{1}{2} \times 1\frac{3}{4} + \frac{1}{4} = 2\frac{7}{8}''$.
Diameter of cotter, $c = T = \frac{3}{8}''$.
Diameter of cotter hole, $h = \frac{3}{8} + \frac{1}{16} = \frac{7}{16}''$.
Diameter of end, $d = \frac{7}{8} \times 1\frac{3}{4} = 1\frac{1}{2}''$.
Length of end, $e = \frac{3}{8} \times 1\frac{3}{4} + \frac{1}{4} = \frac{7}{8}''$.
Length of taper, $t = \frac{1}{2} \times \frac{7}{8} = \frac{7}{16}''$.
Length of grip, $G = 2\frac{1}{2} + \frac{1}{4} = 2\frac{3}{4}''$.
Total length of pin, $L = 2\frac{3}{4} + \frac{3}{8} + \frac{7}{8} = 4''$.

1643. Pin Washers.—In Plate, Title: Highway Bridge: Details III, Figs. 10 to 13, are shown small rings of sizes to fit loosely upon the pins. They are used to fill the vacant places between the various members connecting upon certain pins, and serve to hold the members in their proper positions. These rings are known variously as **pin washers, packing rings,** and **filler rings.** They are, as a rule, formed by bending strips of metal into circular form so that the inside diameters will be about $\frac{1}{8}$ of an inch greater than the diameter of the pins. The metal used is from $\frac{3}{16}$ to $\frac{1}{2}$ inch thick.

In determining the lengths of pin washers, sufficient clearance must be given, making ample allowance for variations in the thickness of the members. If this is not done, and the pin washers do not fit readily into place, they are very liable to be omitted in the process of erecting the bridge. In determining the lengths of the pin washers.

practically the same allowances are made as in rule II, Art. **1637,** for determining the grip of the pin.

EXAMPLE.—What should be the length of the pin washer for the hip joint of the bridge shown on the Mechanical Drawing plates?

SOLUTION.—The distance between the inner surfaces of the pin plates on the end post at the hip joint is $7\frac{3}{8} - 2 \times \frac{7}{16} = 6\frac{1}{2}$ inches. See Plate, Title: Highway Bridge: Details I, Fig. 1. By allowing $\frac{1}{4}$ of an inch on each side for the riveted member and $\frac{1}{16}$ of an inch for each forged bar, the length of the pin washer will be $6\frac{1}{2} - 2 \times \frac{1}{4} - 4 \times \frac{7}{8} - 4 \times \frac{1}{16} = 2\frac{1}{4}$ inches. Ans. See Plate, Title: Highway Bridge: Details III, Fig. 10.

1644. The Shop Drawings.—The details of the members and their various connections, as shown in Plates, Titles: Highway Bridge: Details I, II, III, together with the calculations necessary for proportioning the same, have now all been explained. In the order of procedure for the construction of the bridge, the shop drawings may be considered to be completed.

It will be well, however, to notice that, in many cases, sizes and dimensions which, in the plates referred to, are shown marked as dimensions, are usually simply stated in actual working drawings. The sizes of all plates, bars, and shape iron should be distinctly stated upon the drawings, and such dimensions as will be clearly understood from such statement may be omitted. Thus, for the main tie bar, Plate, title as above, IV, Fig. 1, the marked dimensions 2", $\frac{13}{16}$", $5\frac{7}{8}$", 6", and $\frac{7}{8}$" would usually be omitted in the *drawing*, but under the drawing would be written:

Main Tie Bar, $2'' \times \frac{13}{16}''$; *Heads*, $5\frac{7}{8}'' \times \frac{7}{8}''$, 8 *required.*

This statement gives all necessary information concerning the main tie bar, except the sizes of the pin holes and the length from center to center of the same.

Shop drawings, and, indeed, nearly all bridge drawings are made upon tracing muslin, in order that they may be blue-printed. The drawing is usually made in pencil on detail paper, then traced and finished in ink on the tracing muslin. As many blue-prints should be taken as are necessary for the office and the different shops.

BILLS.

SHOP LISTS.

1645. What is stated above concerning the main tie bar is true also of many other members; in fact, it is true of nearly, if not quite, all forged members. Therefore, instead of showing such members upon the shop drawings, it is the practice in most bridge offices to state the necessary dimensions upon certain blank forms or sheets, thus conveying the same information with much less labor. Such sheets are commonly known as **shop lists.** The following will serve to illustrate the general forms of shop lists. Two somewhat different forms are given for eyebars.

It will be noticed that lists (*A*) to (*E*), as filled out, give all necessary information concerning the forged members shown in Plate, Title: Highway Bridge: Details IV, Figs. 1 to 6. In these lists are given in each case, not only the dimensions of the finished bar, but in the column headed *Ordered Length* is also given the length of the rolled bar necessary to give the required finished dimensions.

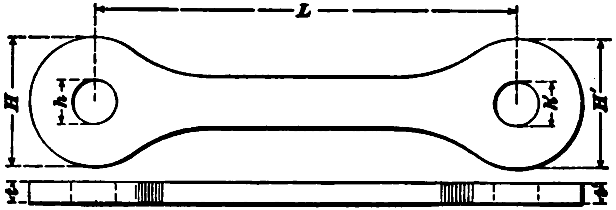
This feature of the shop lists will be more fully explained in the following articles. For the student's convenience, the figure to which each item refers is designated in the left-hand column. Lists (*A*) and (*B*) are simply two different forms which serve the same purpose.

It will be noticed that, in stating the lengths back to end in list (*D*), 4 inches are allowed for the distance between the two screw ends, according to the practice indicated in

(*A*)

FORGE LIST. **EYE-BARS.**

Material, *Wrought Iron.*



Contract No. *578.* Shop No. *735.* Span *90′ 0″.* Sheet No. *1.*

Mark.	Number Required.	Size of Bars.	*H*	*t*	*h*	*L*	*h'*	*t'*	*H'*	Ordered Length.
B c (Fig. 1)	8	2″ × 13/16″	5 7/8″	7/8″	2 25/32″	25′ 5 9/16″	2 25/32″	7/8″	5 7/8″	O, 25′ 9 1/2″

(*B*)

FORGE LIST. **EYE-BARS.**

Material, *Wrought Iron.*

Contract No. *578.* Shop No. *735.* Span *90′ 0″.* Sheet No. *1.*

Mark.	Number Required.	Size of Bar, Inches.	Size of Head, Inches.	Diameter of Pin, Inches.	Length Center to Center of Pin Holes.		Diameter of Pin, Inches.	Size of Head, Inches.	Ordered Length.	
					Feet.	Inches.			Feet.	Inches.
Bc (Fig. 1)	8	$2 \times \frac{13}{16}$	$5\frac{7}{8} \times \frac{7}{8}$	$2\frac{3}{4}$	25	$5\frac{9}{16}$	$2\frac{3}{4}$	$5\frac{7}{8} \times \frac{7}{8}$	O, 25	$9\frac{1}{2}$

(*C*)

FORGE LIST. **BARS WITH WELDED LOOPS.**

Material, *Wrought Iron.*

Contract No. *578.* Shop No. *735.* Span *90′ 0″.* Sheet No. *2.*

Mark.	Number Required.	Size of Bar.	Length of Loop.	Diameter of Pin.	Length Back to Back.	Diameter of Pin.	Length of Loop.	Ordered Length.
Bb (Fig. 3)	8	1″ × 1″	$6\frac{7}{8}''$	$2\frac{3}{4}''$	Ream to $18'\ 2\frac{23}{32}''$	$2\frac{3}{4}''$	$6\frac{7}{8}''$	O, $20'\ 4\frac{3}{4}''$

(*D*)

FORGE LIST. **BARS WITH WELDED LOOPS AND TURNBUCKLES.**

Material, *Wrought Iron.*

Contract No. *578.* Shop No. *735.* Span *90′ 0″.* Sheet No. *3.*

Mark.	Number Required.	Size of Bar.	Length of Loop.	Diameter of Pin.	Length Back of Loop to End of Upset.	Diameter of Upset.	Length of Upset.	Ordered Length.	Length of Turn-buckle.
Cc'	8	$\frac{7}{8}'' \times \frac{7}{8}''$	$4\frac{3}{8}''$	$1\frac{3}{4}''$	$20'\ 3\frac{13}{16}''$	$1\frac{3}{8}''$	$4\frac{1}{4}''$	S, $21'\ 6\frac{1}{4}''$	$9\frac{3}{4}''$
Cc'	8	$\frac{7}{8}'' \times \frac{7}{8}''$	* $6\frac{7}{8}''$	$2\frac{3}{4}''$	$5'\ 0''$	$1\frac{3}{8}''$	$4\frac{1}{4}''$	S, $6'\ 5\frac{1}{2}''$	
(Fig. 2)									
ab_1	4	$1\frac{1}{8}''$ ○	$4\frac{3}{8}''$	$1\frac{3}{4}''$	$19'\ 8\frac{1}{4}''$	$1\frac{1}{2}''$	$4\frac{1}{2}''$	O, $20'\ 11''$	$10''$
ab_1	4	$1\frac{1}{8}''$ ○	$4\frac{3}{8}''$	$1\frac{3}{4}''$	$5'\ 0''$	$1\frac{1}{2}''$	$4\frac{1}{2}''$	O, $6'\ 2\frac{3}{4}''$	
(Fig. 5)									
bc_1	6	$1''$ ○	$4\frac{3}{8}''$	$1\frac{3}{4}''$	$19'\ 7\frac{1}{4}''$	$1\frac{3}{8}''$	$4\frac{1}{4}''$	S, $20'\ 9\frac{3}{4}''$	$9\frac{3}{4}''$
bc_1	6	$1''$ ○	$4\frac{3}{8}''$	$1\frac{3}{4}''$	$5'\ 0''$	$1\frac{3}{8}''$	$4\frac{1}{4}''$	S, $6'\ 2\frac{1}{2}''$	
(Fig. 6)									

* Flatten to $\frac{3}{4}''$ thick.

(*E*)

FORGE LIST. **BARS WITH UPSET SCREW ENDS.**

Material, *Wrought Iron.*

Contract No. *578.* Shop No. *735.* Span *90′ 0″.* Sheet No. *4.*

Mark.	Number Required.	Size of Bar.	Length of Upset.	Diameter of Upset.	Length of Bar End to End.	Diameter of Upset.	Length of Upset.	Ordered Length.	Nuts.
$B\ C_1$	4	1″ O	4¼″	1⅜″	25′ 10⅝″	1⅜″	4¼″	26′ 7⅝″	Stand.
$C\ C_2$ (Fig. 4)	2	1″ O	4¼″	1⅜″	26′ 2½″	1⅜″	4¼″	26′ 11½″	Stand.

Art. **1612.** The lengths of upset ends given in lists (*D*) and (*E*) are also in accordance with the practice there indicated.

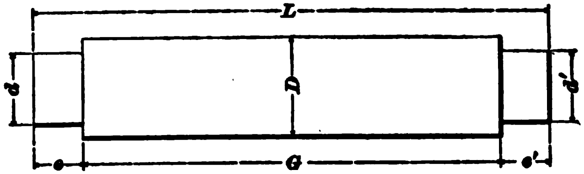
For the details of pin-connected members, the practice in some bridge offices is to indicate the size of each pin hole by stating the size of the *pin*, allowance for the clearance of the pin hole being made in the shop. In other bridge offices the practice is to state the exact diameter actually required for each pin *hole*. Each practice has its advantages.

Either system will obtain the required results if consistently followed, but the latter practice is to be preferred. On the Mechanical Drawing plates and in the shop lists given above, the former practice has been followed, except in list (*A*), in which the diameter of the pin hole is stated.

In lists (*F*) and (*G*) is given all necessary information concerning the pins shown in Plate, Title: Highway Bridge: Details IV, Figs. 9 to 12. In list (*F*) the lengths of all screw ends are given according to formula **227.** In list (*G*), all dimensions of

(*F*)

MACHINE LIST. **CHORD PINS.**

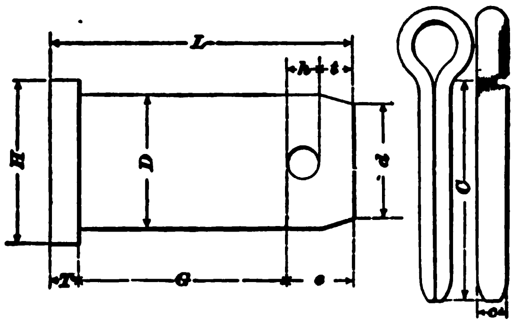


Contract No. *578.* Shop No. *735.* Span *90′ 0″.* Sheet No. *5.*

Mark.	Material.	Number Required.	D	d	e	G	e'	d'	L	Ordered.		Nuts.	Washers.	
										Diam.	Length.		No. Required.	Length
a (Fig. 9.)	M. Steel	4	$2\frac{3}{4}''$	$2''$	$1\frac{1}{4}''$	$11\frac{5}{8}''$	$1\frac{1}{4}''$	$2''$	$14\frac{1}{8}''$	$2\frac{7}{8}''$	O, $17'\,1\frac{3}{8}''$	Rec.	0	
B, *b*, *c* (Fig. 10)	M. Steel	12	$2\frac{3}{4}''$	$2''$	$1\frac{1}{4}''$	$9\frac{3}{4}''$	$1\frac{1}{4}''$	$2''$	$12\frac{1}{4}''$	$2\frac{7}{8}''$	O, $17'\,1\frac{3}{8}''$	Rec.	4 8 8	$2\frac{1}{4}''$ $1\frac{1}{4}''$ $\frac{3}{16}''$
C (Fig. 11)	Iron	4	$1\frac{3}{4}''$	$1\frac{1}{4}''$	$1\frac{1}{8}''$	$9\frac{1}{8}''$	$1\frac{1}{8}''$	$1\frac{1}{4}''$	$11\frac{3}{8}''$	$1\frac{7}{8}''$	O, $10'\,8\frac{3}{8}''$	Rec.	4	$4\frac{1}{4}''$

(*G*)

MACHINE LIST. **COTTER PIN.**



Contract No. *578*. Shop No. *735*. Span *90′ 0″*. Sheet No. *6*.

Mark.	Material.	Num. Required.	*D*	*H*	*T*	*G*	*e*	*d*	*t*	*L*	*h*	*c*	*C*	Ordered.	
														Diam.	Length.
(Fig. 12)	Iron	20	1¾″	2⅛″	⅜″	2¾″	⅞″	1½″	7/16″	4″	7/16″			1⅞″	O, 10′ 8⅜″
(Fig. 6)												⅜″	2⅞″	⅜″	S
Anchor Bolts	Iron	12	1″							10″					

the cotter pin are according to the formulas given in Art. **1641.** For the details of pins, the correct diameters of the finished pins are always stated.

The forms of shop lists, as used in the different bridge offices, of course vary somewhat, but the forms given serve to illustrate the general system. The blank forms for shop lists are generally printed upon tracing muslin or upon a thin bond paper which will blue-print.

IRON ORDER.

1646. General Method of Procedure.—After the shop drawings are completed, the next step in the process of the construction is to make out the iron order. The companies who manufacture bridges do not usually roll the iron themselves. A few of the larger bridge companies own both rolling mills and construction shops, but even in such cases the management of each is generally separate and distinct from the other.

The greater portion of the material for each bridge is ordered from the rolling mill in the exact lengths required, making due allowances for the requirements of manufacture. There is, practically, no waste in such ordered material, beyond that necessary to finish it properly.

1647. Certain common sizes of channels, angles, plates, and bars are carried in stock by most bridge works. Stock iron is ordered in lengths of 30 feet. For iron rolled in lengths greater than 30 feet the prices per pound are somewhat higher. This fact should be borne in mind in making the design for a metal structure; the lengths used should generally not be greater than 30 feet. A list of the material carried in stock, called the **stock list,** should always be kept in the office, and the material checked off as used.

From the shop drawings, a list of the material required to make the various members of the structure is made out. In this list, the material is taken in substantially the same order as it appears upon the shop drawings. This list is known as an **order list.**

The order list is then carefully scrutinized and compared with the stock list, and such sizes as are carried in stock are marked to be taken from stock. When this is done, a second list is made, in which the remaining pieces, not carried in stock, are grouped and classified, the various pieces of each size and shape being assembled together. In this classification, the channels are usually written first, followed by the angles or other shape iron, then by the plates, flat bars, square bars, and round bars, respectively. In this list, also, the lengths of short pieces of plates or bars of the same size, such as pin plates, batten plates, etc., are usually added together, and the aggregate length given in one or more pieces. In such case, the length of each piece thus formed must not be more than 30 feet. This classified list of the material to be ordered is called the **iron order.** It is made out by the engineering department of the bridge works, and then transmitted to the business department, by whom the actual order is placed with the rolling mill.

1648. Ordered Lengths of Material.—In making out the order list for the material for a bridge, certain rules must be observed with reference to the ordered lengths of the material.

Pieces of plates, bars, angles, or other shapes, which are not required to be finished to exact lengths, or to be upset, welded, or forged, are ordered in neat lengths, that is, in the lengths actually shown on the drawings, without any allowance for extra length. When a number of short pieces of the same size are required, however, a single plate, having a length equal to the aggregate length of all such plates, is usually ordered.

When short pieces of plates are to be cut diagonally, or on a bevel, they are usually ordered in such rectangular lengths as will best cut to the required dimensions.

1649. If one or both ends of a rolled or built member are to be planed, in ordering the material a certain amount must be added to its finished length, to provide for the material taken off in planing. This amount varies somewhat with the form of the planer used; but with the machines commonly used in bridge shops, the amount to be added *for each finished end* should generally be as follows:

If the greatest dimension (width) of the member does not exceed 6 inches, add $\frac{1}{8}$ *of an inch.*

If the greatest width of the member exceeds 6 inches, add $\frac{1}{16}$ *of an inch for each additional 6 inches width, or fraction thereof.*

This may also be expressed by the following formula:

$$a = \frac{1}{16} \times \frac{b}{6} + c, \qquad \textbf{(244.)}$$

in which a is the length to be added for each finished end, b is the width of the member in inches, and c is a constant which, for most planers, may be taken at $\frac{1}{16}$ of an inch. In this formula, any fraction in the value of the expression $\frac{b}{6}$ should be considered as unity.

1650. In ordering the material for eye-bars, upon which the heads are to be formed by piling and forging, the length of bar required beyond the center of the pin hole is given for *one* head by the formula

$$a = \frac{H}{2} - \left(\frac{w}{8} + c\right), \qquad \textbf{(245.)}$$

in which a is the additional length of bar beyond the center of the pin hole, H is the diameter of the head, w is the width of the bar, and c is a constant, which may usually be taken equal to $\frac{3}{4}$ of an inch. This formula applies generally to wrought-iron eye-bars.

The additional length a of the bar beyond the center of the pin hole, required to form one eye-bar head by upsetting and forging, is given by the formula

$$a = 3\,(H - w) + c, \qquad \textbf{(246.)}$$

in which H is the diameter of the head, w is the width of the bar, and c is a constant, which must be obtained from the practice of each shop, but may usually be taken equal to

$3 - \frac{w}{4}$. This formula applies to steel eye-bars as generally manufactured.

The additional length a beyond the center of the pin, necessary to form one welded loop upon the end of a round or square bar, is given by the formula

$$a = 3.57\,D + 4.57\,d + c, \qquad \textbf{(247.)}$$

in which D is the diameter of the pin, d is the diameter of the round, or the side of the square bar, and c is a constant, which will here be taken simply equal to zero.

1651. The additional length u necessary to form one upset upon the end of a round bar is given by the formula

$$u = \left(\frac{D^2}{d^2} - 1\right)E + c, \qquad \textbf{(248.)}$$

in which D is the diameter and E the length of the upset end, d is the diameter of the original bar, and c is a constant, which may usually be taken at ¾ of an inch.

The additional length u necessary to form one upset upon the end of a square bar is given approximately by the formula

$$u = .785\,E\left(\frac{D^2}{S^2} - 1.27\right) + c, \qquad \textbf{(249.)}$$

in which S is the side of the square bar, D and E have the same values as in formula **248**, and c is a constant, which may usually be taken as 1 inch.

Results obtained by formula **249** are not generally as satisfactory as those obtained by formula **248**. It is in all cases roughly approximate to take the length required to make each upset end as equal to the length of the upset.

1652. If ordered cut to the required length, the ordered length of a chord pin may be the same as its finished length, as obtained by formula **227** and the preceding rules. This length will be given by the formula

$$L = G + \frac{3}{16}D + c, \qquad \textbf{(250.)}$$

in which L is the length, G is the total calculated grip, and D is the diameter of the pin, while c is a constant, which, for lengths of screw ends as given by formula **227**, will be equal to 2 inches. If all pins of the same size are ordered in one piece of a length sufficient to be cut into the required lengths, the value of c will be $\frac{1}{4}$ of an inch greater, as will be hereafter noticed.

The ordered diameter of a chord pin greater than $1\frac{9}{16}$ inches must always be a multiple of $\frac{1}{8}$ of an inch. The ordered diameter will be at the full eighth of an inch greater than the diameter required for the finished pin. (See Art. **1639.**)

1653. Rivets are made with one head; the second head is formed in driving. For the proportions of rivet heads given in Mechanical Drawing, the length of the rivet before driving, allowing sufficient additional length for forming the head in driving, should be :

For round heads:

$$l = 1\tfrac{1}{16}G + 1\tfrac{1}{4}d + c. \qquad \textbf{(251.)}$$

For countersunk heads:

$$l = 1\tfrac{1}{16}G + \frac{d}{2} + c. \qquad \textbf{(252.)}$$

In both of these formulas, l is the length from the under side of the head to the end of the rivet before driving, d is the diameter, and G is the net grip of the rivet, while c is a constant which may generally be taken at $\frac{3}{8}$ of an inch.

These formulas apply to machine-driven rivets in punched (unreamed) holes, and will be found generally satisfactory. As the additional length of rivet required to make the head is affected by the power used in driving and by the size of the die or cup used to form the head, both of which vary in different shops, it will in some cases be found necessary to use a different value for c.

For hand-driven rivets, no satisfactory formula can be given. The additional length of rivet necessary to form the head will differ in hand riveting done by different men, and the proper amount to allow for this purpose can be learned only from experience.

Formulas **251** and **252** give fair approximations, and will be adopted here.

In stating the lengths of undriven rivets, fractions smaller than eighths of an inch are not commonly used.

1654. If a lattice bar is connected by a single rivet at each end, the total or ordered length L is given by the formula

$$L = l + w + c, \qquad \textbf{(253.)}$$

in which l is the length from center to center of rivets, w is the width of the bar, and c is a constant, which has usually different values for different widths of bar, and also in different shops. It will here be taken as $\frac{5}{8}$ of an inch. Liberal allowance should be made in ordering the material for lattice bars, as there is always more or less waste.

1655. In ordering the material for beam hangers, the length U of that portion of the hanger bar above the center of the pin may be found by the formula

$$U = 1.57\ (D + d), \qquad \textbf{(254.)}$$

in which D is the diameter of the pin, and d is the thickness of the bar.

1656. All the preceding formulas, except formula **254,** contain a constant c, which is in each case a quantity depending upon the methods and conditions of the shop work. It is, therefore, evident that this quantity, as used in any one of the formulas, will probably not have the same value when applied to the practice of different shops. The values here given are believed to represent a fair average of good practice.

For convenience in using formulas **245** to **252,** inclusive, the results given by each formula, as applied to the various sizes, should be tabulated. In actual practice it is essential to have the results arranged in convenient tables, so that, for any common size, the required length may be read at once from the table. In preparing each table, however, the value of the constant *c* should conform to the practice of the shop for which the tables are to be used.

1657. Order Lists.—Two examples of parts of order lists are given below. List (*H*) is the order list for the material required for the end post, Plate, Title: Highway Bridge: Details I, Fig. 1.

The lower end *a* of the end post is not planed; consequently, no allowance of extra length need be made for this end in ordering. The upper end *B* of the end post is planed on a bevel. In such cases, the amount given by formula **244** is usually added to the length of the cover-plate for finishing the upper end; it is not, however, always added to the length of the channels, although this is done in the present case.

Plates with the corners sheared off, as the tongues of the pin plates at the hip joint, are generally listed in rectangular form, without reference to the shearing, which is usually done at the bridge shops. They are almost always ordered as one plate (or more, if necessary) of length equal to the aggregate length of the several plates required of the same size, and are sheared into the required lengths and forms at the bridge shops. For plates sheared to dimensions, the mills charge a higher price than for plates simply cut to ordinary lengths.

As rivet iron is always carried in stock, lists of the shop rivets required are not commonly given on the order lists, although it has been done in the present case. When lists of rivets are given, however, all rivets of the same length are generally given in the same item.

1658. List (*I*) is the order list for the material required to form the members and details shown on Plate, Title: Highway Bridge: Details IV, Figs. 1 to 12. The ordered dimensions are those required to make the finished dimensions given on the shop lists in Art. **1645.** The ordered dimensions of the material should be given on the shop lists and working drawings, in order that the workmen may be able to select easily the proper material for each member.

The ordered lengths of forged bars are seldom expressed by fractions smaller than eighths of an inch, although sixteenths may sometimes be used. In making out order lists, it is well to use, in each case, the eighth of an inch next above the calculated length.

(*H*)

ORDER LIST.

For *4 End Posts.* Material, *Wrought Iron.*

Contract No. *578.* Shop No. *735.* Span *90′ 0″.* Sheet No. *1.*

Mark.	Number R'quir'd.	Name of Shape.	Size. Inches.	W'ght, lb. per Ft.	Length.		To'al Wt.	Ord-der'd
					Ft.	In.		
a B	8	Chans.	8	16	25	$10\frac{11}{16}$		O
a B	4	Angles.	$2\frac{1}{2} \times 2\frac{1}{2}$	4	3	$4\frac{3}{4}$		O
a B	4	Plates.	$12 \times \frac{3}{8}$		25	$10\frac{13}{16}$		O
a B	8	Plates.	$6 \times \frac{7}{16}$		1	$2\frac{1}{2}$		O
a B	8	Plates.	$6 \times \frac{7}{16}$		1	$1\frac{1}{2}$		O
a B	8	Plates.	$12 \times \frac{1}{4}$		1	6		O
a B	180	Bars.	$1\frac{3}{4} \times \frac{1}{4}$		1	$1\frac{3}{8}$		S
Where Used.		***Shop.***	***Rivets***					
Cov. Pl.	512	O heads.	$\frac{5}{8} \times 2\frac{1}{8}$					S
Bat. Pl.	72	O heads.	$\frac{5}{8} \times 2$					S
Lattice.	184	O heads.	$\frac{5}{8} \times 2\frac{1}{4}$					S
P't Ang.	32	O heads.	$\frac{5}{8} \times 1\frac{7}{8}$					S
Pin Pls.	80	O heads.	$\frac{5}{8} \times 2$					S
Pin Pls.	48	O heads.	$\frac{5}{8} \times 1\frac{5}{8}$					S
Pin Pls.	16	C.S.he'ds	$\frac{5}{8} \times 1\frac{5}{8}$					S
		Field.	***Rivets.***					
Hip Cov.	16	O heads.	$\frac{5}{8} \times 2\frac{3}{8}$					S
Hip Cov.	4	O heads.	$\frac{5}{8} \times 1\frac{7}{8}$					S
PortCon.	32	O heads.	$\frac{5}{8} \times 1\frac{3}{4}$					S
PortCon.	16	O heads.	$\frac{5}{8} \times 1\frac{7}{8}$					S

1659. Stock Material and Iron Order.—The sizes and weights of channels and angles, as well as the sizes of plates and flat bars, most commonly used at bridge works, are generally carried in stock.

Round and square bars of one inch dimensions and less, and sometimes the common sizes of much larger bars, are carried in stock.

Rivet iron, turnbuckles, standard and recessed nuts, spikes, nails, wood screws, washers, several sizes of bolts, and all small extras of like nature are always carried in stock.

Small bars, such as the small flat bars used in making the heads for cotter pins or the half round bars used for making the cotters, are carried in stock, and are cut off in small pieces as used. Such material is required to be listed upon the order list only for the purpose of checking off from the stock list, and, therefore, the lengths need not be given with very great accuracy. But the lengths of the larger pieces of stock material, which are to be forged or cut to certain dimensions, must be given accurately.

1660. In making out the iron order, all short pieces of the same size (width and thickness) are assembled in one or more long pieces of the same aggregate length. Short pieces of plates and angles, for such purposes as pin plates, batten plates, lateral-hitches, etc., are simply sheared off, and require no allowance for cutting. But bars for pins are usually cut by turning off, which requires about $\frac{1}{4}$ of an inch to be allowed for each cut. (See formula **250.**)

When the order list has been made out, it is compared with the stock list, and such sizes as are in stock are marked on the order list with a letter *S*, and are checked off from the stock list. The iron order, or list of required material not carried in stock, is then made out from the order list. This material is checked off from the order list by marking either the letter *O* or the date on which the iron order is made out opposite each item that is transferred to the iron order. A copy of the iron order, as thus made out, is generally sent directly to the rolling mills by the business department of the bridge-works; if it is a large

(*I*)

ORDER LIST.

For *Forged Members and Pins.* Material, *Wrought Iron.*
(*Except where otherwise marked.*)

Contract No. *578.* Shop No. *735.* Span *90′ 0″.* Sheet No.

Mark.	Num. Required.	Name.	Size. Inches.	Weight. Pounds per Foot.	Length.		Total W'gh't	Ordered.
					Feet.	Inches.		
$B\,c$	8	Bars.	$2 \times 1\frac{3}{16}$		25	$9\frac{1}{2}$		O
$C\,c'$	8	Bars.	$\frac{7}{8} \times \frac{7}{8}$		6	$5\frac{1}{2}$		S
$C\,c'$	8	Bars.	$\frac{7}{8} \times \frac{7}{8}$		21	$6\frac{1}{4}$		S
$B\,b$	8	Bars.	1×1		20	$4\frac{3}{4}$		O
$B\,C_1$	4	Bars.	1 ″ O		26	$7\frac{5}{8}$		S
$C\,C_2$	2	Bars.	1 ″ O		26	$11\frac{1}{2}$		S
$a\,b_1$	4	Bars.	$1\frac{1}{8}$″ O		6	$2\frac{3}{4}$		O
$a\,b_1$	4	Bars.	$1\frac{1}{8}$″ O		20	11		O
$b\,c_1$	6	Bars.	1 ″ O		6	$2\frac{1}{4}$		S
$b\,c_1$	6	Bars.	1 ″ O		20	$9\frac{3}{4}$		S
Hgr.	8	Bars.	$1\frac{1}{4} \times 1\frac{1}{4}$		6	$11\frac{1}{2}$		O
Hgr.	8	Plates.	$6 \times \frac{3}{4}$		0	9		O
	4	Chans.	8 ″	——		$8\frac{1}{4}$		S
a	4	Bars.	$2\frac{7}{8}$″ O		1	$2\frac{1}{8}$		St'l O
B, b, c	12	Bars.	$2\frac{7}{8}$″ O		1	$0\frac{1}{4}$		St'l O
C	4	Bars.	$1\frac{7}{8}$″ O		0	$11\frac{3}{8}$		O
Lats.	20	Bars.	$1\frac{7}{8}$″ O		0	4		O
Lats.	20	Bars.	$\frac{3}{8} \times \frac{3}{16}$		0	$6\frac{1}{4}$		S
Lats.	20	$\frac{1}{2}$ O's	$\frac{3}{8}$″		0	$6\frac{1}{4}$		S
Nuts	16	Stand.	For $1\frac{7}{8}$″O					S
Nuts	32	Rec.	For 2 ″O					S
Nuts	8	Rec.	For $1\frac{1}{4}$″O					S
$C\,c'$	8	Tbkls.	For $1\frac{3}{8}$″O		0	$9\frac{3}{4}$		S
$a\,b_1$	4	Tbkls.	For $1\frac{1}{2}$″O		0	10		S
$b\,c_1$	6	Tbkls.	For $1\frac{3}{8}$″O		0	$9\frac{3}{4}$		S

order, several copies are usually made, by blue-printing or otherwise, which are sent to various rolling mills, requesting them to quote prices on the order.

List (*K*) is the iron order for the material ordered from order lists (*H*) and (*I*).

(*K*)

IRON ORDER.

Contract No. *578.* Shop No. *735.* Span *90′ 0″.* Sheet No. *1.*

Mark.	No. Required.	Name.	Material.	Size. Inches.	Weight. Pounds per Foot.	Length.		Total W'ght.
						Ft.	Inch.	
a B	8	Chans.	W. Iron	8	16	25	10 1/16	
a B	4	Angles.	W. Iron	2½ × 2½	4	3	4¾	
a B	4	Plates.	W. Iron	12 × ⅝		25	10 3/16	
a B	1	Plates.	W. Iron	12 × ¼		12	0	
a B	1	Plates.	W. Iron	6 × 7/16		18	8	
H	1	Plates.	W. Iron	6 × ¾		6	0	
B c	8	Bars.	W. Iron	2 × 13/16		25	9½	
H	8	Bars.	W. Iron	1¼ × 1¼		6	11½	
B b	8	Bars.	W. Iron	1 × 1		20	4¾	
$a\,b_1$	4	Bars.	W. Iron	1⅛″ ○		6	2¾	
$a\,b_1$	4	Bars.	W. Iron	1⅛″ ○		20	11	
C	1	Bars.	W. Iron	1⅞″ ○		10	11½	
a	1	Bars.	M. Steel	2⅞″ ○		17	3½	

LUMBER BILL.

1661. General Observations Concerning Timber.—Of the bills of material required in the construction of a bridge, the last bill to be made out is the bill of the lumber. The kind of lumber and the essential dimensions of the same to be used for the various purposes are given on the stress sheet, from which information the lumber bill is made out.

As kept in stock by dealers, the lengths of pine lumber are commonly multiples of two feet, but oak lumber can usually be obtained in lengths of odd feet also. When lumber is ordered in lengths of odd feet and can not be furnished as ordered, it is furnished in the next longer lengths of even feet. Lengths not greater than 20 feet are easily obtainable, but greater lengths are difficult to obtain, and are expensive. Therefore, the panel lengths of bridges in which timber joists or stringers are to be used should, if possible, not exceed 20 feet.

As commonly sawed, timber is nearly always a few inches longer than its nominal length. Consequently, if the panel lengths of a bridge are in even feet, the joists for the bridge, if ordered of a nominal length the same as the length of panel, will be long enough to give full bearings upon the floor-beams. It is necessary for the joists in the end panels of a bridge to reach to the extreme end of the bridge, or somewhat beyond the extremities of the shoes; this requires the joists in the end panels to be about a foot longer than those in the intermediate panels. This will be readily understood by reference to Plate, Title: Highway Bridge: General Drawing, Fig. 1.

1662. Relative Heights and Arrangement of Joists.—Timbers of the same nominal width always vary slightly in their actual widths. Consequently, in order that joists may have a uniform height over bearings, they should be sized down to uniform widths at the ends. The uniform width to which the joists in a bridge are to be sized is usually $\frac{1}{2}$ inch less than their nominal width.

That portion of a bridge abutment upon which the shoes and bed-plates rest is called the **bridge seat,** and that portion which extends above the bridge seat for the purpose of a retaining wall is called the **back wall.** By reference to the partial elevation and top view of the abutment shown in Figs. 1 and 2 of the plate just referred to, it will be noticed that a step in the masonry, six inches above the bridge seat, projects out seven inches from the back wall. This is to support the ends of the joists at the expansion end of the bridge, which is necessary in this case, in order that the joists may clear the lateral rod. At the expansion shoe, the top of the lateral rod is $\frac{1}{2} + 2 + \frac{7}{8} + \frac{3}{8} + \frac{3}{8} + 1\frac{1}{8} = 5\frac{1}{4}$ inches above the bridge seat. (See Plate, Title: Highway Bridge: Details III, Figs. 5, 7, 8.)

The top of the roadway floor plank will be $11\frac{1}{2} + 3 = 14\frac{1}{2}$ inches above this step, or just even with the top of the back wall. At the expansion end of the bridge, the center of the shoe pin will be $\frac{1}{2} + 2 + \frac{7}{8} + 5 = 8\frac{3}{8}$ inches above the bridge seat (see same Plate and Figs.), or $8\frac{3}{8} - 6 = 2\frac{3}{8}$ inches above the masonry step that is to support the joists. At the intermediate panel points, or joints, of the lower chord, the tops of the floor-beams will be $4\frac{3}{4}$ inches below the centers of the lower chord pins, or $4\frac{3}{4} - 2\frac{3}{8} = 2\frac{3}{8}$ inches lower, with respect to the lower chord pins, than the top of the masonry step that is to support the ends of the joists at the expansion end of the bridge. Consequently, in order that the joists shall all have the same elevation, with reference to the lower chord pins, it will be necessary to place a bearing plank or riser piece, about $2\frac{3}{8}$ inches thick, upon the top of each floor-beam, for the joists to rest upon. Each bearing plank will be a 10″ × $2\frac{1}{2}$″ oak plank placed lengthwise upon the top of the floor-beam, as shown in Plate, Title: Highway Bridge: General Drawing, Fig. 2.

These planks should be secured to the flange of the floor-beam, either by bolts passing through the flange angles or by steel nails clinched under them. As the bolt holes would cut out a portion of the flange section, the latter expedient will here be employed. Such bearing planks are not very commonly used; the joists being generally placed directly upon the flange of the floor-beam. It is not really essential for all portions of the floor to have the same elevation with reference to the lower chord pins, and, therefore, such bearing planks may be omitted without any real detriment to the structure.

At the expansion end of the bridge, it will be necessary for a notch to be cut in the lower edge of each joist, in order that it may clear the shoe strut. Although this is not a really commendable thing to do, the notch is so near to the end of the joists, that it will probably not weaken them. This could be avoided only by raising the joists 3½ inches higher.

1683. The Bill of Lumber.—For this bridge, the bill of lumber will be as follows (see stress sheet):

BILL OF LUMBER.

For Bridge at .... Contract No. ....
Span, C. to C., ... feet. .... Roadway .... feet, clear width,
Panel length, ... feet. .... Sidewalks .... feet, clear width.

Number of Pieces.	Name.	Size.	Length.	No. of Feet, B. M.	Material.
30	Joists	3½″ × 12″	18′ 0″	1,890	Yellow Pine
20	Joists	3½″ × 12″	19′ 0″	1,330	Yellow Pine
..	Plank	3″ Thick	18′ 0″	4,968	Yellow Pine
12	W. Guards	4″ × 6″	16′ 0″	384	Yellow Pine
	Total	Yellow	Pine	8,572	
4	Bear. Plk.	2½″ × 10″	18′ 0″	150	White Oak

It will be noticed that in the items for the joists, wheel-guards, and bearing planks, all dimensions are given, while for the floor planks only the length and thickness are stated. The floor planks may be of any width not less than 7 nor more than 10 inches (they are usually of different widths), but their aggregate width must be equal to the extreme length of the bridge, which will be about 92 feet. This will be the case if they give the required number of feet board measure.

The wheel-guard timbers may be in any lengths, uniform or varying, but their aggregate length must be sufficient to extend the entire length of the bridge, upon each side of the roadway, with six inches lap at the joints.

As lumber is sold by the thousand feet (or hundred feet), board measure, the number of feet board measure in each item should be calculated and written opposite the item in the column headed *Number of Feet, B. M.*

A foot in board measure is one foot square and 1 inch thick.

SHIPPING BILLS.

1664. Nature of a Shipping Bill.—When the shop drawings, shop lists, order lists, iron order, and lumber bill have been made for a bridge, the next office work in hand is to make out a detailed list of all members, pieces, and materials that are to be shipped to the bridge site and used in erecting and completing the bridge. Such a list is called a **shipping bill.** All things listed upon the shipping bill are weighed out from the shop and shipped to the bridge site.

The shipping bill gives a list and partial description of all finished members, parts of members, and connecting pieces shown on the working drawings and described in the shop lists, and is made principally from these. A brief description of each member is given, with a few essential dimensions, in order that each member may be readily identified and roughly checked. The shipping bill includes also items of material not shown on the drawings nor given in the shop lists, such as the field rivets, bolts with nuts and washers, spikes, nails, lag screws, paint, etc., required to complete the structure, as well as the erection bolts, pilot nuts, and other things required in its erection. It will be well to notice here the requirements for some of these items of material.

1665. Field Rivets.—The diameters, lengths, and number of field rivets required may be obtained from the shop drawings; they will correspond to the vacant holes shown on the drawings. The number of rivets actually required of each size must be increased from ten to fifteen per cent. to provide for waste. The percentage of waste depends to some extent upon conditions at the bridge site, difficulty of erection, etc., and can be estimated only by a very liberal approximation. Twelve per cent. is, perhaps, a good average allowance for this purpose; it is a convenient practice to make the total number of rivets required of each size equal to the multiple of ten nearest to, but generally above, the estimated number with the percentage added.

1666. Bolts and Washers.—Ordinary bolts will be required to bolt the wheel-guards to the floor planks. Bolts ¾ of an inch in diameter should be used for this purpose; each bolt should have two washers. These bolts should, under ordinary conditions, be spaced about 6 feet apart along each wheel-guard. Hence, the number n of bolts required for the wheel-guards in one roadway will be given by the formula

$$n = \frac{L}{3} + 2, \qquad \textbf{(255.)}$$

in which L is the extreme length of span in feet, which may commonly be assumed to be 2 feet longer than the length from center to center of end pins. Any fraction in the expression $\frac{L}{3}$ may be neglected.

In order to afford ample drainage for the roadway, the wheel-guards are elevated from 1 to 2 inches above the floor planks, by means of shims, which consist of pieces of plank about a foot long, placed under the wheel-guard of each bolt. The wheel-guard is laid flat, i. e., with its broadest dimension horizontal. The bolt passes through the wheel-guard, shim, and floor plank. Hence, the length l of the bolt, under head, will be given by the formula

$$l = g + c, \qquad \textbf{(256.)}$$

in which g is the grip of the bolt, equal to the aggregate thickness of the wheel-guard, floor plank, and shim, and c is a constant which, for bolts not more than ¾ of an inch in diameter, may be taken equal to 1 inch.

EXAMPLE.—(*a*) What number and (*b*) what length of bolts will be required for the wheel-guards of the bridge shown on the Mechanical Drawing plates, it being assumed that the wheel-guard is to be blocked up by 2-inch shims?

SOLUTION.—(*a*) By formula **255**, the required number of bolts is $\frac{92}{3} + 2 = 32$ bolts. Ans.

(*b*) The thicknesses of the wheel-guard, shim, and floor plank are 4, 2, and 3 inches, respectively. Hence, the length under head of each bolt will be $4 + 2 + 3 + 1 = 10$ inches. Ans.

1667. Spikes and Nails.—Each roadway plank should be spiked to each joist upon which it rests by two spikes. As the widths of the planks should average about 9 inches, the total number of spikes n required for this purpose will be given by the formula

$$n = \frac{8 L N}{3}, \qquad (257.)$$

in which L is the extreme length of the span, and N is the number of lines of joists.

Some specifications require the spikes used in spiking the roadway planks to the joists to be $\frac{5}{16}$-inch wrought spikes 7 inches long. But it is doubtful whether such large spikes are really required or even advantageous. Common 60-penny nails (i. e., cut spikes, 6 inches long) are frequently used. Steel-wire spikes (round) are very satisfactory for this purpose.

Nails and spikes are always sold and shipped by the pound or keg. The number of spikes required for the floor planks may be found by applying formula **257**, and the number of pounds (or kegs) necessary, in order to give the required number of spikes, may be obtained from Table 43. An allowance of about 10 per cent. should be made for waste.

EXAMPLE.—If steel-wire spikes 6 inches long are used for the floor planks of the bridge shown on the Mechanical Drawing plates, (*a*) how many spikes, and (*b*) how many pounds of spikes will be required?

SOLUTION.—(*a*) As given by formula **257**, the number of spikes to be used is $\frac{8 \times 92 \times 10}{3} = 2,453$. If 10 per cent. be added for waste, the number of spikes required will be $2,453 + 245 = 2,700$. Ans. (*b*) By reference to Table 43, it is found that, for steel-wire spikes 6 inches long, there are 10 spikes to the pound. Hence, the number of pounds required is $\frac{2,700}{10} = 270$ lb. Ans.

TABLE 43.

SIZES AND WEIGHTS OF IRON AND STEEL NAILS AND SPIKES.

Common Nails.					Common (not Railroad) Spikes.							
Nominal Size.	Length in Inches.	Iron Cut Nails.	Steel-Wire Nails.		Steel-Wire Spikes.		Length in Inches.	Wrought Spikes.				
								Number per Keg of 150 Pounds.				
		Number per Pound.	Diameter in Inches.	Number per Pound.	Diameter in Inches.	Number per Pound.		¼ In.	$\frac{5}{16}$ In.	⅜ In.	$\frac{7}{16}$ In.	½ In.
2 *d*	**1**	800	.0524	1060	.1620	41	**3**	2250				
3 *d*	**1¼**	400	.0588	640	.1819	30	**3½**	1890	1208			
4 *d*	**1½**	300	.0720	380	.2043	23	**4**	1650	1135			
5 *d*	**1¾**	200	.0764	275	.2294	17	**4½**	1464	1064			
6 *d*	**2**	150	.0808	210	.2576	13	**5**	1380	930	742		
7 *d*	**2¼**	120	.0858	160	.2893	11	**5½**					
8 *d*	**2½**	85	.0935	115	.2893	10	**6**	1292	868	570		
9 *d*	**2¾**	75	.0963	93	.3249	7½	**6½**					
10 *d*	**3**	60	.1082	77	.3249	7	**7**	1161	662	482	445	306
12 *d*	**3¼**	50	.1144	60	.3648	5	**8**		635	455	384	256
16 *d*	**3½**	40	.1285	48	.3648	4½	**9**		573	424	300	240
20 *d*	**4**	20	.1620	31			**10**			391	270	222
30 *d*	**4½**	16	.1819	22			**11**				249	203
40 *d*	**5**	14	.2043	17			**12**				236	180
50 *d*	**5½**	11	.2294	13								
60 *d*	**6**	8	.2576	11								

Wrought spikes, sometimes called **ship spikes** or **boat spikes**, are usually sold by the keg of 150 pounds, and are shipped by the keg, except where a smaller amount is required. If, in the preceding example, $\frac{5}{16}$-inch wrought spikes 7 inches long were designated, there would be required (see Table 43) $\frac{2,700}{662} = 4$ kegs and 52 pounds.

A few steel-wire nails will be required to fasten the bearing planks upon the tops of the floor-beams. These nails will be driven through the planks outside (on each side) of the floor-beam flanges, then clinched under the flanges. Nails $4\frac{1}{2}$ inches long will be suitable for this purpose. The number of nails may be estimated at 2 nails per foot of beam, making the number of nails to be used equal to $2 \times 18 \times 4 = 144$; and, by adding 10 per cent. for waste, the number of nails required will be equal to $144 + 15 = 159$. If $4\frac{1}{2}$-inch steel-wire nails are used, then there will be required (see Table 43) $\frac{159}{22} = 7$ pounds, near enough.

1668. Erection Bolts.—These bolts are used to secure, temporarily, the various riveted connections, holding the connecting parts in position until the field rivets can be driven. It is, therefore, evident that the sizes and lengths of the erection bolts may be determined from the rivet holes shown vacant on the shop drawings, in much the same manner as the sizes and lengths of field rivets; the number of erection bolts required may be somewhat less than the number of field rivets required. The length of an erection bolt, however, being adjustable, need not be determined with very great accuracy, but the same length of bolt may be used for different grips, varying through a range of $\frac{1}{2}$ inch, or even more. The length, under head l, of an erection bolt may be given by the formula

$$l = g + d + c, \quad \textbf{(258.)}$$

in which g is the grip and d is the diameter of the bolt, and c is a constant which may have a value of from $\frac{1}{8}$ to $\frac{5}{8}$ of an inch.

For Bridge at..
................Span..................feet, C. to C. Ship to...
................Roadway..................feet, clear width. ...
................Sidewalks.................. feet, clear width. Via

Mark.	No. Required.	Name of Member.		Material in M
a B	4	End Posts	2-R 2-L	1-Plate 12″ × ⅜″ 2-8″ Chans. @ 16 lb.
B C	4	Top Chords	2-R 2-L	1-Plate 12″ × ¼″ 2-8″ Chans. @ 11 lb.
C C′	2	Top Chords		1-Plate 12″ × ¼″ 2-8″ Chans. @ 11 lb.
C c	4	Int. Posts		4-3½″ × 2″ L's @ 4.4 lb.
$B\ B_1$	2	Portal Struts with Brackets		4-3½″ × 2½″ L's @ 4.8 l
$C\ C_1$	2	Lateral Struts		2-5″ × 4″ L's @ 10.8 lb.
K B	4	Knee Braces		2-3″ × 2½″ L's @ 4.4 lb.
$a\ a_1$	1	Shoe Strut		1-Plate 6″ × ¼″ 2-3½″ × 2½″ L's @ 4.8 l
F B	4	Floor-Beams		1-Plate 24″ × ⅜″ 4-4″ × 3″ L's @ 8 4 lb.
B H	8	Beam Hangers		1¼″ × 1¼″, 1⅞″ S. Ends w
H P	8	Hanger Pls.		6″ × ¾″
S	4	Separators		8″ × 1¼″
a b	4	Lower Chords		4-3″ × 2½″ L's @ 4.6 lb
b c	4	Lower Chords		4-3¼″ × 2″ L's @ 5.8 lb
C C′	2	Lower Chd. Bars		2 Bars 4½″ × 13/16″ Lattic
B c	8	Tie Bars		2″ × 13/16″
B b	8	Hip Vert. Rods		1″ × 1″
C c′	8	Counters		⅞″ × ⅞″, 1⅜″ S. Ends
C c′	8	Counters		⅞″ × ⅞″, 1⅜″ Ends, 9
$a\ b_1$	4	Lateral Rods		1⅛″ ○, 1½″ S. Ends
$a\ b_1$	4	Lateral Rods		1⅛″ ○, 1½″ S. Ends, 1c
$b\ c_1$	6	Lateral Rods		1″ ○, 1⅜″ S. Ends
; c_2	6	Lateral Rods		1″ ○, 1⅜″ S. Ends, 9
$B\ C_1$	4	Lateral Rods		1″ ○, 1⅜″ S. Ends
$C\ C_2$	2	Lateral Rods		1″ ○, 1⅜″ S. Ends
a	4	Chord Pins		2¼″ ○, 11⅝″ Grip
B, b, c	12	Chord Pins		2¼″ ○, 9¾″ Grip
C	4	Chord Pins		1¾″ ○, 9⅛″ Grip
L	20	Lateral Pins		1¾″ ○, 2¼″ Grip, ⅜″ C
C	4	Pin Washers		For 1¾″ Pin
B	4	Pin Washers		For 2¼″ Pin
b	8	Pin Washers		For 2¼″ Pin
b	8	Pin Washers		For 2¼″ Pin
	2	Anchor Shoes		Stand. 7″ × 3½″ × 5 Pl. 17½″ × ⅞″ × 13″
	2	Roller Shoes		Stand. 7″ × 3½″ × 5 Pl. 17½″ × ⅞″ × 13
	2	Nests Rollers		2″ ○, 17¼″ × 14″
	2	Bed-Plates		24″ × ½″ × 14″
	2	Bent Anch. Pls.		2″ × ⅜″ × 3⅜″
	12	Anchor Bolts		1″ ○ × 10 with Wed
	20	Field Rivets		⅝″ ○ × 2½″ U. H.
	20	Field Rivets		⅝″ ○ × 2⅜″ U. H.
	30	Field Rivets		⅝″ ○ × 2⅛″ U. H.
	50	Field Rivets		⅝″ ○ × 1⅞″ U. H.
	150	Field Rivets		⅝″ ○ × 1¾″ U. H.
	32	Bolts		¾″ ○ × 10″ U. H.
		Spikes		6″ Steel Wire
		Nails		4½″ Steel Wire
		Paint		
	160	Erection Bolts		⅝″ ○ × 2″ U. H.
	40	Erection Bolts		⅝″ ○ × 1½″ U. H.
		Pilot Nuts		For 2¼″ Pin
		Pilot Nuts		For 1¾″ Pin

NG BILL.

.... Contract No. ....

....

....

....

ember.	No. and Size of Holes.	Length.	Shipping Weight.
	2-2$\frac{25}{32}$"	25′ 5$\frac{5}{8}$" C. to C.	
	1-2$\frac{23}{32}$"	17′ 8$\frac{1}{8}$" C. to E.	
	2-2$\frac{25}{32}$"	18′ 0$\frac{3}{8}$" C. to C. 18′ 8$\frac{7}{8}$" E. to E.	
Latticed	1-2$\frac{25}{32}$"	17′ 7$\frac{1}{8}$" C. to E.	
. Latticed		20′ 0" E. to E.	
		20′ 0" E. to E.	
		4′ 3$\frac{7}{16}$" B. to B.	
		16′ 10$\frac{1}{8}$" E. to E.	
.			
		19′ 8" E. to E.	
ith Check Nuts	1-2$\frac{25}{32}$"	2′ 11$\frac{1}{8}$" B to E.	
	2-2"	0′ 4$\frac{3}{4}$" C. to C.	
	2-2"	0′ 4$\frac{3}{4}$" C. to C.	
Latticed	2-2$\frac{25}{32}$"	17′ 11$\frac{31}{32}$" C. to C.	
Latticed	2-2$\frac{25}{32}$"	17′ 11$\frac{31}{32}$" C. to C.	
ed	2-2$\frac{25}{32}$"	17′ 11$\frac{31}{32}$" C. to C.	
	2-2$\frac{25}{32}$"	25′ 5$\frac{9}{16}$" C. to C.	
	2-2$\frac{25}{32}$"	18′ 2$\frac{23}{32}$" B. to B.	
	1-1$\frac{25}{32}$"	20′ 3$\frac{3}{16}$" B. to E.	
" Turnbuckle	1-2$\frac{25}{32}$"	5′ 0" B. to E.	
	1-1$\frac{25}{32}$"	19′ 8$\frac{1}{4}$" B. to E.	
′ Turnbuckle	1-1$\frac{25}{32}$"	5′ 0" B. to E.	
	1-1$\frac{25}{32}$"	19′ 7$\frac{1}{4}$" B. to E.	
" Turnbuckle	1-1$\frac{25}{32}$"	5′ 0" B. to E.	
	St. Nuts	25′ 10$\frac{5}{8}$" E. to E.	
	St. Nuts	26′ 2$\frac{1}{2}$" E to E.	
	R. Nuts	1′ 2$\frac{1}{8}$" E. to E.	
	R. Nuts	1′ 0$\frac{1}{4}$" E. to E.	
	R. Nuts	0′ 11$\frac{3}{8}$" E to E.	
tters		0′ 4" E. to E.	
		0′ 4$\frac{1}{4}$" E. to E.	
		0′ 2$\frac{1}{4}$" E. to E.	
		0′ 1$\frac{1}{4}$" E. to E.	
		0′ 0$\frac{3}{16}$" E. to E.	
" L's vith Lat. con.	2-2$\frac{25}{32}$" 1-1$\frac{13}{16}$"		
" L's hoe St. & Lat. con.	2-2$\frac{25}{32}$" 1-1$\frac{13}{16}$"		
	1-1$\frac{1}{8}$"		
es			
			270 7

1669. Pilot Nuts.—One pilot nut is required for each size of the pin, and it is always well to have extra pilot nuts of each size, as they are liable to be lost.

Erecting bolts and pilot nuts are used simply during the erection of a bridge. They do not remain in the structure, but may be used again in the erection of other structures. Erecting foremen usually keep supplied with a limited number of pilot nuts and erecting bolts remaining from structures previously erected, and carried as a part of their erecting outfit. Erection bolts and nuts should, however, be listed upon the shipping bill.

1670. Lumber Bill.—The bill of lumber required for the bridge proper is usually ordered of a lumber mill or dealer, and by him delivered directly at the bridge site. In such cases, it, of course, does not appear upon the shipping bill, the latter being a bill of the material shipped from the bridge works.

The lumber and other material required for false work in the erection of the structure, as well as the outfit of tools required, are matters usually left in charge of the erecting foremen, and will not be considered here.

1671. Shipping Bill.—The accompanying complete shipping bill is for the bridge shown on the Mechanical Drawing plates. In order to show a form of blank sheet suitable for the shipping bill, the blanks in the heading of the sheet are not filled out.

THE ERECTION DIAGRAM.

1672. A sketch or diagram showing the proper positions of the various members in a bridge should be made for the guidance of the erecting foreman. Such a sketch is called an **erection diagram.** By the aid of it the erector may readily ascertain the correct position of each member and piece entering into the construction of the bridge.

The erection diagram is simply a skeleton diagram of the truss and lateral systems, somewhat similar to the stress sheet, having written along each member a description of the material composing its section, with its length, and giving also the dimensions of each pin and such other information as may be essential. A partial plan of one lower chord is also drawn to an exaggerated scale, showing the arrangement of the chord bars and the manner in which the chord bars, tie bars, counters, and beam hangers are packed upon each lower chord pin, with the dimensions and positions of the pin washers used. It is often not necessary to show more than the mere *arrangement* of the various members as packed upon each pin. If the truss is symmetrical, both with reference to the center of its span and the center line of its roadway, it will not be necessary for more than one-half of one truss, and but little more than a quarter of each lateral system, to be shown in the erection diagram.

Fig. 336 is the erection diagram for the bridge we have been considering. It will be readily understood.

ESTIMATES.

THE APPROXIMATE, OR PRELIMINARY, ESTIMATES.

1673. General Methods and Process.—At some point during the process of making the design for a bridge, and previous to its actual construction, it is necessary to make an estimate of the weight of the metal required in its construction. An approximate estimate of the weight is usually made directly after the completion of the stress sheet. This estimate is often called a **preliminary estimate,** although in some cases a rough estimate has been previously made. If no drawings of the details are made before the letting of the contract, the preliminary estimate, as made from the stress sheet, forms the basis for the estimate of cost in submitting the proposal and making the contract for the construction of the bridge. In many cases the preliminary estimate is the only estimate of weight made. Although not usually a close estimate, yet, when

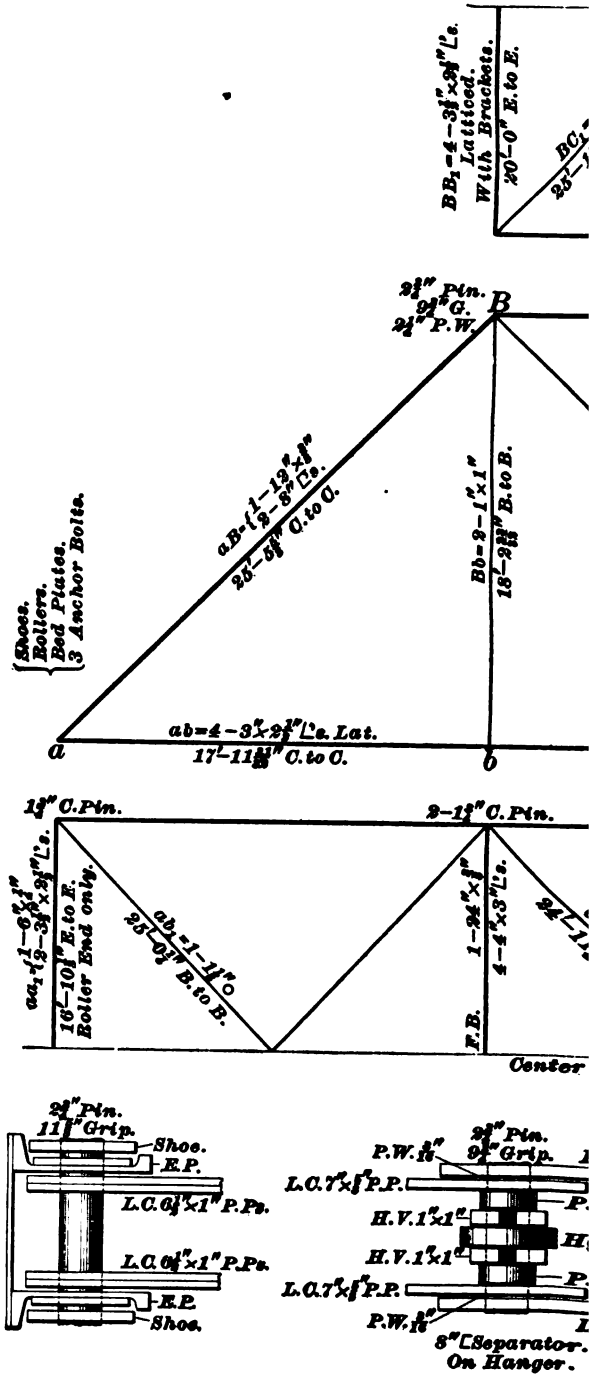


FIG.

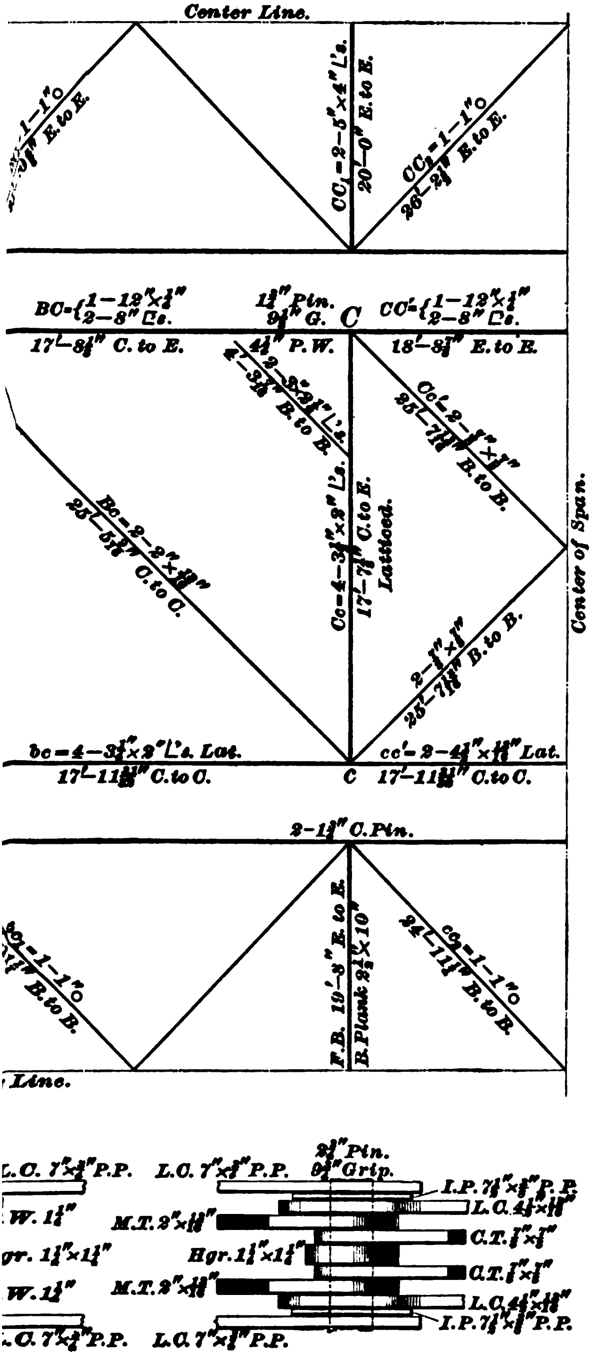
Erection

Contract No. 578. *Location, Jo*

Span C. to C. . . . . = 90 feet.

Height of Truss . . . . = 18 feet.

Clear Roadway. . . . . = 18 feet.



336.

Diagram.

es' Crossing, Sycamore County, Pa.

loor Plank........3" Yellow Pine.

oists, 10 lines, 3½" × 12" Yellow Pine.

Vheel-Guard, 2 lines 4" × 6" Yellow Pine.

made by an experienced bridge engineer, a preliminary estimate will often approximate quite near to the actual weight of the structure. If the detail drawings of the structure have been made previous to the making of the estimate, the latter will, of course, be much closer.

The preliminary estimate usually gives the same information that is given upon the stress sheet, together with an approximate detailed estimate of the weight of the material. In fact, in many cases, after the stresses have been determined, they are copied upon the estimate sheet, the material is proportioned and also written upon it, and the estimate is made before making the stress sheet. The stress sheet is then made from the estimate sheet, to be submitted with the proposal or to form a part of the contract; the estimate sheet is filed for office reference.

1674. In making the preliminary estimate, the weight per foot of each member is estimated according to the material shown on the stress (or estimate) sheet, and this weight multiplied by the length of the member, in feet, for the total weight of the member. The length of each member, always expressed in feet and decimals of a foot, is taken from center to center of the intersecting members. An additional amount is commonly added to the center-to-center length of each member, to allow for the weight of certain details. This is the common method of providing for the weight of eye-bar heads, loops, upset screw ends, and turnbuckles, in estimating the weight of forged tension members. For riveted members, the additional weight of the details is approximately estimated. Some engineers, however, add certain percentages of the estimated weight of the main members, to provide for the weight of all details.

1675. Assumed Lengths and Equivalents.—It is not possible to give a system of rules for approximate estimates that will be found satisfactory in all cases. The estimator must necessarily be guided by his judgment and experience. The following general rules are, perhaps, as nearly satisfactory for ready approximations as any that can be proposed. These rules give a length for each member, the weight of which, estimated according to the material of its regular section as shown on the stress sheet, is assumed to be approximately equivalent to the weight of the member, including certain details:

The length L of certain members hereafter designated, which is to be used in estimating their weight, may be taken as given by the formula

$$L = l + c, \qquad \textbf{(259.)}$$

in which l is the length of the member from center to center of the connecting members, and c is a constant which, for the various members, may generally have the following average values:

1. For each end post .... $c = 1\frac{1}{2}$ feet
2. For each end panel of top chord......... $c = 1\frac{1}{4}$ feet
3. For each intermediate panel of top chord.. $c = \frac{1}{2}$ foot
4. For each intermediate post .... $c = 2$ feet
5. For each riveted tension member........ $c = 5$ feet
6. For each lateral strut .... $c = 1$ foot
7. For each forged eye-bar................ $c = 8$ times the width of the bar.
8. For each adjustable bar having two screw ends with nuts .... $c = 2$ feet
9. For each bar having two loop ends ...... $c = 2\frac{1}{2}$ feet
10. For each adjustable bar having two loop ends and a turnbuckle .... $c = 5\frac{1}{2}$ feet

In each of the first three items, the weight of the additional length c will be approximately equal to the weight of the *pin plates*, while in the fourth and fifth items, it will be approximately equal to the weight of the pin plates and other *connecting plates*. In the sixth item, the weight of the additional length c will be approximately equal to the connections for the lateral strut. In each of the remaining items, the weight of the additional length c will be approximately equal to the weight of all details of the members; that is, to the amount which the weight of the finished member exceeds the weight of the plain bar having a length *l*.

It must be remembered that these values of *c*, though fair average values, are but roughly approximate and may be modified according to the conditions of each special case.

1676. *Latticing.*—For each single system of latticing in which the lattice bars make an angle of about 60 degrees with the axis of the member, the aggregate length of the lattice bars will be approximately equal to twice the center-to-center length of the member.

For each single system of latticing in which the lattice bars make an angle of about 45 degrees with the axis of the member, the aggregate length of the lattice bars will be approximately equal to 1.4 times the center-to-center length of the member.

These lengths are independent of the *width* of the member. If a double system of latticing is used, or if the member is latticed on both sides with a single system, the aggregate length of lattice, as obtained by the above rules, should be doubled.

1677. *The lengths of the batten plates* are governed by the requirements of the specifications. If Cooper's specifications are used, the length of *each* batten plate will be one and one-half times the width of the member, or the aggregate length of the batten plate upon each end post or chord member, having a cover-plate, will be three times the width of the cover-plate.

1678. *Each knee brace* may be assumed to be 5 feet long.

1679. *The weight of each lateral hitch* composed of short pieces of angles, similar to that shown in Plate, Title: Highway Bridge: Details I, Fig. 4, may be taken equal to the weight of 2½ feet of the lateral rod connecting upon it.

1680. *If the diameters of the chord pins have not been determined*, in making the estimate, the diameter of all lower chord pins and of the hip pin may be assumed equal to about three-quarters the width of the widest eye-bar in the lower chord; the diameter of the remaining pins in the upper chord may be assumed to be two-thirds as great as or equal to one-half the width of the widest chord bar.

1681. *The lengths of all chord pins* may be assumed equal to 1¼ times the width of the cover-plate on the upper chord or end post, making no allowance for the reduced diameters of the screw ends. The weight of the pins thus estimated may be considered to include the weight of the nuts and washers.

1682. *If a portal is latticed* with ordinary 2″ × ¼″ bars, the weight of the *latticing* (including weight of rivet heads), in pounds per lineal foot of portal, may be taken equal to five times the depth of the portal in feet.

The aggregate weight of both portal brackets and the portal connections for each portal may be taken equal to the total weight of 8 feet in length of the portal proper; consequently, the length of portal having parallel flanges may be assumed to be equal to the clear width of roadway plus 8 feet.

1683. *The weight of all rivet heads in each line of rivets* in a member may be assumed as follows:

For ½-inch rivets, ⅓ pound per lineal foot of member.
For ⅝-inch rivets, ⅔ pound per lineal foot of member.
For ¾-inch rivets, $\frac{8}{10}$ pound per lineal foot of member.
For ⅞-inch rivets, 1⅓ pounds per lineal foot of member.

With this assumption, the two lines of rivets connecting a system of latticing may generally be considered as a single line of rivets.

1684. *In estimating the weight of the floor-beams* in a bridge having a single roadway and no sidewalks, the length of flanges, web-plate, and rivet lines in the flanges may be assumed to be equal to 1⅙ times the clear width of roadway plus the width of both chords. This will provide for the approximate weight of the stiffeners, fillers, and other details.

Estimated Weight of Bridge at..........
Length of Span, C. to C. ....ft. Live
Height of Truss, C. to C. ....ft. Live
Length of Panel ....ft. Dead
..........Roadway ....ft., clear width. Panel
..........Sidewalks ....ft., clear width. Panel
Material, ... Lengt
Specifications ....

Member.	Stress.	Description of Material.
End Post	+ 45,800	1-Pl., 12" × ⅜"
a B	+ 19,600	2-8" Chans. @ 16 lb.
		Lattice, 1¾" × ¼"
		Batten Pls., 12" × ¼" × 18"
		⅝" Rivet Heads
Top Chord	+ 48,600	1-Pl., 12" × ¼"
B B′	+ 20,800	2-8" Chans. @ 10 lb.
		Lattice Bars, 1¾" × ¼"
		Batten Pls., 12" × ¼" × 18"
		⅝" Rivet Heads
Int. Post	+ 9,700	4-3½" × 2" L's @ 4.4 lb.
C c	+ 2,300	Lattice, 1¾" × ¼"
		Batten Pls., 7" × ¼"
		⅝" Rivet Heads
Low. Chd.	− 32,400	4-3" × 2½" L's @ 4.6 lb. N. S. = 4.03
a b	− 13,900	Lattice Bars, 1¾" × ¼"
		Batten Pls., 6" × ¼"
		⅝" Rivet Heads
b c	− 32,400	4-3¼" × 2" L's @ 5.8 lb. N. S. = 4.7
	− 13,900	Lattice Bars, 1¾" × ¼"
		Batten Pls., 6" × ¼"
		⅝" Rivet Heads
c c′	− 48,600	2-Bars, 4½" × 13/16". N. S. = 6.39
	− 20,800	Lattice Bars, 1¾" × ¼"
		⅝" Rivet Heads
H. Vert.	− 16,200	2-Bars, 1" × 1"
B b	− 4,600	
Ties	− 27,500	2-Bars, 2" × 13/16"
B c	− 9,800	
C c′	− 13,750	2-Bars, ⅞" × ⅞"
Pins		8-3" ○ × 15" long (steel)
		2-2" ○ × 15" long
		6-Lateral Hitches for 1" ○
		Total Weight of one Truss,
		Total Weight of two Trusses
BC_1	− 3,700	4-Rods, 1" ○
CC_2		2-Rods, 1" ○
ab_1	− 14,900	4-Rods, 1⅛" ○
bc_1	− 8,200	4-Rods, 1" ○
CC_2	− 2,200	2-Rods, 1" ○
BB_1		4-3½" × 2½" L's @ 4.8 lb.
		Latticed 2 feet deep
CC_1		2-5" × 4" L's @ 10.8 lb.
K B		2-3" × 2½" L's @ 4.4 lb.
aa_1		1-Pl., 6" × ¼"
		2-3½" × 2½" L's @ 4.8 lb.
4-Fl. Bms.	− 53,700	Web Pl., 24" × ⅜" × 20′-0"
		4-4" × 3" L's @ 8.4 lb., 26′-0"
		⅝" Rivet Heads
8-Hangers		1¼" × 1¼"
		Shoes, Rollers, and Bed Pl.
		Bolts, Spikes, and Nails
		Total Weight of Metal,

E SHEET.

Load per sq. ft. { Roadway ....lb.
{ Sidewalks ....lb.

Load ....lb. per lin. ft., trusses.

l Load ....lb. per lin. ft., trusses.

l Live Load, one truss, ...lb.

:l Dead Load, one truss, ...lb.

;th of Diagonal, 1st ....2d ....

	Area Given.	Weight per Foot.	Aggregate Length.	Total Weight.
	14.10	47.0	54	2,538
	.44	1.46	102	149
	3.00	10.	6	60
		.67	153	102
	9.0	30.	57	1,710
	.44	1.46	108	158
	3.0	10.	9	90
		.67	162	108
	5.28	17.6	40	704
	.44	1.46	72	105
	1.75	5.83	3.5	20
		.67	36	24
.03		18.4	46	846
	.44	1.46	72	105
	1.5	5.	3	15
		.67	36	24
.7		23.2	46	1,067
	.44	1.46	72	105
	1.5	5.	3	15
		.67	36	24
	7.31	24.37	20.7	504
	.44	1.46	36	53
		.67	18	12
	2.0	6.67	41	273
	3.25	10.83	53.7	582
	1.53	5.1	62	316
		24.00	10	240
		10.45	2.5	26
		2.61	15	39
				10,014
				2
				20,028
	.79	2.61	112.8	294
	.79	2.61	56.4	147
	.99	3.31	126.8	420
	.79	2.61	126.8	331
	.79	2.61	63.4	165
	5.76	19.2 10.	52	1,518
	6.48	21.6	40	864
		8.8	20	176
	4.38	14.6	20	292
	9.0	30.	92	2,760
		33.6	92	3,091
		1.33	92	123
	1.56	5.2	96	499
				810
				324
				31,842

1685. *In estimating the weight of the forged beam hangers*, the length of the rod forming each hanger may be assumed to be equal to twice the nominal depth of the floor-beam plus 8 feet. This will include the approximate weight of upset ends, nuts, and hanger plate.

1686. *The total weight W of the shoes, bed-plates, rollers, and anchor bolts* for one span may be roughly approximated by the formula

$$W = c\,S\,R, \qquad \textbf{(260.)}$$

in which S is the span, R is the clear width of the roadway, and c is a variable coefficient which may be taken equal to $\frac{S}{200}$, with a minimum value of $\frac{1}{2}$.

1687. *The total weight of bolts, spikes, and nails* required for the roadway floor may also be roughly estimated by formula **260** by giving c a value of .2.

In estimating the weights of structures, the tables of the weights per foot of the various sizes of round, square, and rectangular sections, given in structural hand-books, will be found very convenient, if not indispensable. The sectional areas of most of the members are given on both the stress sheet and the estimate sheet, and, for such members, the weights per foot may be most quickly obtained from the sectional areas. This can not be done, however, in the case of riveted tension members for which the net section only is given.

1688. **The Estimate of Weight.**—The approximate estimate of the weight of the metal in the bridge is given on the accompanying sheet. This estimate is made wholly from the information given upon the stress sheet shown in Fig. 306, Art. **1452.** In this estimate, the weights per foot of rectangular bars, and of those members for which the gross sectional areas are given, are obtained from the areas. The weights of round bars are obtained by formula **153,** Art. **1487.** This formula applies to steel, and, therefore, the results must be reduced 2 per cent. for iron. The lengths of all members, as used in this estimate, are the lengths obtained by the preceding formulas for assumed equivalents.

1689. Accuracy of the Dead Load.—When the estimate of the weight of the metal in the structure has been made, the designer can readily ascertain whether the proper amount of dead load was assumed for the structure in calculating the stresses. In the present case, the total estimated weight of the metal work is 31,842, or, practically, 31,800 pounds. Hence, the weight of the metal work *per lineal foot* of the bridge is $\frac{31,800}{90} = 354$ pounds.

As estimated by formula **90,** Art. **1298,** the weight per lineal foot of the structure, exclusive of the floor, was found to be 352 pounds. This is only 2 pounds per lineal foot in excess of the weight as estimated, and the assumed dead load may, therefore, be considered correct.

It will be well to notice, however, that had the unnecessary latticing been omitted from the center panel of the lower chord, and the remaining panels of the lower chord been formed of eye-bars latticed similar to the center panel, instead of being made up with angles, the weight of the structure would have been somewhat less.

THE CLOSE ESTIMATE.

1690. In some cases the detail drawings for a bridge are made before the contract is let for its construction. If the structure is designed by a consulting engineer, complete drawings are usually made showing all details clearly. In such cases it is possible to make a very close preliminary estimate of the weight of the structure by calculating the weight of each separate piece and estimating the weight of the rivet heads.

The weights per hundred of the rivet heads, as ordinarily formed upon rivets of the four sizes most commonly used, are as follows:

Diameter of rivets	½ in.	⅝ in.	¾ in.	⅞ in.
Weight in pounds per 100 heads	5.7	10.9	13.4	22.2

Countersunk and flattened heads are commonly counted the same as full heads, though countersunk heads should really not be counted.

The length of a lattice bar having two semicircular ends, each connected by a single rivet, may be taken equal to the length from center to center of rivets plus the width of the bar.

Where the length of a piece is expressed in feet and inches, in estimating the weight it will be found convenient to reduce the inches to decimals of a foot.

Results are always taken to the nearest pounds; fractions of pounds are not used in the results or total weights.

1691. As an example of a portion of a close estimate, the weight of the end post, estimated from the shop drawing, Fig. 1 of Plate Title: Highway Bridge: Details I, is given below:

No.	Material.	Size.	Length.	Weight per Foot.	Total Weight.
1	Cover Plate,	12″ × ⅜″	25′ 10⅝″	15.0	388
2	8″ Channels,	@ 16 lb.	25′ 10½″	32.0	828
1	2½″ × 2½″ Angle,	@ 4 lb.	3′ 4¾″	4.0	14
2	Pin Plates,	6″ × $\frac{7}{16}$″	1′ 2½″	17.5	21
2	Pin Plates,	6″ × $\frac{7}{16}$″	1′ 1½″	17.5	20
2	Batten Plates,	12″ × ½″	1′ 6″	20.0	30
45	Lattice Bars,	1¾″ × ¼″	1′ 1″	1.46	71
560	Rivet Heads,	For ⅝″ Rivets		10.9	61
	Total				1,433

In the approximate estimate given in Art. **1688,** the aggregate weight of two end posts was found to be $2,538 + 149 + 60 + 102 = 2,849$ pounds, or $\frac{2,849}{2} = 1,425$ pounds for each end post. This differs by only 8 pounds from the above close estimate.

Where ends are beveled, or corners are cut off, it is, in most cases, customary to estimate them as though full and square, making no deduction for the metal cut away. This is because the material cut away is simply wasted and must be paid for the same as any other portion of the bill of material. For the same reason, no deduction is made for pin holes and vacant rivet holes. Indeed, for the same general reason, it is customary in some bridge offices to estimate the weight of the *rivets* required, instead of the rivet *heads*, as it is desired to arrive at the weight of material required to manufacture the structure, rather than the weight of the finished structure. In other bridge offices, however, it is customary to estimate the weight of the finished members. The latter practice is the most common, and, with the exceptions noticed above, is followed here.

In this connection, it will be well to remark that, in rolling the material, the rolling mills usually demand to be allowed a variation in weight of 2½ per cent. from the specified weight. It is thus evident that it is useless to attempt to be exceedingly exact in making estimates of weight. Beyond a reasonable degree of accuracy, any effort in the direction of exact estimates is time wasted.

METAL JOISTS AND STRINGERS.

1692. Description.—Thus far in the study of bridge designing, wood joists are the only ones that have been noticed. Metal joists, however, are often used for bridges, especially in cities and where the traffic is heavy. They are generally solid rolled beams having a cross-section of a form somewhat similar to the letter I; such beams are known as **I-beams** (sometimes written eye-beams). I-beam joists usually rest upon the upper flanges of the floor-beams, though they are sometimes supported upon angle lugs riveted to the web of the floor-beam; they are riveted to the flanges, or to the lugs, as the case may be. The floor is laid directly upon the I-beams; if a plank floor, it is usually fastened by steel-wire spikes driven each side of the joists and clinched under the flanges.

1693. Strength of I-Beams.—The various properties of I-beams of the different sizes rolled, together with convenient tables of the safe uniform loads for the same, will be found in any structural hand-book. From the tables of safe loads, the size of I-beams required for any ordinary load and span (panel length) can be selected at once, without calculation.

For concentrated or unusual loading, it is necessary to find the bending moment upon the I-beam. The student has learned how to find the bending moment upon a simple beam under any system of loads. Having obtained the bending moment, in inch pounds, it is only necessary to select, from a table of the properties of I-beams, the size of beam which will give a resisting moment equal to the bending moment found.

The moment of resistance R of a solid beam is (formula **73**, Art. **1243**)

$$R = \frac{SI}{c}.$$

In the case of steel or wrought-iron I-beams having equal flanges, c is equal to one-half the depth of the beam. As the moment of resistance R must in all cases be equal to the bending moment M, the value of the latter quantity, in inch-pounds, should be used for R.

By dividing the moment of resistance, as expressed by the above formula, by S, the numerical values of the resulting quotient may be readily tabulated for different sizes of I-beams. The quotient $\frac{R}{S}$, which will be designated here by Q, has been called the **section modulus.** This appears to be an appropriate name, and will here be used. The value of Q may be readily obtained by dividing both terms of the preceding equation by S, as follows:

$$Q = \frac{R}{S} = \frac{I}{c}. \qquad \textbf{(261.)}$$

1694. The following table gives various properties of the steel I-beams rolled by the Carnegie Steel Company, Limited :

TABLE 44.

PROPERTIES OF STEEL I-BEAMS.

Depth of Beam.	Weight per Foot.	Area of Section.	Thickness of Web.	Width of Flange.	Increase of Thickness of Web for each Pound Increase of Weight.	Moment of Inertia, Neutral Axis Perpendicular to Web at Center.	Section Modulus, Neutral Axis as before.	Radius of Gyration, Neutral Axis as before.	Moment of Inertia, Neutral Axis Coincident with Center Line of Web.	Radius of Gyration, Neutral Axis as before.
In.	Lb.	Sq. In.	In.	In.	In.	I	Q	r	I'	r'
24	80	23.5	.50	6.95	.0123	2059.3	171.6	9.42	41.6	1.34
20	80	23.5	.60	7.00	.015	1449.2	144.9	7.85	45.6	1.39
20	64	18.8	.50	6.25		1146.0	114.6	7.80	27.3	1.20
15	80	23.5	.77	6.41	.020	785.9	104.8	5.82	42.2	1.35
15	60	17.6	.54	6.04		644.0	85.9	6.04	30.4	1.32
15	50	14.7	.45	5.75	.020	529.7	70.6	6.00	21.0	1.20
15	41	12.0	.40	5.50		424.1	56.6	5.94	14.0	1.08
12	40	11.7	.39	5.50	.025	281.3	46.9	4.90	16.8	1.20
12	32	9.4	.35	5.25		222.3	37.0	4.85	10.3	1.04
10	33	9.7	.37	5.00	.029	161.3	32.3	4.08	11.8	1.10
10	25	7.5	.31	4.74		122.5	24.5	4.06	7.27	0.99
9	21	6.2	.27	4.50	.033	84.3	18.7	3.70	5.56	0.95
8	18	5.3	.25	4.25	.037	57.8	14.4	3.30	4.35	0.91
7	15	4.4	.21	3.98	.042	38.0	10.86	2.92	3.42	0.87
6	13	3.8	.23	3.50	.049	23.5	7.83	2.48	2.27	0.77
5	10	3.0	.22	3.00	.059	12.4	4.96	2.05	1.29	0.66
4	7	2.1	.17	2.59	.074	5.7	2.85	1.66	0.72	0.59
3	6	1.8	.20	2.26	.098	2.6	1.74	1.21	0.47	0.51

The properties of I-beams given in this table are for the minimum weight to which *each pattern* can be rolled. It will be noticed that, in several cases, more than one pattern is used for the same depth of beam.

From formula **261** it is evident that, having obtained the bending moment M in inch-pounds, it is only necessary to divide it by the allowed unit stress, and select from the table a size of I-beam having a value of Q equal to the quotient.

In case the value of $\frac{M}{S}$ does not correspond to any value of Q given in the table, but is intermediate between any two values of Q there given for the same depth of beam, the *next lower* value of Q should be selected from the table and subtracted from $\frac{M}{S}$, calling the remainder Q_1. This is expressed by the formula

$$Q_1 = \frac{M}{S} - Q. \qquad \textbf{(262.)}$$

Then, the additional weight per foot W_1 necessary to give the required value of Q $\left(\text{equal to } \frac{M}{S}\right)$, will be given by the formula

$$W_1 = \frac{Q_1}{.049\,d}, \qquad \textbf{(263.)}$$

in which d is the depth of the beam, and W_1 is the required *additional* weight per foot above the tabulated weight. That is, W_1 is the weight per foot necessary to give a section modulus equal to Q_1. From formula **261**, $Q + Q_1 = \frac{M}{S}$. If W_1 be added to the tabular weight per foot for which the value of Q was taken, the sum will be the weight per foot of the I-beam giving the required moment of resistance. It will sometimes be found, however, that the weight per foot as thus obtained will be greater than the minimum weight of the next larger size of I-beam. In such cases it will be more economical to use the larger size of the beam, although it will give a value of Q, and, consequently, a moment of resistance somewhat in excess of that required. Table 44 and formula **263** apply to steel I-beams only.

For iron I-beams,

$$W_1 = \frac{Q_1}{.05\,d}. \qquad \textbf{(264.)}$$

But iron I-beams are little used at present.

EXAMPLE.—What will, according to Cooper's specifications, be the size and weight per foot of an I-beam composed of medium steel, required to resist a bending moment of 32,340 foot-pounds?

SOLUTION.—According to item (*b*) of the specifications (Art. **1399**), the unit stress allowed on a solid wrought-iron rolled beam is 12,000 pounds per square inch, and, according to Art. **1455**, medium steel may be used for rolled beams with an allowance of 20 per cent. increase above the unit stress allowed on wrought iron. Hence, in the present case, the allowed unit stress will be $12,000 + .20 \times 12,000 = 14,400$ pounds, requiring a value of Q equal to $\frac{32,340 \times 12}{14,400} = 26.95 = \frac{M}{S}$. By reference to column Q of Table 44, it will be found that the nearest value of Q below this required value is 24.5, which is the section modulus of a 10" I-beam weighing 25 pounds per foot. By formula **262**, $Q_1 = 26.95 - 24.5 = 2.45$, and by formula **263**, $W_1 = \frac{2.45}{.049 \times 10} =$ 5 pounds. Hence, a moment of the resistance of 32,340 foot-pounds will be given by a 10" I-beam weighing $25 + 5 = 30$ pounds per foot. Ans.

POSITIONS AND CONNECTIONS OF FLOOR-BEAMS.

1695. In a through bridge, the floor-beams are sometimes supported just below the lower chord, and sometimes just above it. These two positions of floor-beams with reference to the chord are commonly designated as **beams below chord** and **beams above chord**, respectively.

1696. Beams Below Chords, Single Forged Hangers.—In the bridge we have been designing, the floor-beams are supported below the chord by a single forged beam hanger or stirrup at each lower chord joint. This method of supporting floor-beams has been very extensively employed, though it is not now as much used as formerly. One of its disadvantages is that when the diagonal members of the lower lateral system are attached to the floor-beams, which act as lateral struts, effective connections are not afforded between the lateral diagonals and the lower chords, which also act as the chords of the lateral system. This is especially the case when the lateral rods are attached to the webs of the floor-beam, as is sometimes done.

1697. Double Forged Hangers.—In order to decrease the bending moments upon the pins, two hangers at each support, called **double beam hangers,** are sometimes used. As these can usually be placed adjacent to the ties which support the pin, they will evidently produce less bending moment upon the pin than a single hanger at the center. If floor-beams were absolutely rigid, double beam hangers could be employed to advantage. But the material in floor-beams possesses elasticity, and, as a beam deflects under its load, it throws the greater part of the load upon the inner hanger, and, consequently, upon the inner tie bar. Certain devices may be used to equalize the load upon the hangers, but the cost of such devices will counterbalance the saving effected by the use of double hangers, and it will probably always be found more advantageous to use single hangers. No better connections between the lateral diagonals and the chords are afforded by double hangers than by single hangers.

A disadvantage common to all forged hangers is that, as they have screw ends, the nuts are liable to work loose. This tendency, however, may be somewhat obviated by the use of check nuts, or some one of the various devices known as nut-locks, nearly all of which are more or less effective. The best forms of beam hangers, however, have no screw ends.

1698. Plate Hangers.—An excellent form of beam hanger is composed of one or more plates attached to the end

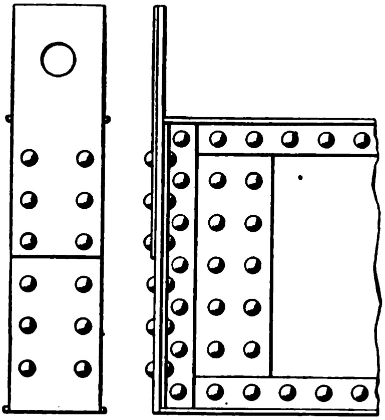


Fig. 337.

of the floor-beam, as shown in Fig. 337. The plate forming the hanger is riveted to connecting angles, which also serve as end stiffeners for the floor-beam. The hanger should preferably consist of a single plate. It must be thick enough to give sufficient bearing upon the pin, and, in compliance with formula **196** of Art. **1598**, should never be less than $\frac{5}{8}$ of an inch thick. In some cases, in order to avoid an excessive thickness of plate, it will be found expedient to attach a shorter reinforcing plate to the upper portion of the hangers, as shown. In order to prevent bending moment on the rivets attaching the connecting angles to the web of the floor-beam, and in order better to distribute the stresses upon the latter, the filler plates under the connecting angles should be attached to the web-plate by one or two rows of rivets besides those in the connecting angles.

In designing the hanger, it is necessary to provide sufficient bearing surface upon the pin, sufficient metal above the pin, and sufficient sectional area on each side of the pin hole to resist the tensile stress in compliance with item (*f*) of the specifications (Art. **1399**); enough rivets must also be used to connect the hanger to the connecting angles and to connect the latter to the web. The student has learned to do this, and it will not be necessary to give an example.

The length of the floor-beam, from out to out of connecting angles, will be equal to the distance from center to center of chords minus the thickness of one beam hanger. The length of the flange angles and web-plate should be about $\frac{1}{2}$ of an inch less than this. The plate hanger is, in some respects, the best beam hanger yet devised. It does not, however, form any better connection between the lateral diagonals and the chords than the forged hanger, when the lateral diagonals are connected to the floor beam.

1699. Stiff Beam Hangers.—Floor-beams of Pratt trusses are sometimes supported below the chord by riveting them to the angles, channels, or other shapes forming the vertical members, which are extended below the chords for this purpose. This forms an excellent attachment for the floor-beams, and affords efficient connections for the lateral diagonals, but it is not economical, and, for this reason, not much used in highway bridges.

The objection noticed with regard to double hangers, namely, that when the beam deflects under its load the inner bearings will take the greater part of the load, to some extent applies to this form of support also. This condition is, however, somewhat counteracted by the stiffness of the posts, and may be further obviated by the use of deep (and, consequently, stiff) floor-beams. In designing this form of support, the bearing upon the pins and attachment of pin plates must be sufficient to provide for both the stress upon the post and the load upon the beam.

1700. Beams Above Chords.—In through Pratt truss bridges, the floor-beams are not uncommonly supported above the lower chords. Where the head-room is sufficient to allow it, this is the best position for the floor-beam; it affords excellent connective details and is economical. With this detail, the lateral rods may be attached to the lower flanges of the floor-beams; in which case, stress in the lateral rods will neutralize a small amount of the tension in these flanges.

The floor-beams are simply riveted to the vertical members, the form of the connection depending upon the form of the member. If each of the posts consists of four angles, latticed in the form shown in Plate, Title: Highway Bridge: Details I, Fig. 3, the floor-beam is usually riveted between the angles forming the post, as shown in Fig. 338. In this detail the pin plates must be proportioned to take not only the stress upon the post, but also one-half the total load upon the beam. In order that the beam may be placed near the lower ends of the angles forming the post, the metal along the center line of the inner pin plate must be cut away through the upper portion of the plate, in order to straddle the web-plate of the beam. The metal is cut away by punching a row of contiguous holes with a square punch, making a continuous cut for the proper distance down the center of the plate, beginning at the upper end. There is some objection to thus cutting a slot down the center of the inner pin plate, but if it is not done it will be necessary to elevate the beam sufficiently to clear the tops of the pin plates, which would further decrease the head-room and also require a stay plate in the post below the floor-beam.

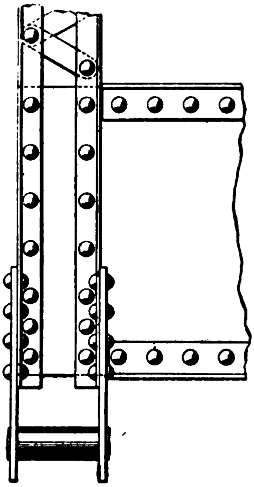


FIG. 338.

The condition of unequal loading upon the pin plates due to the deflection of the beam, noticed with regard to double hangers and the attachment to the post below the chord, will, to some extent, be present in this detail also, and will have the further disadvantage of producing a bending moment upon the post. To obviate this, floor-beams are sometimes made to connect upon a pin placed in the axis of the post. Various other devices are used to deliver the floor-beam load equally upon the pin plates. But, if the floor-beam is made deep enough to prevent excessive deflection, the simple detail shown in Fig. 338 is believed to be about as efficient as any that can be devised. It is to be noticed that the deflection produced in the post is usually in the plane of its greatest strength, and that, while slight deflection is produced in the post, it is by the beam also supported against further deflection.

The detail represented in Fig. 338 is for connecting the floor-beam shown in Plate, Title: Highway Bridge: Details II, Fig. 4, to the post shown in Plate I, same title, Fig. 3.

NAME PLATES AND RAILINGS.

1701. These two features of the design of bridges have thus far been entirely neglected. The design of either the name plates or the railing does not involve any real principles of engineering, and will, therefore, require but a passing notice.

Name Plates.—Upon nearly all metal bridges are attached cast-iron name plates giving the name of the company constructing the bridge, with date of construction, and sometimes other data, such as the names of the officials purchasing the bridge, etc. This information is given by raised letters cast upon a plate of suitable size. The names are formed by attaching the letters in the proper positions upon the wooden pattern used in casting the plate. Letters are made especially for this purpose and are usually kept in stock; they can be purchased at almost any hardware store. The name plates are attached by rivets or bolts to conspicuous portions of the bridge; if a truss bridge, they are usually attached to the portals or to the end posts.

It would be well if the guaranteed capacity of the bridge, as well as the name of the builder and date of construction, were required to be shown on the name plate.

1702. Wood Hub Guards.—Railings are used for two general purposes, namely, for closing the openings between the web members of the trusses along the outer edges of the roadway, and for hand railings along the outer edges of the sidewalk. Railings along the outer edges of the roadway, for the purpose of closing the openings between the members of the truss, may serve also to protect the members from the hubs of passing vehicles. In fact, this is commonly understood to be the principal office of these railings, and they are generally known by the name of **hub guards.**

For many county bridges each hub guard consists simply of a line of plank attached to the posts of the truss at the height of an ordinary wagon hub above the roadway floor. This is commonly a 2″ × 10″ plank, dressed on one side and painted, and bolted to the posts so that its center will be about 2½ feet above the roadway floor. In such cases no attention need be given to the hub guard in making the shop drawings, further than to show bolt holes in the posts at the proper distance above the lower chord pins for attaching the hub guard, and some arrangement for attaching it to the hip verticals. If the hip verticals consist simply of round or square bars, this attachment will usually be a **U**-shaped bolt or a bolt made in the form of a hook, so that it will hook around the hip vertical rod.

Wood railings, of the form shown in Fig. 339, are sometimes placed along the outer edges of the roadway. Many of the highway bridges in the central, western, and southern portions of the United States have short trestle approaches, and in such cases this railing extends along the outer edges of the roadway across the entire bridge and approach. These railings are not attached to the trusses, but the posts are halved upon and bolted to the outer lines of roadway joists, as shown. The construction will be readily understood from the figure. Although commonly constructed as here shown, these railings are sometimes made lighter, the posts being 4″ × 4″, the upper railing pieces 2″ × 4″, and the lower railing pieces 2″ × 8″. Although this may be called a hand rail, it is here noticed under the head of wooden hub guards.

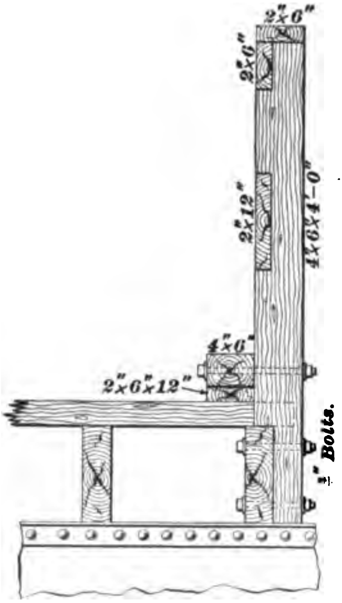


FIG. 339.

1703. Iron Hub Guards.—On some county bridges, and on most city bridges, iron hub guards are used. In some cases, each hub guard consists simply of two lines of gas pipe, about 2 inches in diameter, attached to the posts of the truss. When gas pipe railing is used it is necessary only to designate the required size and lengths of the pipe, with the couplings, and show the arrangement of the fastenings by which it is attached to the posts. Usually, some kind of standard fastenings are used, which simply require certain holes to be shown in the posts.

What is probably the best form of hub guard consists of two angle bars latticed. This railing is usually from a foot to 18 inches in depth; that is, one angle is placed from a foot to 18 inches above the other. A double system of latticing is used, composed of light lattice bars making an angle of 45 degrees with the angles. The general construction of such hub guards, with the splices and fastenings, must be shown on the shop drawings. In Fig. 340 is shown a portion of a latticed hub guard, with the manner in which it is attached to the intermediate post; it is composed of two of the lightest 3″ × 2″ angles, double latticed with 1¼″ × ⅛″ lattice bars. As here shown, it is attached to the intermediate post by means of short pieces or clips of the same size of angle. The design of the hub guard is merely a matter of detail; no further explanation will be required.

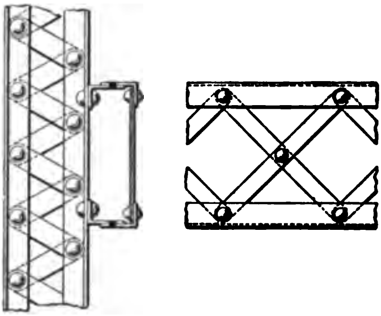


FIG. 340.

1704. Hand Rails.—Many different designs are used for hand rails, all being more or less ornamental. All hand rails, however, consist essentially of an upper and lower horizontal rail, connected by a webbing of lattice or other light ornamental work. Sometimes a lighter rail is also placed about midway between the upper and lower rails.

The lower rail will generally consist of two angles, while if a middle rail is used it will usually be a single angle.

The upper rail is made in various forms and is often quite ornamental. The top of the upper rail is usually about 3 feet 9 inches above the sidewalk floor, and the lower rail should not be more than 6 inches above the floor. The openings in the lower part of the lattice work should not be more than 6 inches square.

The rails are supported by posts attached to the ends of the floor-beams. They should usually be braced at intervals of about 8 feet. Both rails are connected and supported at each post, but it is quite commonly the case that the lower rail only is braced at intermediate points. It will not be necessary to show here any designs of hand rails, but it will be well to show the manner in which the posts are connected to the ends of the floor-beams, because this connection forms a part of the engineering design of a structure. The posts quite commonly consist of two angles each, but sometimes consist of four angles each, and sometimes of other forms.

1705. Attachment of Hand-Rail Posts.—Posts are sometimes riveted rigidly to the ends of the floor-beams, as shown in Fig. 341. This forms an excellent attachment, so far as the attachment itself is concerned, but it is very inconvenient for getting the upper rail in straight line. The attachment not being adjustable, if the rail is not put in perfect line before the posts are riveted, or if from any cause it should afterwards get out of line, it would be very difficult, if not impossible, to put the rail in perfect alinement. For this reason various adjustable attachments are used, by means of which the rail can be lined up without difficulty.

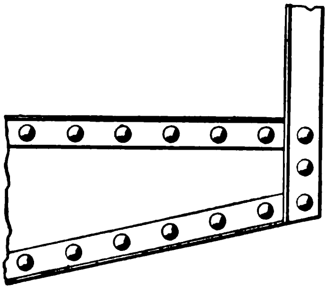


FIG. 341.

A very common adjustable attachment is shown in Fig. 342. It consists of a short brace connecting the post with the projecting end of one of the lower flange angles. The brace passes through a hole in the flange angle, and the attachment is made adjustable by means of two nuts upon the brace screwing against the flange. By means of this adjustment the top of the post may be readily thrown in or out and the railing be put substantially in line before the last rivet attaching the post to the floor-beam is driven. But even after the post is riveted, its top can, to some extent, be sprung in or out by this adjustment. A simple, compact, and efficient adjustable attachment is shown in Fig. 343. In this attachment, the two angles of the post, passing upon each side of the web of the floor-beam, are attached directly to the web by a single bolt or rivet, and, by means of a short headless bolt carrying two nuts at each end, the lower end of each angle is attached to a short angle clip riveted upon the lower portion of the beam. As will be clearly understood from the figure, the position of the post can be readily adjusted by means of these bolts.

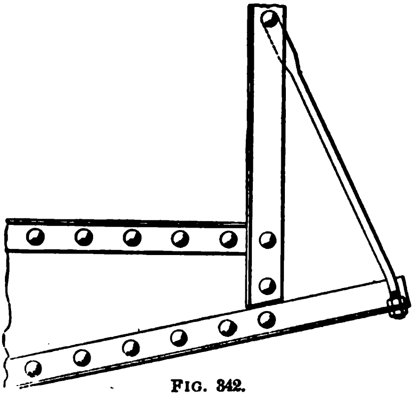


FIG. 342.

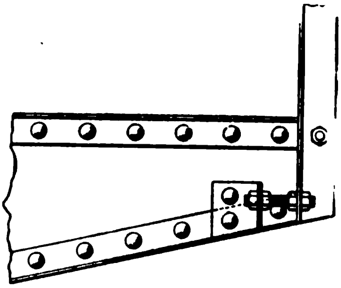


FIG. 343.

PAINT AND PAINTING.

1706. General Considerations.—The thorough painting of metal bridges is of great importance. The members of iron or steel bridges do not commonly wear out from use, although they may be somewhat impaired; but they deteriorate very materially through the action of rust. Iron or steel exposed to the action of the atmosphere will soon begin to oxidize or rust, and the process, once begun, will continue rapidly. In order to preserve the metal from rusting, it must be well protected by paint or other impervious coating. A paint that will preserve iron work from rusting must afford a coating to the surface of the metal that is absolutely impenetrable to air, moisture, or gases; it must protect the metal perfectly from the destructive influence of oxygen, which will combine with it to form rust (iron oxide) whenever moisture is present. A paint consists of a pigment or coloring body, ground and carried in suspension in linseed oil or other suitable carrier. The value of the pigment will be the greater the more finely it is ground; the greater the proportion of finely ground and well-mixed pigment the paint contains, the better it will generally be.

1707. The Kinds of Paint Used.—The kinds of paint most commonly used for metal bridges are red lead, white lead, asphaltum, iron oxide, graphite, and various proprietary or patented mixtures. The relative merits of these various paints will be briefly noticed. The amount of reliable information on this subject is not great, and there exists much difference of opinion concerning it, but the statements made below are believed to be fair and, as nearly as possible, reliable.

1708. *Red lead* is a pure oxide of lead (triplumbic tetroxide $Pb_3 O_4$), and, when mixed with pure linseed oil, appears to be the best paint known for preserving iron work from rust. When subjected to the severest test this paint has shown good results. It is extensively used for painting important structures, and is quite generally conceded to be superior to all other kinds of paint as a priming coat for metal work.

1709. *White lead* (75 per cent. lead carbonate and 25 per cent. hydrated lead oxide), when pure and mixed with pure linseed oil, probably deserves to be classed next to red lead as a protective paint. As, however, one of the constituents of white lead is a hydrated oxide, it is not as suitable a paint for iron work as for wood work; it is not extensively used for painting metal work. White lead is very commonly adulterated with white clay.

1710. *Asphaltum paint* is generally a mixture of asphalt (a solid bitumen or natural pitch) with certain forms of lead, minerals, and coloring matter, ground in linseed oil. This paint is quite extensively used for painting metal structures. Though not generally considered equal to red lead as a coating for metal, pure Trinidad asphalt, properly prepared and ground in pure linseed oil, forms a durable paint for metal work and is commended by its cheapness. It is, however, disadvantageously affected by the extremes of heat and cold. It is often adulterated with coal tar, which is not a good material for paint.

1711. *Iron oxide* has been very extensively used for painting metal structures. It is used in several forms and mixtures, which are known by various names, such as *iron clad paint*, *armor paint*, *iron ore paint*, *mineral paint*, *magnetic paint*, etc. With regard to the chemical composition of iron oxide paint, the following may be stated:

There are three well-defined oxides of iron which occur in considerable quantities in a natural state, and may be produced artificially; nearly, if not quite, all commercial oxides are mixtures of these. The magnetic oxide is generally black in color, on which account it is not extensively used as a pigment. The anhydrous sesquioxide (red hematite) varies in color from a bright red to a dark brown. This is the most common oxide of iron. The hydrated sesquioxide (brown hematite) is a compound containing a large percentage of water; it often contains also sulphur and other organic acids, as impurities. Iron rust is a hydrated oxide. All iron oxide paint probably contains, or partly consists of, hydrated oxide or, practically, iron rust. Moreover, the percentage of earthy materials and other impurities in iron oxide paint is large.

It is believed by many that when iron oxide paint is applied to iron it tends to superinduce corrosion of the surface coated. Whether this is really the case or not, iron oxide paint can not be said to possess great merit as a preservative of iron.

1712. *Graphite* is to some extent used as a paint for metal structures. This material is sometimes called "black lead," but it is not lead in any form nor similar to lead; it is carbon. Graphite is, however, a permanent and neutral substance, and is the most indifferent to chemical influence of any known pigment. But, as a practical paint, graphite possesses certain decided disadvantages. It is so light that it is impossible to get any considerable percentage of it into the oil, and it is also flaky and greasy, partaking rather of the nature of a grease than of a true paint. Although graphite doubtless possesses considerable merit as a paint, it is said not to be impervious to water nor have sufficient body to be a really efficient preservative of metal work.

1713. *Proprietary or patented paints* are generally not to be relied upon. As their ingredients are kept secret, little can be said concerning them, except that all secret mixtures are to be looked upon with suspicion until their merit or lack of merit has been fully proved. For iron work, at least, it is a good rule never to use paint purchased upon the market in a ready-mixed form as prepared paint. (Asphaltum paint may, however, be considered as a possible exception to this.) Several mixtures of this kind, when analyzed, have been found to contain large percentages of such adulterants as carbonate of lime, red clay, etc. It must be conceded, however, that certain proprietary paints have been found quite efficient for protecting iron work.

From what has been said it may be concluded that the most reliable paints for metal work are red lead and asphaltum. Of these, red lead is the better, while asphaltum is, in its first cost, the cheaper. Red lead is believed to be the best paint that can be used for the priming coat on iron work. Other paints, however, are sometimes preferred for the outside or finishing coat.

1714. Covering Capacity of Paints.—When used under ordinary conditions, the covering capacity of the common kinds of paint is reasonably well known. By the term **covering capacity** is meant the amount of surface that can be covered by a certain amount of paint; as usually expressed, it is the area, in square feet, that can be covered by a gallon of paint. For the various kinds of paint mentioned above, used on new work with the average conditions of surface, the covering capacities will generally be about as given in the following table:

TABLE 45.

AREA COVERED BY ONE GALLON OF PAINT.

Kind of Paint.	Area in Square Feet.	
	First Coat.	Second Coat.
Red lead	700	1,000
White lead, iron oxide, or graphite	500	700
Asphalt	300	500

The covering capacity given for red lead is for the mixture known as the B. & O. Railroad formula; namely, ten ounces of lampblack to twelve pounds of red lead. The resulting paint is a very dark brown, nearly black. For repainting old work, the covering capacity of the first coat will generally be about half as much as for the first coat on new work.

1715. The Use of Red Lead.—Red lead should always be purchased as a dry paint, mixed with pure linseed oil, and *used when freshly mixed.* A mixture of pure red lead and linseed oil partakes somewhat of the nature of a soap or cement; it will "take a set" in a few days. This "setting" of red lead mixed with linseed oil is, by some authorities, spoken of as **saponification,** but Professor Spennroth, of Aix-la-Chapelle, states that he has proved by numerous experiments that saponification does not take place with any of the usual pigments. However this may be, on account of its setting quality, red lead can not be sold as a mixed paint; it should always be mixed fresh each day as used. Red lead paint a day old, or which has begun to set, should not be used.

Red lead combines more perfectly with linseed oil when the raw (unboiled) oil only is used, but, on account of the weight of the lead, such paint must be mixed very thick in order to prevent the particles of lead from settling in the vessel or "running" on the painted surface. It should be used as thick as may be expedient. When the mixture is desired to be thinner than is obtainable with raw oil alone, a small quantity of japan should be used, or boiled oil may be used instead of raw oil.

1716. Proportions for Mixing Red Lead Paint.—For painting iron work, the proportion of red lead to linseed oil may be from 3 to 5 parts of red lead to 1 part of linseed oil *by weight*, or say from 24 to 40 pounds of red lead to each gallon of linseed oil. About 4 to 1 by weight, or say 30 pounds of red lead to 1 gallon of linseed oil, is a good average mixture. If mixed in about this proportion, 20 pounds of red lead in 5½ pounds (.7 of a gallon) of linseed oil will make a gallon of paint. When japan is used as a dryer, there should be, by either weight or measure, about one part of japan to 10 parts of linseed oil.

The color of red lead mixed with pure linseed oil may be described as a bright brick red, or about midway between a bright vermilion and a terra cotta. When a darker shade is desired, it can be obtained by adding a small amount of either lampblack or graphite. One-sixteenth of an ounce of lampblack to one pound of red lead will produce a rich chocolate color, while an ounce of lampblack to a pound of red lead will produce a very dark brown, scarcely distinguishable from black.

1717. Amount of Painted Surface.—In bridges of different forms, the amount of surface to be painted will vary through a considerable range; it will not be exactly or very closely proportioned to any fixed dimension of the bridge, and, therefore, can not be accurately expressed by a formula. Some forms of trusses will have an area of painted surface much greater than others of the same span and capacity. The most reliable practice is to estimate the actual amount of surface to be painted in each case, although this involves an amount of labor that is not usually warranted by the importance of the result.

For ordinary Pratt truss bridges, however, carrying a single roadway, the following formulas are probably as satisfactory as any that can be proposed.

For highway bridges of 16 feet roadway and a live load capacity of 90 pounds per square foot:

$$A = \frac{L}{3}(L - 20) + 400. \qquad \textbf{(265.)}$$

For single track railroad bridges of 80 feet span and over:

$$A = 4L\left(\frac{L}{10} + 7\right). \qquad \textbf{(266.)}$$

In both of which A is the total area of painted surface in square feet, and L is the length of span in feet.

1718. Preparing Iron for Painting.—Before iron work is painted it should be thoroughly cleaned of all rust, scale, and dirt. *This is of great importance, and should never be neglected.* It is probable that an ordinary quality of paint on well-cleaned iron will more effectually preserve the metal than the best quality of paint upon a rusty surface. The progress of the oxidizing or rusting process, when once begun, may be somewhat checked by the application of paint, but it can not be stopped by it. The rusting process will continue slowly under the paint until the latter will peel off with a layer of rust. Although the progress of the rusting, when once begun, can not be prevented by paint, it will probably be as well opposed by red lead as by any paint.

New iron work, before being painted, should be well cleaned. All loose scale, dirt, and rust should be removed.

Wire brushes are used for this purpose. Cooper's specifications require that "all iron work before leaving the shop shall be thoroughly cleaned from all loose scale and rust, and be given one good coating of pure raw linseed oil, well worked into all joints and open spaces." This specification is often but indifferently complied with.

Pure raw linseed oil is specified for the priming coat because to some extent it allows the condition of the surface of the iron beneath it to be seen during the construction of the work and before the application of the second coat. Except that it removes all opportunity for subsequently inspecting the condition of the surface of the metal, red lead mixed with raw linseed oil forms a much better priming coat for iron work than the linseed oil alone.

A surface of rusting iron, or one covered with old decomposing paint (on which more or less rust is always present), will require more thorough treatment. Such a surface may sometimes be cleaned by a thorough and vigorous application of wire brushes. But, if the surface is considerably rusted, it may be necessary to use kerosene or a dilute solution of sulphuric or muriatic acid in connection with the wire brush treatment. A badly rusted surface should be well scrubbed with water containing about 25 per cent. of muriatic acid or about 3 per cent. of sulphuric acid, after which it should be well washed with lime water to neutralize the acid, then thoroughly dried before painting.

IMPERFECT DESIGN AND INCONSISTENT REQUIREMENTS.

1719. The design of all details of the bridge shown on the Mechanical Drawing plates, and certain features of bridge design not there shown, have been explained. Many of the details shown in those drawings are not the best details possible, and they do not, in all cases, comply with the requirements of the specifications. This design was made for the purpose of instruction in Mechanical Drawing, and for convenience was used to illustrate the principles of bridge design as taught in this Course; it is not given as an example of perfect design. It will be very instructive to notice now a few of the imperfections of this design.

1720. Light Weights and Thin Metal.—In this design, the lightest weights of 8-inch channels (10 pounds per foot) are used for the upper chord. The metal in the webs of these channels is about $\frac{7}{32}$ (.21) of an inch in thickness, whereas, according to the specifications, no metal should be less than $\frac{1}{4}$ of an inch in thickness. These channels should be of such weight as would give $\frac{1}{4}$ of an inch thickness of metal in the web. The increase of the weight, corresponding to the required increase in the thickness of the web, may be calculated by formula **174,** Art. **1555.** The required increase in the thickness of the web is $.25 - .21 = .04$ of an inch, and, by applying this formula in the present case, we shall have the equation $.04 = \frac{.3y}{8}$, from which $y = 1.07$ pounds. The weight per foot of each channel in the upper chord should, therefore, be $10 + 1.07 = 11.07$, or say 11 pounds per foot.

In making a design, it is a good plan to use, so far as possible, shapes in which the thickness of metal in the lightest weights rolled is not less than the minimum thickness allowable, which is usually $\frac{1}{4}$ of an inch. But when the thickness of metal used is less than $\frac{1}{4}$ of an inch, it should not be counted as $\frac{1}{4}$ of an inch in computing the bearings upon the pins and rivets, as is not infrequently done. It is exceedingly bad practice to use metal less than $\frac{1}{4}$ of an inch in thickness for any important part of a member.

1721. Countersunk Rivets.—Rivets should not be countersunk in metal having a thickness less than one-half the diameter of the rivet; rivets $\frac{5}{8}$ of an inch in diameter should not be countersunk in metal less than $\frac{5}{16}$ of an inch thick. As the webs of the channels in the upper chord are less than $\frac{5}{16}$ of an inch in thickness, the rivets in them should not be countersunk. At the hip joint, the rivets nearest to the pin in the pin plate should be located at a sufficient distance from the pin to clear the ends of the pin plates on the end post without being countersunk. Countersunk rivets are not, in any case, as reliable as rivets with full heads, and are more expensive; they should be avoided where possible.

1722. Unsymmetrical Sections.—Symmetrical sections should be used for compression members whenever possible. It is a very general practice, however, to use unsymmetrical sections for top chords and end posts, and there are some advantages in using them. In some cases symmetrical sections composed of two channels latticed on both sides are used for these members, but much more commonly a cover-plate is attached to the upper flanges of the channels, while the lower flanges are stayed by lattice bars, thus forming an unsymmetrical section. In case the section is so heavy that it has to be composed of plates and angles, the angles forming the lower flanges can often be made enough heavier than the angles of the upper flanges to counterbalance the section of the cover-plate. This will give a section which, though not symmetrical in form, is not eccentric with reference to the position of its center of gravity, and may be treated as a symmetrical section.

In any compression member connecting upon pins, the pins should be located in such position that the stress upon the member will be uniformly distributed over its section. If the member is not vertical, the bending stress due to the weight of the member itself must be considered in fixing the position of the pins, as was explained in Art. **1498.** When this is done, further consideration of the bending moment, due to the weight of the member only, may be neglected.

1723. Eccentricity of Pin Plates.—One serious disadvantage of chord sections in which the pins are not at, or very near, the centers of the channels or side plates, is that the bearings upon the pin plates are generally eccentric with reference to the rivets attaching the pin plates. This produces a sidewise bending movement upon the pin plate, which, although it has no very injurious effect upon the plate, increases the stresses upon some of the rivets. It is possible to obtain a satisfactory solution for the intensities of the stresses upon the rivets, but, as the solution is rather complex, it is doubtful whether it would be worth while to apply it to the rivets of ordinary pin plates, and it will not, therefore, be given here. In practice, the number of rivets in a pin plate should be increased somewhat when the pin bearing upon the pin plate is eccentric. When the pins in a member are but slightly eccentric, the positions of the rivets in the pin plate, with reference to the pin, can usually be so arranged that the pin bearing will not be eccentric with reference to the rivets.

1724. Pitch of Rivets at Ends of Members.—The specifications require that "the pitch of rivets at ends of compression members shall not exceed four diameters of the rivets for a length equal to twice the width of the member."

This requirement was not strictly complied with in the rivet spacing on the chord and end post of the bridge considered. The rivet spacing on these members could be considerably improved by being made more uniform and being also made to comply with the requirements of the specifications. All rivet spacing should be as regular and uniform as possible; irregular spaces should be avoided.

1725. Requirements for Fatigue of Metals.—In Art. **1466,** it was noticed that some specifications take the fatigue of metal into consideration in determining the allowed unit stresses. It appears from the experiments of Spangenberg that it is in the case of alternating stresses only that the material fails from repetition of stresses below the elastic limit; the material does not fail from repeated stresses of the same character, so long as they do not exceed the elastic limit. As the unit stresses allowed are always well within the elastic limit of the material, it is evident that for stresses of the same character the fatigue of the metal may be neglected.

But for stresses alternating between tension and compression, the material is found to fail under stresses considerably below the elastic limit, if repeated a sufficiently great number of times. Consequently, for this condition the fatigue of the metal should always be considered.

1726. Requirements for Flat-Ended Columns.—In some specifications greater unit stresses are allowed for flat-ended columns than for pin-ended columns, and this is a very common practice. Theoretically, flat-ended columns are stronger than pin-ended or round-ended columns of the same dimensions; from experiments also this is found to be the case when the bearings of the flat-ended columns are perfect, and the load upon them is symmetrical with reference to their neutral axes. But when the bearings of a flat-ended column are imperfect, or when from any reason the load upon the column becomes eccentric, the flat-ended column may be even weaker than a round-ended column of the same dimensions. A pin-ended column is practically round ended with reference to a plane perpendicular to the axis of the pin.

The load upon a properly constructed round-ended column will be applied at its neutral axis, and the stresses will be uniformly distributed over its section, while the load upon a flat-ended column will nearly always be more or less eccentric. The eccentricity of the load produces a bending moment upon the column; the latter, must, therefore, resist the bending moment as well as the actual compression.

Thus, it is seen that while a flat-ended column is stronger than a round-ended column when perfect conditions of loading are attained, it may be weaker when perfect conditions are not attained. As the conditions of the bearings in flat-ended compression members are usually such that the members are loaded more or less eccentrically, it is evident that flat-ended compression members can not consistently be allowed greater unit stresses than pin-ended members.

In a bridge, some of the compression members may be allowed higher unit stresses than others, according to the general form of their connections, their liability to eccentric loading, and the nature of their stresses. For bridge work it is probably the best practice to treat all columns as pin-ended, and, in specifying the allowed unit stresses, designate the members to which they apply.

CONCLUDING REMARKS.

1727. The design of a structure should be simple; it should, as far as possible, conform to the requirements of the stresses, and be complete in every detail. Each part and detail of a structure should be strong enough to bear alone the stress for which it is intended. The structure as a whole can not be stronger than its weakest part, and it is useless to make one part relatively stronger than another.

Each detail should not only have the required strength, but it should be simple and practical. At each joint where several members connect, the connections should, if possible, be so designed that the axes of all the connecting members shall intersect at a common point. The designing of such details requires skill, judgment, and experience, as well as careful consideration of all the conditions attending each case. Each detail should be well studied and made as simple and practical as possible, having in mind the economical requirements of the shop work and erection. The simplest details are the most difficult to design.

Where there are several similar details they should, if possible, be made alike, as the shop work is thereby much facilitated. Familiar details should generally be used. In nearly every bridge office certain standards of construction are adopted. These should be used whenever practical, because the shop men are familiar with them, but a good detail should never be sacrificed for the sake of using a standard.

The shop drawings should be so made as to be readily understood by the workmen. All letters, words, and figures written on them should be plain and easily read. In making shop drawings it must be borne in mind that they are for the purpose of conveying exact information to the shop men, who are not supposed to have, and generally have not, any knowledge concerning the structure other than that obtained from the drawings. The information given on the drawings must, therefore, be complete and explicit.

Information once given should generally not be repeated; dimensions or rivet spacing shown on a top or bottom view should not be again shown on the side elevation. Such repetitions not only require unnecessary labor upon the drawings, but tend to confuse the workmen. Where two members are exactly alike except that the positions of certain corresponding dimensions are reversed, as the portal connections on two end posts, it is sufficient to show the dimensions in one position and mark the members *right* and *left*. It is generally sufficient to show half top and half bottom views of chords and end posts; in such cases they are shown together in one half top and half bottom view, both being shown *as seen from above*. The lower ends of verical and diagonal members should generally be at the left on the drawing.

No computations whatever should be left for the workmen to do; every necessary calculation relating to a structure should be performed in the office. The workmen will do well if they properly carry out the construction according to the drawings. Although the dimensions for any part of a bridge are never taken by scale from the working drawings, the scale, together with the title and date, should, for convenience in the office, be given in the lower right-hand corner of each drawing.

A SERIES

OF

QUESTIONS AND EXAMPLES

RELATING TO THE SUBJECTS
TREATED OF IN THIS VOLUME.

It will be noticed that the questions and examples contained in the following pages are divided into sections corresponding to the sections of the text of the preceding pages, and that each section has a headline which is the same as the headline of the section to which the questions refer. No attempt should be made to answer any questions or to work any examples until the corresponding part of the text has been carefully studied.

STRENGTH OF MATERIALS.

(ARTS. 1184–1266.)

EXAMINATION QUESTIONS.

(640) (*a*) What are the advantages and disadvantages of cast iron as a material for construction purposes ? (*b*) Of steel ? (*c*) Of wrought iron ? (*d*) For what purposes is each of the above largely used ?

(641) Define the following: (*a*) Stress; (*b*) strain; (*c*) elasticity; (*d*) coefficient of elasticity; (*e*) ultimate strength.

(642) In how many ways may stress be applied to a body? Give an example of each.

(643) How much will a wrought-iron rod 2" in diameter and 10 feet long be elongated by a pull of 40 tons ?

Ans. .12223 in.

(644) A steel rod 7½" long and ½" in diameter is elongated .009" by a pull of 7,000 pounds; what is the coefficient of elasticity ? Ans. 29,708,853 lb. per sq. in.

(645) What force is required to produce an elongation of .006" in a cast-iron bar 1½" × 2" and 9 feet long ?

Ans. 2,500 lb.

(646) A wooden rod 3" in diameter is elongated .05" by a force of 2,000 pounds; what was its original length ?

Ans. 265.07 in.

(647) What is the minimum diameter of a wrought-iron bolt which is to withstand a steady pull of 6 tons with safety ? Ans. 1.054 in.

(648) How long must a cast-iron bar be if supported vertically at its upper end, to break under its own weight ?

Ans. 6,400 ft.

(649) The diameter of a wrought-iron bolt is ¾"; what should be the thickness of the bolt head in order that the bolt may be equally strong in tension and in shear; in other

For notice of copyright, see page immediately following the title page.

words, what should be the thickness of the head in order that the force required to pull the bolt apart shall be just equal to the force required to strip the head from the bolt ?

Ans. .206 in.

(650) What safe load may be borne by a brick pier, the cross-section of which is 2½ × 3½ feet ? Ans. 105 tons.

(651) In Fig. 28, the force *P* is 1¾ tons and acts at an angle of 30° with the horizontal. The width *b* of the timber tie-rod is 4 inches. Find the safe length *a* necessary to prevent shearing on the surface *a b*.

Ans. 10 1 in.

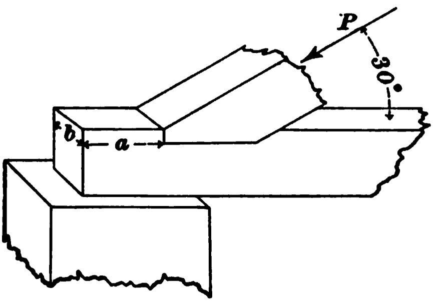


FIG. 28.

(652) Show that a cylindrical boiler is twice as liable to rupture along a longitudinal seam as along a transverse seam.

(653) Assuming that the strength of the boiler plate is reduced 40% owing to the area lost by the rivet holes, what should be the thickness of the plate of a wrought-iron boiler 4 ft. in diameter which sustains a steam pressure of 120 lb. per sq. in. ? Assume factor of safety for steady stress.

Ans. .35 in.

(654) What should be the thickness of a 6″ cast-iron water pipe to carry a steady pressure of 200 lb. per sq. in. ?

Ans. .18 in.

(655) What should be the thickness of a wrought-iron boiler tube 3″ in diameter,. 12 ft. long, and exposed to an external steam pressure of 130 lb. per sq. in. ? Use a factor of safety of 10. Ans. .272 in.

(656) A cast-iron cannon of 4″ bore is subjected to an internal pressure of 2,000 lb. per sq. in. on being fired; what should be the thickness of the metal in order that it shall not be subjected to a stress of more than 2,800 lb. per sq. in. ?

Ans. 5 in.

(657) A structure of the nature of a simple beam is 100 feet long between the supports, and carries three equal loads of 1,200 lb. each, at distances from the left support of 40, 60, and 80 ft. (*a*) Find the maximum bending moment; (*b*) the shear at a distance of 30 ft. from the left support; (*c*) the maximum shear.

Ans. { (*a*) 748,800 in.-lb.
(*b*) 1,440 lb.
(*c*) – 2,160 lb.

(658) A beam is loaded as in Fig. 29. Find the reaction of the supports, and draw the shear diagram.

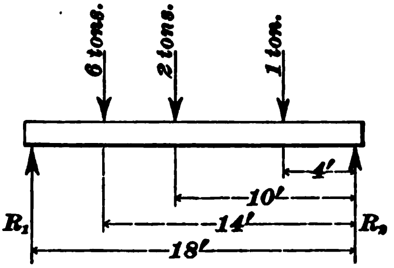


FIG. 29.

(659) Define the following: Simple beam; cantilever; restrained beam; continuous beam.

(660) A rectangular white pine beam 22 feet long carries a load of 4 tons, 8 feet from one end. Assuming the depth of the beam to be double the width, what should be its dimensions to safely support the load?

Ans. Depth = 17⅓ in.; breadth = 8⅔ in.

(661) The moment of inertia of a wrought-iron I-beam, 12" deep, is 280; what uniform load will it sustain? The stress is assumed to be steady. The beam is 20 ft. long and rests upon two supports. Ans. 875 lb. per ft.

(662) A hollow circular cast-iron beam, 8 feet long, rests upon two supports. The inside diameter is 5 in. and the outside diameter 6½ in.; what is the maximum safe load that may be concentrated at its center? Ans. 4,624 lb.

(663) A hemlock floor-beam is 2" × 10" and 16 feet long; (*a*) what distributed load will it carry? (*b*) What distributed load would it carry if laid with the 10" side horizontal?

Ans. { (*a*) 78.12 lb. per ft.
(*b*) 15.6 lb. per ft.

(664) Find the deflections of the beams of (*a*) example 661, (*b*) example 662, and (*c*) example 663 (*a*).

$$\text{Ans.} \begin{cases} (a) & .45 \text{ in.} \\ (b) & .1 \text{ in., nearly.} \\ (c) & .461 \text{ in.} \end{cases}$$

(665) The piston of a steam engine is 14 in. in diameter; the steam pressure is 80 lb. per sq. in. Assuming that the total pressure on the piston comes on the crank-pin at the dead points, and considering the crank-pin as a cantilever uniformly loaded, what should be its diameter if 4 inches long and made of wrought iron? Take a factor of safety of 10. Ans. 3.82 in.

(666) What load can be safely sustained by a cast-iron column 14 feet long and 6″ in diameter, with flat ends?

Ans. 120,872 lb.

(667) What should be the dimensions of a piece of timber 30 ft. long, with flat ends, to support a load of 7 tons, the cross-section being in the form of a square?

Ans. $9\frac{5}{8}$ in. square.

(668) A cylindrical steel connecting-rod $7\frac{1}{2}$ ft. long is subjected to a maximum stress of 21,000 pounds. Assuming it to be a column with both ends hinged and subjected to shocks, determine its diameter to the nearest $\frac{1}{8}$″. Ans. $2\frac{5}{8}$ in.

(669) A wrought-iron piston rod has a diameter of 2″ and a length of 4 feet. Assuming it to be a column with one end flat and the other rounded, what is the allowable diameter of the piston if the steam pressure is 60 pounds per sq. in.? The rod is subjected to shocks. Ans. $15\frac{7}{8}$ in.

(670) A beam is 6″ × 8″ and 10 feet long, the 8″ side being vertical. Another beam of the same material is 4″ × 12″ and 16 feet long, the 12″ side being vertical. (*a*) What is the ratio between the maximum loads the two beams are capable of supporting? (*b*) What is the ratio between the deflections in the two cases, the manner of loading being the same?

$$\text{Ans.} \begin{cases} (a) & 1\frac{1}{18} : 1. \\ (b) & .549 : 1. \end{cases}$$

(671) (*a*) What should be the diameter of a round

wrought-iron shaft to transmit 40 H. P. at 120 R. P. M.? (*b*) To transmit 80 H. P. at 100 R. P. M.? Ans. { (*a*) 3.739 in. (*b*) 4.65 in.

(672) What must be the diameter of a steel engine shaft to transmit 4,000 H. P. at 50 R. P. M.? Ans. 14.22 in.

(673) What H. P. can be transmitted by a steel shaft 4" square, making 80 R. P. M.? Ans. 71.775 H. P.

(674) What horsepower can be transmitted by a hollow wrought-iron shaft making 100 R. P. M., the outside diameter being 7½ in. and the inside diameter 5 in.? Ans. 717.7 H. P.

(675) (*a*) What weight may be safely sustained by a hemp rope 8" in circumference? (*b*) What should be the diameter of an iron-wire rope of 7 strands to safely sustain a stress of 6,000 pounds? (*c*) What should be the circumference of a steel hoisting rope which is required to lift a load of 6⅔ tons? Ans. { (*a*) 6,400 lb. (*b*) 1.054 in. (*c*) 3.65 in.

(676) (*a*) An open-link chain is made of ⅞" iron; what safe load will it sustain? (*b*) What should be the diameter of the iron composing the links of a stud-link chain which is to sustain a load of 4 tons? Ans. { (*a*) 9,187.5 lb. (*b*) .667 in.

(677) A steel shaft 2" in diameter is supported by hangers 10 feet apart; if a pulley be placed midway between the hangers, what should be the maximum allowable belt pull on the pulley in order that the shaft shall not deflect more than ⅛"? Ans. 327.25 lb.

SUGGESTION.—Assume the shaft to be a restrained beam, loaded in the middle.

(678) An engine shaft rests on bearings 5 feet apart; midway between the bearings, the shaft carries a fly-wheel weighing 3 tons. (*a*) Find the size of the shaft to withstand this load. (*b*) Find the size of the shaft on the assumption that the engine makes 80 R. P. M. and develops 75 H. P. The shaft is made of steel. Factor of safety 10. Ans. { (*a*) 4¼ in., nearly. (*b*) 4⅝ in., nearly.

(679) A white oak cantilever is loaded as shown in Fig. 30, and has in addition a uniform load of 80 pounds per foot. (*a*) Determine graphically the shear diagram and the maximum bending moment. (*b*) Find the necessary dimensions of the cross-section of the beam, assuming the depth to be 2¼ times the breadth and the stress to be steady.

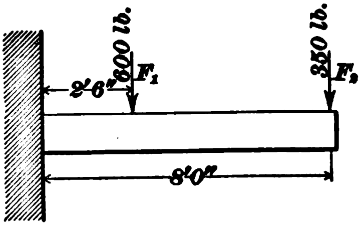


FIG. 30.

Ans. { (*a*) 82,320 in.-lb.
(*b*) $b = 3.7$ in.; $d = 9\frac{1}{4}$ in.

(680) A steel beam of the cross-section shown in Fig. 31 rests upon two supports 35 feet apart; (*a*) what concentrated load will it sustain at the center, the stress being considered as varying? (*b*) What total uniform load will it sustain, the stress being steady?

Ans. { (*a*) 7,246 lb.
(*b*) 20,290 lb.

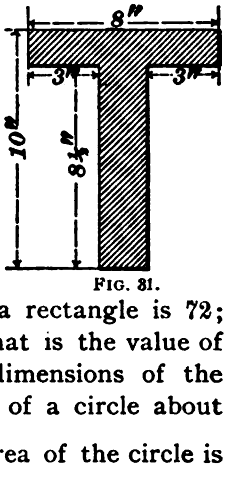


FIG. 31.

(681) (*a*) The moment of inertia of a rectangle is 72; the area of the rectangle is 24 sq. in.; what is the value of r in formula **72**? (*b*) What are the dimensions of the rectangle? (*c*) The moment of inertia of a circle about an axis through its center is $\frac{\pi d^4}{64}$; the area of the circle is $\frac{\pi d^2}{4}$; what is the radius of gyration of the circle?

Ans. { (*a*) 1.732.
(*b*) $b = 4$ in.; $d = 6$ in.
(*c*) $\frac{1}{4}d$.

(682) A cast-iron sphere 8″ in diameter is subjected to a steady internal pressure of 100 lb. per sq. in.; what should be its minimum thickness to safely withstand this pressure?

Ans. .06 in.

(683) A beam 28 feet long, uniformly loaded with 60 pounds per foot, rests upon two supports which are placed

5 feet from each end. Determine graphically the maximum shear and the maximum bending moment.

Ans. Bending moment = 20,160 in.-lb.

(684) Assuming the above beam to be made of white pine, find the dimensions of its cross-section, the stress being steady. The beam is to be rectangular.

(685) A wrought-iron boiler tube 2½″ in diameter and 9 feet long has a thickness of .2″; what external pressure can the tube withstand, using a factor of safety of 10?

Ans. 106 lb.

(686) A crane chain is required to lift 5 tons; what should be the diameter of the iron composing the links of the chain, which is of the open-link variety? Ans. .913 in.

(687) A gear-wheel 4 feet in diameter is keyed to a shaft. The force acting tangent to the pitch circle of the gear-wheel is 350 pounds; what should be the diameter of the shaft if made of steel? Ans. 2.84 in., say 2⅞ in.

(688) The head of an engine cylinder, 12″ in diameter, is fastened on by 10 wrought-iron bolts. In order to make the joint steam tight, a safe stress of only 2,000 pounds per sq. in. is allowed. The steam pressure being 90 pounds per sq. in., what should be the minimum diameter of the bolts; that is, the diameter at the root of the thread? Ans. .8 in.

(689) A beam 18 feet long has a load of 1 ton placed at each end. The supports are placed 5 feet from each end. (See Fig. 32.) (*a*) Draw the shear diagram and find the maximum bending moment. (*b*) If made of cast iron and circular in cross-section, what should be its diameter, the stress being steady?

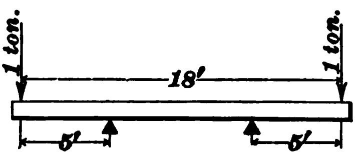


FIG. 32.

Ans. { (*a*) 120,000 in.-lb.
(*b*) 5.784 in.

(690) A beam 2″ × 6″ in cross-section and 18 feet long deflects .3″; how much will a beam 3″ × 8″ and 12 feet long

deflect under the same load? The long side is placed vertically in both cases. Ans. .025 in.

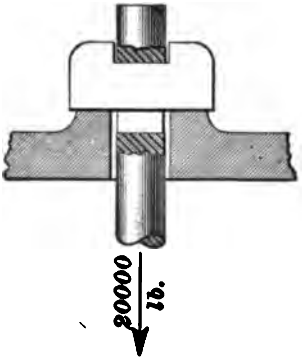


FIG. 33.

(691) A wrought-iron key or cotter is used to fasten a rod, as shown in Fig. 33. There is a pull of 20,000 pounds on the rod. Assuming the breadth of the key to be four times the thickness, find its dimensions to safely resist the stress. Use a factor of safety of 10.

Ans. { Breadth, 2.828″.
Thickness, .707″.

(692) A steel engine shaft 12″ in diameter, resting in bearings 54″ apart, carries a fly-wheel, weighing 30 tons, midway between the bearings; what is the deflection of the shaft? Consider the shaft as a simple beam. Ans. .006446 in.

(693) (*a*) A 7-strand steel-wire rope has a diameter of 1¼″; what load will it carry safely? (*b*) Find the circumference of a hemp rope to carry 1¾ tons.

Ans. { (*a*) 14,062.5 lb.
(*b*) 5.92 in.

(694) What stress would be required to rupture a cast-iron cylinder which is 8″ in diameter and 6″ thick? Ans. 12,000 lb.

(695) A beam which is 100 feet long carries four equal loads of 1,200 pounds each at distances of 20, 40, 60, and 80 feet from the left support. (*a*) Find the maximum bending moment and (*b*) the shear at each support.

Ans. { (*a*) 864,000 in.-lb.
(*b*) 2,400 lb.

ANALYSIS OF STRESSES.

(ARTS. 1267-1335.)

EXAMINATION QUESTIONS.

(696) What is a truss?

(697) What is (*a*) the essential truss element and (*b*) the simplest possible form of a truss?

NOTE.—Questions 698 to 713, inclusive, refer to the same structure.

(698) The data assumed for a truss of 90 ft. span are the same as used in Art. **1327**; the height of truss is 15 ft., the truss is divided into 6 panels, and the dead load per lineal foot is the same as that obtained in Art. **1299.** For one truss what is the panel dead load? Ans. 5,775 lb.

(699) Using the panel dead load obtained for the preceding question, construct to a scale of 4,000 lb. to the inch the complete stress diagram, obtaining the dead load stresses.

Ans.
For $a\ B$ + 20,420 lb.
For $B\ C$ + 23,100 lb.
For $C\ D$ + 25,990 lb.
For $a\ b$ − 14,400 lb.
For $b\ c$ − 14,400 lb.
For $c\ d$ − 23,100 lb.
For $B\ b$ − 5,780 lb.
For $B\ c$ − 12,250 lb.
For $C\ c$ + 2,890 lb.
For $C\ d$ − 4,080 lb.
For $D\ d$ − 0,000 lb.

NOTE.—In the answers to questions the same notation is used to designate the members of the truss that is used throughout the text. By remembering that capital letters are used to designate the upper chord joints, and lower case letters the lower chord joints of the truss, the student will readily recognize the position of each member.

In order to obtain uniformity in the graphical work, the student is requested to use the scale suggested in each question. Results will usually be given to the nearest ten pounds, in order that the student may be able to judge of the degree of accuracy which he is able to attain. His answers, however, need be obtained only to the nearest hundred pounds.

(700) What are the corrected dead load stresses for the vertical members in the same?

Ans.
For $B\ b$ − 3,850 lb.
For $C\ c$ + 4,810 lb.
For $D\ d$ + 1,930 lb.

For notice of copyright, see page immediately following the title page.

(701) For one truss of the same, what is (*a*) the panel live load and (*b*) the live load stress in the hip vertical?

Ans. (*a*) 13,500 lb.
(*b*) − 13,500 lb.

(702) What are the live load stresses in the chords and end posts of the same?

Ans. For $a\ B$ + 47,730 lb.
For $B\ C$ + 54,000 lb.
For $C\ D$ + 60,750 lb.
For $a\ b$ − 33,750 lb.
For $b\ c$ − 33,750 lb.
For $c\ d$ − 54,000 lb.

NOTE.—The maximum live load web stresses for this truss were obtained in Art. **1329.**

(703) Assuming the wind force against the upper and lower chords to be 150 lb. and 300 lb. per lineal foot, respectively, what is (*a*) the panel (live or dead) wind load for the lower chord, and (*b*) the panel wind load for the upper chord?

Ans. (*a*) 2,250 lb.
(*b*) 2,250 lb.

(704) Construct to a scale of 2,000 lb. to the inch the stress diagram, obtaining the dead load wind stresses in the members of the lower lateral system.

NOTE.—In this and the following questions the distance between centers of trusses, used in finding the stresses in the lateral systems, portal bracing, and floor-beams, will always be considered to be 1 foot greater than the clear width of roadway.

(705) Determine the maximum live wind stresses in the web members of the same lateral system by the method explained in Art. **1327,** using a scale of 2,000 lb. to the inch.

(706) Combine the live and dead load wind stresses, and give the total wind stress for each member in the left half of the lower lateral system. (Notice that the chord stresses are the same for the live wind load stresses as for the dead wind stresses.)

Ans. For $a\ b$ + 8,880 lb.
For $b\ c$ + 14,220 lb.
For $c\ d$ + 15,980 lb.
For $b_1 c_1$ − 8,880 lb.
For $c_1 d_1$ − 14,220 lb.
For $a\ b_1$ − 14,330 lb.
For $b\ b_1$ + 11,260 lb.
For $b\ c_1$ − 9,080 lb.
For $c\ c_1$ + 7,130 lb.
For $c\ d_1$ − 4,300 lb.
For $d\ d_1$ + 4,500 lb.

(707) Construct to a scale of 2,000 lb. to the inch the stress diagram, obtaining the wind stresses in the upper lateral system.

Ans. $\begin{cases} \text{For } BC + 2{,}660 \text{ lb.} \\ \text{For } CD + 3{,}550 \text{ lb.} \\ \text{For } C_1D_1 - 2{,}660 \text{ lb.} \\ \text{For } BC_1 - 4{,}300 \text{ lb.} \end{cases}$ $\begin{cases} \text{For } CC_1 + 3{,}380 \text{ lb.} \\ \text{For } CD_1 - 1{,}430 \text{ lb.} \\ \text{For } DD_1 + 2{,}250 \text{ lb.} \end{cases}$

(708) What is the wind pressure (*a*) on the windward side and (*b*) on the leeward side of the portal?

Ans. $\begin{cases} (a)\ 4{,}500 \text{ lb.} \\ (b)\ 1{,}125 \text{ lb.} \end{cases}$

(709) What is the (so-called) vertical reaction (*a*) at the foot of the windward and (*b*) at the foot of the leeward end post, calling the length of the end post 21.5 ft.; and (*c*) what is the laterally horizontal reaction at the foot of each end post?

Ans. $\begin{cases} (a)\ - 6{,}365 \text{ lb.} \\ (b)\ + 6{,}365 \text{ lb.} \\ (c)\ \ 2{,}812.5 \text{ lb.} \end{cases}$

(710) Assuming the effective depth of the portal to be 18 in. and the horizontal and vertical depth of each bracket to be 30 in., what is the stress in (*a*) the upper flange and (*b*) the lower flange at a distance of 3 ft. from the center of the windward end post? What is the stress at (*c*) the center of the upper and (*d*) the center of the lower flange?

Ans. $\begin{cases} (a)\ + 29{,}270 \text{ lb.} \\ (b)\ - 27{,}583 \text{ lb.} \end{cases}$ $\begin{cases} (c)\ + 1{,}688 \text{ lb.} \\ (d)\ \ 0{,}000 \text{ lb.} \end{cases}$

(711) The width of each chord is 12 in.; the floor is yellow pine. Assuming the total depth of the floor-beam to be 22.5 in., what is the total amount of live and dead load supported by one beam? Ans. 34,080 lb.

(712) What is the maximum flange stress in the same, assuming the effective depth of the beam to be 1½ in. less than its total depth? Ans. 48,686 lb.

(713) For the structure to which the fifteen preceding

questions refer, make a stress sheet so far as the same relates to the data and the stresses. See Fig. 283, Art. **1322.**

NOTE.—Questions 714 to 731, inclusive, refer to the same structure.

(714) A 7-panel Pratt truss through bridge of 112 ft. span carries a roadway 14 ft. wide, clear width. The live load assumed for the bridge is 100 lb. per square foot. For one truss what is the panel load of live load? Ans. 11,200 lb.

(715) With a white oak floor, what is the estimated amount of dead load per lineal foot for this bridge?

Ans. 679 lb.

(716) Assuming the dead load per lineal foot to be 680 lb., what is (*a*) the panel dead load and (*b*) the dead load reaction for one truss?

Ans. { (*a*) 5,440 lb.
(*b*) 16,320 lb.

(717) Assuming the height of truss to be 18 ft., construct the stress diagram, determining the dead load stresses in the members in the left half of the truss due to the left reaction. Use a scale of 2,000 lb. to the inch.

Ans. {
For $B\ C$ + 24,180 lb.
For $C D$ + 29,010 lb.
For $D E$ + 29,010 lb.
For $a\ \ b$ – 14,510 lb.
For $b\ \ c$ – 14,510 lb.
For $c\ \ d$ – 24,180 lb.
For $d\ \ e$ – 29,010 lb.

For $a\ B$ + 21,830 lb.
For $B\ b$ – 5,440 lb.
For $B\ c$ – 14,560 lb.
For $C\ c$ + 5,440 lb.
For $C\ d$ – 7,280 lb.
For $D d$ 0,000 lb.

(718) What are the corrected dead load stresses for the vertical members?

Ans. {
For $B\ b$ – 3,630 lb.
For $C\ c$ + 7,250 lb.
For $D d$ + 1,810 lb.

(719) Compute the live load stresses in the chords and end posts from the corresponding dead load stresses.

Ans. {
For $a\ B$ + 44,950 lb.
For $B\ C$ + 49,780 lb.
For $C D$ + 59,730 lb.
For $D E$ + 59,730 lb.

For $a\ \ b$ – 29,870 lb.
For $b\ \ c$ – 29,870 lb.
For $c\ \ d$ – 49,780 lb.
For $d\ \ e$ – 59,730 lb

(720) Find the maximum live load web stresses by the method explained in Art. **1327.** Use a scale of 4,000 lb. to the inch.

Ans.
For $a\ B + 44{,}950$ lb.
For $B\ c - 32{,}110$ lb.
For $C\ c + 16{,}000$ lb.
For $C\ d - 21{,}410$ lb.
For $D\ d + 9{,}600$ lb.
For $D\ e - 12{,}840$ lb.

(721) Construct the moment and shear diagrams, obtaining the live load chord stresses and web stresses. Use a scale of 5,000 lb. to the inch.

(722) Assuming the wind force against the lower chord to be 300 lb. per lineal foot, what are the wind stresses in the lower lateral system? Use a scale of 2,000 lb. to the inch.

Ans.
For $a\ b + 15{,}360$ lb.
For $b\ c + 25{,}600$ lb.
For $c\ d + 30{,}720$ lb.
For $d\ e + 30{,}720$ lb.
For $b_1 c_1 - 15{,}360$ lb.
For $c_1 d_1 - 25{,}600$ lb.
For $d_1 e_1 - 30{,}720$ lb.
For $a\ b_1 - 21{,}060$ lb.
For $b\ b_1 + 14{,}400$ lb.
For $b\ c_1 - 14{,}540$ lb.
For $c\ c_1 + 9{,}940$ lb.
For $c\ d_1 - 8{,}520$ lb.
For $d\ d_1 + 5{,}830$ lb.
For $d\ e_1 - 3{,}010$ lb.

(723) Assuming the wind force against the upper chord to be 150 lb. per lineal foot, what are the wind stresses in the upper lateral system? Use a scale of 2,000 lb. to the inch.

Ans.
For $B\ C + 5{,}120$ lb.
For $C\ D + 7{,}680$ lb.
For $D\ E + 7{,}680$ lb.
For $C_1\ D_1 - 5{,}120$ lb.
For $D_1 E_1 - 7{,}680$ lb.
For $B\ C_1 - 7{,}020$ lb.
For $C\ C_1 + 4{,}800$ lb.
For $C\ D_1 - 3{,}510$ lb.
For $D\ D_1 + 2{,}400$ lb.
For $D\ E_1$ 0,000 lb.

(724) What is wind pressure (*a*) at the windward side and (*b*) at the leeward side of the portal?

Ans.
(*a*) 6,000 lb.
(*b*) 1,200 lb.

(725) What is the so-called vertical reaction at (*a*) the foot of the windward and (*b*) the foot of the leeward end post? What is (*c*) the laterally horizontal reaction at the foot of each end post?

Ans.
(*a*) $-$ 12,000 lb.
(*b*) + 12,000 lb.
(*c*) 3,600 lb.

(726) Assuming the effective depth of the portal to be 2 ft., with brackets each 3 ft. deep, horizontally and vertically, what are the stresses in the upper and lower horizontal flanges at points *a*, *b*, *c*, *d*, *e*, *f*, and *g*, distant, respectively, 1.5, 2.5, 3.5, 4.5, 5.5, 6.5, and 7.5 ft. from the center of the windward end post?

Ans.	Upper Flange.	Lower Flange.
	At *a* + 20,400 lb.	At *a'* − 25,460 lb.
	At *b* + 22,400 lb.	At *b'* − 28,280 lb.
	At *c* + 26,400 lb.	At *c'* − 24,000 lb.
	At *d* + 20,400 lb.	At *d'* − 18,000 lb.
	At *e* + 14,400 lb.	At *e'* − 12,000 lb.
	At *f* + 8,400 lb.	At *f'* − 6,000 lb.
	At *g* + 2,400 lb.	At *g'* 0,000 lb.

(727) What may be considered to be an economical depth for the floor-beams of this bridge?

(728) Assuming the web-plate of the floor-beam to be 18 in. deep, what may be considered as (*a*) the total depth? (*b*) The effective depth of the beam?

Ans. (*a*) 18.5 in.
(*b*) 17.0 in.

(729) What is the total amount of live and dead load (including the white oak floor) supported by one floor-beam, assuming each chord to be 1 ft. in width? Ans. 28,991 lb.

(730) Calling the total load 29,000 lb., what is (*a*) the maximum flange stress and (*b*) the maximum shear in the floor-beam?

Ans. (*a*) 40,940 lb.
(*b*) 14,500 lb.

(731) Make a stress sheet (so far as the same relates to the data and the stresses) for the structure to which the seventeen preceding questions refer.

(732) What condition of uniform load produces the maximum stresses in the chords and end posts of a truss?

(733) Under what condition of uniform load is the maximum stress in the diagonal web member in any panel of a truss produced by the left reaction?

(734) What other member receives its maximum stress with the same condition of load?

(735) A 4-panel low truss bridge of 60 ft. span carries a 14-ft. roadway and has a capacity of 100 lb. per square foot. Assuming the floor to be of yellow pine, what is the estimated dead load per lineal foot? Ans. 557 lb.

(736) Calling the dead load 560 lb. per lineal foot, and assuming the bridge to be a Howe truss 8 ft. in height, construct, to a scale of 2,000 lb. to the inch, the stress diagram, determining the dead load stresses for one truss.

Ans.
For aB + 13,390 lb.
For BC + 11,810 lb.
For $a\ b$ − 11,810 lb.
For $b\ c$ − 15,750 lb.
For Bb − 6,300 lb.
For bC + 4,460 lb.
For Cc − 4,200 lb.

(737) What are the maximum live load stresses in the members of one truss?

Ans.
For aB + 33,480 lb.
For BC + 29,530 lb.
For $a\ b$ − 29,530 lb.
For $b\ c$ − 39,380 lb.
For Bb − 15,750 lb.
For bC + 16,730 lb.
For Cc − 10,500 lb.
For cD + 1,120 lb.

NOTE.—Notice that the maximum stress in Cc can not be less than one panel load.

(738) Assuming the lateral bracing to be of the Howe truss type and the wind force to be 300 lb. per lineal foot (150 lb. considered as live and 150 lb. as dead load), what are the wind stresses in the members of the lateral system? Use a scale of 1,000 lb. to the inch.

NOTE.—If the student has no scale of 10 or 100 divisions to the inch, he may simply double each load and lay off the load line to a scale of 2,000 lb. to the inch (scale of 20). Measure the stresses to the same scale and divide the results by two. Or by just the opposite process a scale of 500 lb. to the inch (scale of 50) may be used.

Ans.
For $a_1 b_1$ − 6,760 lb.
For $b_1 c_1$ − 9,000 lb.
For $b\ c$ + 6,760 lb.
For $a_1 b$ + 9,540 lb.
For $b\ b_1$ − 2,820 lb.
For $b_1 c$ + 3,980 lb.

(739) Assuming the total depth of the floor-beam to be 18.5 in., what is the total amount of live and dead load supported by one beam? Ans. 26,576 lb

(740) Calling the total load 26,600 lb., what is (*a*) the maximum flange stress and (*b*) the maximum shear in the floor-beam?

Ans. (*a*) 37,550 lb.
(*b*) 13,300 lb.

(741) Make a stress sheet (so far as it relates to the data and the stresses) for the structure to which the six preceding questions refer.

(742) Construct to a scale of 2,000 lb. to the inch the stress diagram, obtaining the dead load stresses for the truss of a Warren girder, using the same data that were given in Questions 735 and 736.

Ans.
For $B\,C$ + 11,810 lb.
For $C\,D$ + 15,750 lb.
For $a\ \ b$ − 5,910 lb.
For $b\ \ c$ − 13,780 lb.
For $a\,B$ + 8,640 lb.
For $B\,b$ − 8,640 lb.
For $b\,C$ + 2,880 lb.
For $C\,c$ − 2,880 lb.

(743) What are the maximum live load stresses for the same? Use a scale of 4,000 lb. to the inch.

Ans.
For $B\,C$ + 29,530 lb.
For $C\,D$ + 39,380 lb.
For $a\ \ b$ − 14,780 lb.
For $b\ \ c$ − 34,450 lb.
For $a\ B$ + 21,600 lb.
For $B\ \ b$ − 21,590 lb.
For $b\ \ C$ + 10,790 lb. / − 720 lb.
For $C\ \ c$ − 10,790 lb. / + 720 lb.
For $c\ D$ + 720 lb. / − 10,790 lb.
For $D\ d$ − 720 lb. / + 10,790 lb.

(744) With the wind force the same as was assumed in Question 738, what are the wind stresses in a lateral system of the Pratt truss type for the same structure? Use a scale of 1,000 lb. to the inch.

Ans.
For $a\ \ b$ + 6,750 lb.
For $b\ \ c$ + 9,000 lb.
For b_1c_1 − 6,750 lb.
For $a\ b_1$ − 9,550 lb.
For $b\ b_1$ + 6,750 lb.
For $b\ c_1$ − 3,980 lb.
For $c\ c_1$ + 4,500 lb.

(745) Make a stress sheet (so far as it relates to the data and the stresses) for the structure to which the three preceding questions refer.

(746) To what character of load, with reference to a bridge or similar structure, is the term live load applied?

(747) To what is the term dead load applied, with reference to the same?

(748) What name is often applied to the inclined end posts of a bridge?

(749) (*a*) What is a through bridge? (*b*) A deck bridge? (*c*) A half-deck bridge?

(750) (*a*) What is a high-truss bridge? (*b*) A low-truss bridge?

(751) (*a*) What is a pin-connected bridge? (*b*) A riveted girder?

(752) To what is the panel load equal for one truss of a bridge having two trusses?

(753) An 8-panel Pratt truss through bridge of 140 ft. span supports a roadway 16 ft. wide, clear width; the height of each truss is 24 ft. The assumed live load is 100 lb. per square foot. What is the panel live load?

Ans. 14,000 lb.

(754) Assuming the panel dead load for the same to be 7,200 lb., construct to a scale of 4,000 lb. to the inch the stress diagram, obtaining the dead load stresses in the members of the truss, correcting the stresses for the vertical members.

Ans.
For $a\,B$ + 31,190 lb.
For $B\,C$ + 31,500 lb.
For $C\,D$ + 39,380 lb.
For $D\,E$ + 42,000 lb.
For $a\,b$ – 18,380 lb.
For $b\,c$ – 18,380 lb.
For $c\,d$ – 31,500 lb.
For $d\,e$ – 39,380 lb.
For $B\,b$ – 4,800 lb.
For $B\,c$ – 22,280 lb.
For $C\,c$ + 13,200 lb.
For $C\,d$ – 13,370 lb.
For $D\,d$ + 6,000 lb.
For $D\,e$ – 4,450 lb.
For $E\,e$ + 2,400 lb.

(755) What are the maximum live load stresses in the members of the same? For the special construction for the web stresses use a scale of 5,000 lb. to the inch.

Ans.
For $a\ B$ $+$ 60,650 lb.
For $B\ C$ $+$ 61,250 lb.
For $C\,D$ $+$ 76,560 lb.
For $D\,E$ $+$ 81,670 lb.
For $a\ \ b$ $-$ 35,730 lb.
For $b\ \ c$ $-$ 35,730 lb.
For $c\ \ d$ $-$ 61,250 lb.
For $d\ \ e$ $-$ 76,560 lb.
For $B\,b$ $-$ 14,000 lb.
For $B\,c$ $-$ 45,480 lb.
For $C\,c$ $+$ 26,250 lb.
For Cd $-$ 32,490 lb.
For Dd $+$ 17,500 lb.
For $D\,e$ $-$ 21,660 lb.
For $E\,e$ $+$ 6,900 lb.
For Ed' $-$ 8,540 lb.

(756) Assuming the wind force against the lower chord to be 300 lb. per lineal foot, what are the wind stresses in the lower lateral system? Use a scale of 2,000 lb. to the inch.

Ans.
For $a\ b$ $+$ 18,920 lb.
For $b\ c$ $+$ 32,430 lb.
For $c\ d$ $+$ 40,530 lb.
For $d\ e$ $+$ 43,240 lb.
For b_1c_1 $-$ 18,920 lb.
For c_1d_1 $-$ 32,430 lb.
For d_1e_1 $-$ 40,530 lb.
For $a\ \ b_1$ $-$ 26,370 lb.
For $b\ b_1$ $+$ 18,380 lb.
For $b\ c_1$ $-$ 19,310 lb.
For $c\ \ c_1$ $+$ 13,450 lb.
For $c\ \ d_1$ $-$ 12,710 lb.
For $d\,d_1$ $+$ 8,860 lb.
For $d\ e_1$ $-$ 6,590 lb.
For $e\ \ e_1$ $+$ 5,250 lb.

(757) Assuming the wind force against the upper chord to be 150 lb. per lineal foot, what are the wind stresses in the upper lateral system? Use a scale of 2,000 lb. to the inch.

Ans.
For $B\ \ C$ $+$ 6,760 lb.
For $C\ \ D$ $+$ 10,810 lb.
For $D\ \ E$ $+$ 12,160 lb.
For C_1D_1 $-$ 6,760 lb.
For D_1E_1 $-$ 10,810 lb.
For $B\ \ C_1$ $-$ 9,420 lb.
For $C\ C_1$ $+$ 6,560 lb.
For $C\ D_1$ $-$ 5,650 lb.
For $D\,D_1$ $+$ 3,940 lb.
For $D\,E_1$ $-$ 1,880 lb.
For $E\,E_1$ $+$ 2,630 lb.

(758) Make a stress sheet (so far as the same relates to the data and those stresses which have been found), for the structure to which the five preceding questions refer. Write each stress to the nearest hundred pounds.

ANALYSIS OF STRESSES.

(ARTS. 1336–1375.)

EXAMINATION QUESTIONS.

(759) What is the most economical angle of inclination for the diagonal web members of a truss?

(760) Name more than one expedient by means of which the diagonal members may be given economical inclinations when the center height of the truss considerably exceeds the panel length.

(761) A 9-panel Pratt truss through bridge of 162 ft. span supports a 20-ft. roadway. The live load assumed for the trusses is 90 lb. per square foot of roadway; the roadway floor is of yellow pine. What is the estimated dead load per lineal foot? Ans. 982 lb.

(762) Calling the dead load per lineal foot 980 lb., what is (*a*) the panel dead load and (*b*) the panel live load for one truss?

Ans. { (*a*) 8,820 lb.
(*b*) 16,200 lb.

(763) The height of the truss at the vertical members $B\,b$, $C\,c$, $D\,d$, and $E\,e$ are 18, 24, 26, and 26 ft., respectively. Construct to a scale of 5,000 lb. to the inch the stress diagram determining the dead load stresses in the members in the left half of the truss, and correct the stresses in the vertical members.

Ans. {
For $a\ B$ + 49,900 lb.
For $B\ C$ + 48,800 lb.
For $C\ D$ + 55,300 lb.
For $D\ E$ + 61,100 lb.
For $E\ E'$ + 61,100 lb.
For $a\ b$ − 35,300 lb.
For $b\ c$ − 35,300 lb.
For $c\ d$ − 46,300 lb.
For $d\ e$ − 55,000 lb.

For $e\ e'$ − 61,100 lb.
For $B\ b$ − 5,900 lb.
For $B\ c$ − 15,600 lb.
For $C\ c$ + 5,100 lb
For $C\ d$ − 14,400 lb.
For $D\ d$ + 5,600 lb.
For $D\ e$ − 10,700 lb.
For $E\ e$ + 2,900 lb.

For notice of copyright, see page immediately following the title page.

(764) For the same, what are the maximum live load stresses in the chords and end post?

Ans.
For $a\,B$ + 91,600 lb.
For $B\,C$ + 89,600 lb.
For $C\,D$ + 101,600 lb.
For $D\,E$ + 112,200 lb.
For $E\,E'$ + 112,200 lb.

For $a\,b$ − 64,800 lb.
For $b\,c$ − 64,800 lb.
For $c\,d$ − 85,100 lb.
For $d\,e$ − 100,900 lb.
For $e\,e'$ − 112,200 lb.

(765) Check the live load stresses by formulas **102** and **103,** Arts. **1345** and **1346.**

(766) What are the maximum live load web stresses for the same?

Ans.
For $B\,b$ − 16,200 lb.
For $B\,c$ − 35,600 lb.
For $C\,c$ + 18,900 lb.
For $C\,d$ − 36,300 lb.
For $D\,d$ + 20,800 lb.
For $D\,e$ − 32,800 lb.
For $E\,e$ + 18,000 lb.
For $E\,e'$ − 21,900 lb.
For $E'\,d'$ − 2,400 lb.

(767) For the same structure, what is the distance from center to center of chords to be assumed in determining the stresses in the lateral systems? Ans. 21.35 ft.

(768) Assuming for the same structure, a wind force of 300 lb. per lineal foot against the lower chord, what are the wind stresses in the lower lateral system? Use a scale of 2,000 lb. to the inch.

Ans.
For $a\ b$ + 18,200 lb.
For $b\ c$ + 31,900 lb.
For $c\ d$ + 41,000 lb.
For $d\ e$ + 45,500 lb.
For $e\ f$ + 45,500 lb.
For $b_1\,c_1$ − 18,200 lb.
For $c_1\,d_1$ − 31,900 lb.
For $d_1\,e_1$ − 41,000 lb.
For $e_1\,f_1$ − 45,500 lb.

For $a\ b_1$ − 28,300 lb.
For $b\ b_1$ + 21,600 lb.
For $b\ c_1$ − 21,600 lb.
For $c\ c_1$ + 16,500 lb.
For $c\ d_1$ − 15,300 lb.
For $d\,d_1$ + 11,700 lb.
For $d\ e_1$ − 9,400 lb.
For $e\ e_1$ + 7,200 lb.
For $e\ f_1$ − 3,900 lb.

(769) **Assuming a wind force of 150 lb. per lineal foot against the upper chord of the same structure, what are the wind stresses in the upper lateral system, neglecting the inclination of the chord?**

Ans.
For $B\ C$ + 6,800 lb.
For $C\ D$ + 11,400 lb.
For $D\ E$ + 13,700 lb.
For $E\ E'$ + 13,700 lb.
For $C_1 D_1$ – 6,800 lb.
For $D_1 E_1$ – 11,400 lb.
For $E_1 E_2$ – 13,700 lb.
For $B\ C_1$ – 10,600 lb.
For $C\ C_1$ + 8,100 lb.
For $C\ D_1$ – 7,100 lb.
For $D\ D_1$ + 5,400 lb.
For $D\ E_1$ – 3,500 lb.
For $E\ E_1$ + 2,700 lb.

(770) Make a stress sheet (so far as the same relates to the data and the stresses which have been found) for the bridge to which the nine preceding questions refer.

(771) If the same dead load (980 lb. per lineal foot) be assumed for a bridge having the same dimensions as that to which the ten preceding questions refer, except that the heights of truss at the vertical members $B\ b$, $C\ c$, $D\ d$, and $E\ e$ are 18, 22, 24, and 26 ft., respectively, what are the dead load stresses? Use a scale of 5,000 lb. to the inch, and give the corrected stresses for the vertical members.

Ans.
For $a\ B$ + 49,900 lb.
For $B\ C$ + 51,700 lb.
For $C\,D$ + 59,900 lb.
For $D\,E$ + 61,400 lb.
For $E\,E'$ + 61,100 lb.
For $a\ b$ – 35,300 lb.
For $b\ c$ – 35,300 lb.
For $c\ d$ – 50,500 lb.
For $d\ e$ – 59,500 lb.
For $e\ e'$ – 61,100 lb.
For $B\ b$ – 5,900 lb.
For $B\ c$ – 21,500 lb.
For $C\ c$ + 9,400 lb.
For $C\,d$ – 14,200 lb.
For $D\,d$ + 5,100 lb.
For $D\ e$ – 2,500 lb.
For $E\ e$ – 3,800 lb.

(772) An 8-panel Pratt truss through bridge of 140 ft. span has the same dimensions in all respects as that for which the stress diagrams were explained in Arts. **1338** and following, except that the height of truss at the hip vertical is 19.5 ft.; the center height is 24 ft., and the

inclination of the upper chord is uniform between the hip and the center, as in that case. Assuming the same dead load (7,200 lb. per panel per truss), construct to a scale of 4,000 lb. to the inch the stress diagram, obtaining the dead load stresses, and give the corrected stresses for the vertical members.

Ans.
For $a\ B$ + 33,900 lb.
For $B\,C$ + 36,100 lb.
For $C\,D$ + 42,200 lb.
For $D\,E$ + 42,200 lb.
For $a\ b$ – 22,600 lb.
For $b\ c$ – 22,600 lb.
For $c\ d$ – 36,000 lb.
For $d\ e$ – 42,000 lb.
For $B\ b$ – 4,800 lb.
For $B\ c$ – 20,000 lb.
For $C\ c$ + 10,100 lb.
For $C\ d$ – 9,400 lb.
For $D\,d$ + 2,400 lb.
For $E\ e$ – 4,800 lb.

(773) Assuming the same live load as in Art. **1340** (14,000 lb. per panel per truss), what are the maximum live load stresses in the chords and end post of the same?

Ans.
For $a\ B$ + 65,800 lb.
For $B\ C$ + 70,300 lb.
For $C\,D$ + 82,000 lb.
For $D\,E$ + 82,000 lb.
For $a\ b$ – 44,000 lb.
For $b\ c$ – 44,000 lb.
For $c\ d$ – 70,000 lb.
For $d\ e$ – 81,700 lb.

(774) Check the live load chord stresses by formulas **102** and **103**, given in Arts. **1345** and **1346.**

(775) Determine the maximum live load web stresses by the means of special construction explained in Art. **1341,** using a scale of 5,000 lb. to the inch.

Ans.
For $B\ b$ – 14,000 lb.
For $B\ c$ – 42,300 lb.
For $C\ c$ + 22,500 lb.
For $C\ d$ – 27,300 lb.
For $D\ d$ + 14,000 lb.
For $D\,e$ – 16,600 lb.
For $E\,e$ – 14,000 lb.; + 3,100 lb.
For $E\,d'$ – 17,300 lb.

(776) For the same structure, what is the distance from center to center of trusses to be used in determining the lateral stresses? Ans. 17.17 ft.

(777) Assuming the lateral wind force against the lower chord to be 300 lb. per lineal foot, what are the wind stresses

in the lower lateral system ? Use a scale of 4,000 lb. to the inch.

Ans.
For $a\ b$ + 18,800 lb.
For $b\ c$ + 32,200 lb.
For $c\ d$ + 40,300 lb.
For $d\ e$ + 42,900 lb.
For $b_1\ c_1$ – 18,800 lb.
For $c_1\ d_1$ – 32,200 lb.
For $d_1\ e_1$ – 40,300 lb.
For $a\ b_1$ – 26,300 lb.
For $b\ b_1$ + 18,400 lb.
For $b\ c_1$ – 19,300 lb.
For $c\ c_1$ + 13,500 lb.
For $c\ d_1$ – 12,700 lb.
For $d\ d_1$ + 8,900 lb.
For $d\ e_1$ – 6,600 lb.
For $e\ e_1$ + 5,250 lb.

(778) Assuming a lateral wind force of 150 lb. per lineal foot against the upper chord, what are the wind stresses in the upper lateral system ? Use a scale of 2,000 lb. to the inch.

Ans.
For $B\ C$ + 6,700 lb.
For $C\ D$ + 10,700 lb.
For $D\ E$ + 12,000 lb.
For $C_1\ D_1$ – 6,700 lb.
For $D_1\ E_1$ – 10,700 lb.
For $B\ C_1$ – 9,400 lb.
For $C\ C_1$ + 6,600 lb.
For $C\ D_1$ – 5,600 lb.
For $D\ D_1$ + 3,900 lb.
For $D\ E_1$ – 1,900 lb.
For $E\ E_1$ + 2,600 lb.

(779) Make a stress sheet (so far as it relates to the data and stresses which have been found) for the bridge to which the seven preceding questions refer.

(780) A Whipple truss through bridge of 245 ft. span carries a 24-ft. roadway. The live load assumed for the trusses is 75 lb. per square foot of roadway. What is the estimated amount of dead load per lineal foot, exclusive of the floor ? Ans. 716.6 lb.

(781) It is desired that the panel length in this bridge shall not exceed 18.5 ft. Making the panel length as long as practicable within this limit, and assuming the floor to be yellow pine, what is (*a*) the total estimated load per lineal foot and (*b*) the panel dead load for one truss ?

Ans. (*a*) 1,263.7 lb.
(*b*) 11,057 lb.

(782) Taking the height of truss at one-seventh the span, and calling the panel dead load 11,000 lb. (which is slightly less than 1,260 lb. per lineal foot), construct to a scale of 5,000 lb. to the inch the stress diagrams determining the

dead load stresses, combine the chord stresses, and correct the stresses in the vertical members.

Ans.
For aB + 79,900 lb.
For BC + 79,800 lb.
For CD + 101,800 lb.
For DE + 118,300 lb.
For EF + 129,300 lb.
For FG + 134,800 lb.
For GH + 134,800 lb.
For ab − 35,800 lb.
For bc − 35,800 lb.
For cd − 52,300 lb.
For de − 79,800 lb.
For ef − 101,800 lb.
For fg − 118,300 lb.
For gh − 129,300 lb.
For Bb − 7,300 lb.
For Bc − 36,900 lb.
For Bd − 38,900 lb.
For Cc + 25,700 lb.
For Ce − 31,100 lb.
For Dd + 20,200 lb.
For Df − 23,300 lb.
For Ee + 14,700 lb.
For Eg − 15,600 lb.
For Ff + 9,200 lb.
For Fh − 7,800 lb.
For Gg + 3,700 lb.
For Hh + 3,700 lb.

(783) For one truss what is the amount of live load per panel? Ans. 15,750 lb.

(784) What are the maximum live load stresses in the chords and end post?

Ans.
For aB + 114,500 lb.
For BC + 114,200 lb.
For CD + 145,700 lb.
For DE + 169,300 lb.
For EF + 185,100 lb.
For FG + 192,900 lb.
For GH + 192,900 lb.
For ab − 51,200 lb.
For bc − 51,200 lb.
For cd − 74,800 lb.
For de − 114,200 lb.
For ef − 145,700 lb.
For fg − 169,300 lb.
For gh − 185,100 lb.

(785) What are the maximum live load stresses in the web members? Use a scale of 10,000 lb. to the inch.

Ans.
For Bb − 15,750 lb.
For Bc − 53,500 lb.
For Bd − 56,500 lb.
For Cc + 34,300 lb.
For Ce − 48,500 lb.
For Dd + 27,600 lb.
For Df − 39,000 lb.
For Ee + 23,100 lb.
For Eg − 32,600 lb.
For Ff + 17,400 lb.
For Fh − 24,700 lb.
For Gg + 14,100 lb.
For Gg' − 19,900 lb.
For Hh + 4,100 lb.
For Hf' − 5,700 lb.

(786) What is the estimated width center to center of chords to be used in determining the lateral stresses?

Ans. 26.04 ft.

(787) Calling the width center to center of chords 26 ft., and assuming a lateral wind force of 300 lb. per lineal foot against the lower chord, make to a scale of 4,000 lb. to the inch the constructions determining the wind stresses in the lower lateral system, the latter being of the Pratt truss type.

Ans.
For a b + 23,000 lb.
For b c + 42,400 lb.
For c d + 58,300 lb.
For d e + 70,700 lb.
For e f + 79,500 lb.
For f g + 84,800 lb.
For g h + 86,600 lb.
For b_1 c_1 – 23,000 lb.
For c_1 d_1 – 42,400 lb.
For d_1 e_1 – 58,300 lb.
For e_1 f_1 – 70,700 lb.
For f_1 g_1 – 79,500 lb.
For g_1 h_1 – 84,800 lb.
For a b_1 – 41,100 lb.
For b b_1 + 34,100 lb.
For b c_1 – 35,000 lb.
For c c_1 + 29,100 lb.
For c d_1 – 29,200 lb.
For d d_1 + 24,200 lb.
For d e_1 – 23,500 lb.
For e e_1 + 19,500 lb.
For e f_1 – 18,100 lb.
For f f_1 + 15,000 lb.
For f g_1 – 12,900 lb.
For g g_1 + 10,700 lb.
For g h_1 – 7,900 lb.
For h h_1 + 6,600 lb.

(788) Assuming a lateral wind force of 150 lb. per lineal foot against the upper chord, construct to a scale of 2,500

lb. to the inch the stress diagram determining the wind stresses in the upper lateral system, assuming the latter to be of the Pratt truss type.

Ans.
For B C $+$ 9,700 lb.
For C D $+$ 17,700 lb.
For D E $+$ 23,900 lb.
For E F $+$ 28,300 lb.
For F G $+$ 30,900 lb.
For G H $+$ 31,800 lb.
For C_1 D_1 $-$ 9,700 lb.
For D_1 E_1 $-$ 17,700 lb.
For E_1 F_1 $-$ 23,900 lb.
For F_1 G_1 $-$ 28,300 lb.
For G_1 H_1 $-$ 30,900 lb.
For B C_1 $-$ 17,400 lb.
For C C_1 $+$ 14,400 lb.
For C D_1 $-$ 14,200 lb.
For D D_1 $+$ 11,800 lb.
For D E_1 $-$ 11,100 lb.
For E E_1 $+$ 9,200 lb.
For E F_1 $-$ 7,900 lb.
For F F_1 $+$ 6,600 lb.
For F G_1 $-$ 4,700 lb.
For G G_1 $+$ 3,900 lb.
For G H_1 $-$ 1,600 lb.
For H H_1 $+$ 2,600 lb.

(789) For the same structure the portal bracing is to consist of two horizontal struts connected by diagonal sway rods, as in Art. **1366.** Locating the position of the substrut b b' at the center of the end post, determine the stresses in the struts B B' and b b', and in the sway rod b B'.

Ans.
For B B' $+$ 24,300 lb.
For b b' $+$ 17,100 lb.
For b B' $-$ 43,100 lb.

(790) Assuming for the floor system a live load of 100 lb. per square foot, and taking the total depth of the floor-beam at 32.5 in., what is the total amount of live and dead load supported by one floor-beam?

Ans. 53,250 lb.

(791) What is (*a*) the maximum flange stress and (*b*) the maximum shear in the floor-beam?

Ans.
(*a*) 72,100 lb.
(*b*) 26,600 lb.

(792) Make a stress sheet (so far as it relates to the data and the stresses) for the bridge to which the twelve preceding questions refer.

(793) For a Baltimore truss, having panels subdivided by means of struts, except in the double panel at the center, where the sub-ties will be used, assuming the same data that have been used for the structure to which the thirteen preceding questions refer, construct to a scale of 10,000 lb. to the inch the stress diagram determining the dead load stresses, and correct the stresses in the vertical members.

Ans.

For $a\ B_1$	$+$ 101,100 lb.
For $B_1\,C$	$+$ 93,300 lb.
For $C\ E$	$+$ 110,000 lb.
For $E\ G$	$+$ 132,000 lb.
For $G\ G'$	$+$ 137,500 lb.
For $a\,b$, $b\,c$, $c\,d$, and $d\,e$	$-$ 71,500 lb.
For $e\,f$ and $f\,g$	$-$ 115,500 lb.
For $g\,h$	$-$ 132,000 lb.
For $B_1\,b$, $D_1\,d$, $F_1\,f$, and $H_1\,h$	$-$ 7,300 lb.
For $B_1\,c$, $D_1\ c$, and $F_1\,e$	$+$ 7,800 lb.
For $C\ c$	$-$ 16,500 lb.
For $C\,D_1$	$-$ 62,200 lb.
For $D_1\,e$	$-$ 54,400 lb.
For $E\ e$	$+$ 29,300 lb.
For $E\ F_1$	$-$ 31,100 lb.
For $F_1\,g$	$-$ 23,300 lb.
For $G\ g$	$+$ 12,800 lb.
For $G\,H_1$	$-$ 7,800 lb.

(794) For the same truss what are the maximum live load stresses in the chords and end post?

Ans.

For $a\ B_1$	$+$ 144,800 lb.	For $a\,b$, $b\,c$, $c\,d$, and $d\ e$	$-$ 102,400 lb.
For $B_1\ C$	$+$ 133,600 lb.	For $e\ f$ and $f\,g$	$-$ 165,400 lb.
For $C\ E$	$+$ 157,500 lb.	For $g\ h$	$-$ 189,000 lb.
For $E\ G$	$+$ 189,000 lb.		
For $G\,G'$	$+$ 196,900 lb.		

(795) Check by formula the live load chord stresses.

(796) What are the maximum live load web stresses for the same? Use for the special construction a scale of 10,000 lb. to the inch.

Ans.
For $B_1 b$, $D_1 d$, $F_1 f$, and $H_1 h$ $-$ 15,800 lb.
For $B_1 c$, $D_1 c$, and $F_1 e$ $+$ 11,100 lb.
For $C c$ $-$ 31,500 lb.
For $C D_1$ $-$ 93,900 lb.
For $D_1 e$ $-$ 87,500 lb.
For $E e$ $+$ 42,800 lb.
For $E F_1$ $-$ 60,500 lb.
For $F_1 g$ $-$ 57,300 lb.
For $G g$ $+$ 31,500 lb.
For $G H_1$ $-$ 44,500 lb.
For $H_1 g'$ $-$ 25,600 lb.
For $G' F_2$ $-$ 500 lb.

(797) Using double panels for the upper lateral system, i. e., using lateral struts only at the joints of the full vertical members, thus dividing the upper lateral truss into five panels, and assuming a lateral wind pressure against the upper chord of 150 lb. per lineal foot, construct to a scale of 4,000 lb. to the inch the complete stress diagram determining the wind stresses in the upper lateral system.

Ans.
For $C E$ $+$ 14,100 lb.
For $E G$ $+$ 21,200 lb.
For $G G'$ $+$ 21,200 lb.
For $E_1 G_1$ $-$ 14,100 lb.
For $G_1 G_2$ $-$ 21,200 lb.
For $C E_1$ $-$ 17,600 lb.
For $E E_1$ $+$ 10,500 lb.
For $E G_1$ $-$ 8,800 lb.
For $G G_1$ $+$ 5,300 lb.

(798) A braced portal consisting of two horizontal lateral struts connected by diagonal sway rods, as in Art. **1366,** will be used. Locating the position of the sub-strut $b\, b'$ at the centers of the end posts, determine the stresses in the struts $B B'$ and $b\, b'$ and in the sway rod $b B'$.

Ans.
For $B B'$ $+$ 21,000 lb.
For $b\, b'$ $+$ 15,750 lb.
For $b\, B'$ $-$ 43,700 lb.

(799) Make a stress sheet (so far as it relates to the data and the stresses which have been found) for the bridge

to which the six preceding questions refer. Use for the lower lateral system and floor-beam the stresses that were obtained in the solutions of Questions 787 and 791.

(800) A truss bridge of 245 feet span carries a 24-ft. roadway and two 4-ft. sidewalks. Assuming the live load for the trusses to be 70 lb. per square foot of roadway and sidewalk, what is the estimated weight of the iron per lineal foot? Ans. 716.3 lb.

(801) Assuming the roadway and sidewalk floors to be of white oak, what is (*a*) the estimated weight per lineal foot of floor and (*b*) the total estimated dead load per lineal foot?

Ans. { (*a*) 765.6 lb.
(*b*) 1,482 lb.

(802) The truss has 14 equal panels. Calling the dead load 1,486 lb. per lineal foot, what is the panel dead load for one truss? Ans. 13,000 lb.

(803) Fixing the heights of the truss at the vertical members $B\ b$, $D\ d$, $F\ f$, and $H\ h$ at 27.75 ft., 36.5 ft., 40 ft., and 40 ft., respectively, and effecting the subdivisions for the intermediate panels by means of sub-ties (i. e., making a half-hitch truss), construct to a scale of 20,000 lb. to the inch, the stress diagram, determining the dead load stresses, and correct the stresses in the vertical members.

Ans.

For $a\ B$	+ 99,900 lb.	For $C_1\ d$	− 63,300 lb.
For $B\ D$	+ 112,400 lb.	For $C_1\ D$	− 10,200 lb.
For $D\ F$	+ 134,300 lb.	For $D\ d$	+ 35,000 lb.
For $F\ H$	+ 145,000 lb.	For $D\ E_1$	− 44,500 lb.
For $a\ b$, $b\ c$, and $c\ d$	− 53,300 lb.	For $E_1\ f$	− 36,300 lb.
For $d\ f$	− 102,800 lb.	For $E_1\ F$	− 9,100 lb.
For $f\ h$	− 128,000 lb.	For $F\ f$	+ 21,900 lb.
For $B\ b$, $C_1\ c$, $E_1\ e$, and $G_1\ g$	− 8,700 lb.	For $F\ G_1$	− 25,900 lb.
For $B\ C_1$	− 71,200 lb.	For $G_1\ h$	− 17,300 lb.
		For $G_1\ H$	− 8,600 lb.
		For $H\ h$	+ 21,700 lb.

(804) What are the maximum live load stresses in the chords and end post?

Ans.

For $a\ B$	$+$ 150,700 lb.
For $B D$	$+$ 169,500 lb.
For $D F$	$+$ 202,500 lb.
For $F H$	$+$ 218,700 lb.
For $a b$, $b c$, and $c d$	$-$ 80,300 lb.
For $d\ f$	$-$ 155,100 lb.
For $f\ h$	$-$ 192,900 lb.

(805) Check by formula the chord stresses obtained as answers to the preceding question.

(806) Construct to a scale of 20,000 lb. to the inch the combined stress diagrams determining the maximum live load stresses in the web members.

Ans.

For $B b$, $C_1 c$, $E_1 e$, and $G_1 g$	$-$ 19,600 lb.	For $E_1\ F$	$-$ 13,700 lb.
For $B\ C_1$	$-$ 112,600 lb.	For $F\ \ f$	$+$ 41,400 lb.
For $C_1\ d$	$-$ 100,600 lb.	For $F\ G_1$	$-$ 67,000 lb.
For $C_1 D$	$-$ 15,400 lb.	For $G_1\ h$	$-$ 54,000 lb.
For $D\ d$	$+$ 52,600 lb.	For $G_1 H$	$-$ 13,000 lb.
For $D E_1$	$-$ 83,300 lb.	For $H\ h$	$+$ 24,300 lb.
For $E_1 f$	$-$ 71,000 lb.	For $H G_2$	$-$ 30,400 lb.
		For $G_2 f'$	$-$ 2,000 lb.

(807) Dividing the upper lateral system into six double panels of the Pratt truss type by placing lateral struts at the joints of the upper chord at which the main vertical members attach, and assuming a lateral wind pressure against the upper chord of 150 lb. per lineal foot, construct to a scale of 4,000 lb. to the inch the stress diagram determining the wind stresses in the upper lateral system.

Ans.

For $B\ D$	$+$ 18,200 lb.	For $D\ D_1$	$+$ 13,100 lb.
For $D\ F$	$+$ 28,900 lb.	For $D\ F_1$	$-$ 13,200 lb.
For $F\ H$	$+$ 32,400 lb.	For $F\ F_1$	$+$ 7,900 lb.
For $D_1 F_1$	$-$ 18,200 lb.	For $F\ H_1$	$-$ 4,400 lb.
For $F_1 H_1$	$-$ 28,900 lb.	For $H H_1$	$+$ 5,250 lb.
For $B\ D_1$	$-$ 22,400 lb.		

(808) The position of the lower strut $b\ b'$ of a braced portal for this bridge is located at one-third the assumed

length of the end post from the top of the same. What are the stresses in the struts $B\,B'$, $b\,b'$, and in the sway rod $b\,B'$?

Ans. $\begin{cases} \text{For } B\,B' + 32{,}200 \text{ lb.} \\ \text{For } b\ \ b' + 25{,}600 \text{ lb.} \\ \text{For } b\ \ B' - 55{,}600 \text{ lb.} \end{cases}$

(809) What is (*a*) the total weight of the floor upon one floor-beam and (*b*) the floor weight per lineal foot of the loaded portion of the beam, i. e., of that portion of the beam over which the floor extends? See Question 801.

Ans. $\begin{cases} (a)\ 13{,}398 \text{ lb.} \\ (b)\ 418.7 \text{ lb.} \end{cases}$

(810) Assuming the live load for the floor-beam to be 100 lb. per square foot upon the roadway and 80 lb. per square foot upon the sidewalks, and fixing the depth of the beam at 32.5 inches, what is (*a*) the total estimated weight of the floor-beam and (*b*) the total estimated amount of dead load per lineal foot of the loaded portion of the same?

Ans. $\begin{cases} (a)\ 1{,}938 \text{ lb.} \\ (b)\ 479.3 \text{ lb.} \end{cases}$

(811) Assuming the total amount of dead load per lineal foot of the loaded portion of the floor-beam to be 480 lb., what is the total amount of live and dead load flange stress in the beam (*a*) at each support and (*b*) at the center?

Ans. $\begin{cases} (a)\ 8{,}700 \text{ lb.} \\ (b)\ 70{,}300 \text{ lb.} \end{cases}$

(812) What is the maximum shear (*a*) in that portion of the beam under the sidewalk and (*b*) in that portion of the beam under the roadway?

Ans. $\begin{cases} (a)\ 7{,}500 \text{ lb.} \\ (b)\ 26{,}800 \text{ lb.} \end{cases}$

(813) What is (*a*) the correct live load stress and (*b*) the correct dead load stress in each of the vertical suspenders $B\,b$, $C_1\,c$, $E_1\,e$, and $G_1\,g$ which support the floor-beam directly?

Ans. $\begin{cases} (a)\ 26{,}600 \text{ lb.} \\ (b)\ 7{,}700 \text{ lb.} \end{cases}$

(814) The bridge is to carry an electric car track. Each car, when loaded, weighs 16 tons, and one-fourth of this load, or 8,000 lb., is assumed to be supported upon each of

the four wheels. The two wheels upon each rail are 7.5 ft. apart, center to center. Two lines of stringers (one under each rail) are to be proportioned to carry the car. In what position of the car does the maximum bending moment in the stringer obtain?

Ans. With either wheel at a distance of 6.875 ft. from the corresponding support.

(815) What is the maximum bending moment in inch-pounds produced in the stringer by this load?

Ans. 518,600 in.-lb.

(816) With one wheel at the center of the stringer, what is the bending moment in inch-pounds at that point?

Ans. 480,000 in.-lb.

(817) Assume the total load upon the same two stringers to be 15 tons, supported upon two pairs of wheels which are 11 ft. apart, center to center; the load upon the forward pair of wheels is 6 tons, and the load upon the two rear wheels is 9 tons. What position of the load upon either stringer will produce the maximum bending moment in it?

(818) What is the maximum bending moment in inch-pounds produced in each stringer by this load?

Ans. 472,500 in.-lb.

(819) Make a stress sheet (so far as it relates to the data and the stresses) for the bridge to which the nineteen preceding questions refer. Omit the stringers, and use for the lower lateral system the stresses that were obtained in the solution of Question 787.

(820) A stringer of 20 ft. span is to be designed to carry in any position a system of loads consisting of 3 loads of 10,000 lb. each. The distance from the center load to each of the other two is 4 ft. In what position of this system of loads will the maximum bending moment in the stringer be produced?

(821) What is the maximum bending moment in inch-pounds produced in the stringer? Ans. 1,320,000 in.-lb.

(822) A stringer of 16 ft. span is to be designed to carry in any position a system of 4 loads. The two end or outer loads are each 6,000 lb., and the two intermediate loads are each 12,000 lb. The distance between each two adjacent loads is 6 ft. In what position of this system of loads will the maximum bending moment be produced in the stringer?

Ans. When the center of the span is between the two heavier loads and at a distance of .6 ft. from either, that is, when either of the lighter loads is 1.4 ft. from the corresponding end of the span.

(823) What is the maximum bending moment in inch-pounds produced in the stringer by this system of loads?

Ans. 800,100 in.-lb.

(824) Assuming all conditions for a 9-panel truss to be the same as in Question 763, except that the heights of the truss at *B b*, *C c*, *D d*, and *E e* are 17.14 ft., 24 ft., 27 ft., and 27 ft., respectively, construct to a scale of 5,000 lb. to the inch the stress diagram, obtaining the dead load stresses, and correct the stresses in the vertical members.

Ans.

For *a B* + 51,200 lb.
For *B C* + 49,600 lb.
For *C D* + 53,600 lb.
For *D E* + 58,800 lb.
For *E E'* + 58,800 lb.
For *a b* − 37,100 lb.
For *b c* − 37,100 lb.
For *c d* − 46,300 lb.
For *d e* − 52,900 lb.

For *e e'* − 58,800 lb.
For *B b* − 5,900 lb.
For *B c* − 12,800 lb.
For *C c* + 2,900 lb.
For *C d* − 11,000 lb.
For *D d* + 2,900 lb.
For *D e* − 10,600 lb.
For *E e* + 2,900 lb.

PROPORTIONING THE MATERIAL.

(ARTS. 1376–1507.)

EXAMINATION QUESTIONS.

(825) What materials are commonly used in the construction of bridge superstructures?

(826) What is the usual requirement for (*a*) the elastic limit and (*b*) the ultimate tensile strength of wrought iron as used in bridges?

(827) What are the common requirements for the elongation of wrought iron before rupture?

(828) What cold bending tests are required?

(829) At what value is the coefficient of elasticity of wrought iron commonly taken?

(830) What are the three grades of structural steel commonly manufactured?

(831) What are the maximum and minimum ultimate strengths and the elastic limits as usually required for each grade of steel?

(832) What are the minimum elongation and minimum reduction of area at fracture specified for each grade of steel?

(833) What is the cold bending test required for each grade of steel?

(834) What grades of steel are commonly used in the construction of bridges?

(835) When are the specific gravities of wrought iron and steel the greater, when rolled in large sections or in small bars?

(836) What is the weight per cubic foot generally used in estimating the weight of wrought iron?

For notice of copyright, see page immediately following the title page.

(837) (*a*) What is the weight of a bar of wrought iron 1 inch square and 1 yard long? (*b*) What is the weight of a wrought-iron plate 1 foot long, having a section of 12″ × 1″?

(838) (*a*) In estimating the weight of structural steel, how much heavier than wrought iron is it considered to be? (*b*) On this basis, what is the estimated weight per cubic foot of structural steel?

(839) (*a*) What is the weight of a bar of steel 1 in. square and 1 ft. long? (*b*) What is the weight of a steel plate 1 yard long, having a section of 12″ × ¼″?

Ans. { (*a*) 3.4 lb.
(*b*) 30.6 lb.

(840) What is the weight per foot of round bars of steel (*a*) 1 in., (*b*) 1½ in., (*c*) 2⅜ in., and (*d*) 3¼ in. in diameter?

Ans. { (*a*) 2.67 lb.
(*b*) 6.00 lb.
(*c*) 15.04 lb.
(*d*) 28.17 lb.

(841) What quantities are considered in determining the sectional area of a tension member?

(842) What other conditions must be considered in proportioning the section of a compression member?

(843) Upon what variable quantities do the values of the unit stresses allowed for compression members depend, as determined by modern formulas?

(844) Using Rankine's formula for compression, what is the ultimate strength per square inch of a wrought-iron column 18 ft. 4 inches long, composed of two 6-inch channels latticed in the form shown at *F*, Fig. 303, Art. **1407**, having (*a*) two square bearings, (*b*) one square and one pin bearing, and (*c*) two pin bearings?

Ans. { (*a*) 31,610 lb.
(*b*) 28,610 lb.
(*c*) 26,120 lb.

NOTE.—In the solution of questions of this kind the values of compression formulas will be obtained to the nearest ten pounds. In using straight line formulas, the value of l, if a mixed number, will be taken to the nearest inch above its calculated value, while the value of r will be expressed to the second decimal place only. In order to afford definite and explicit comparisons of the results, the weights per foot of channels, angles, and other rolled shapes, as well as the areas of sections, will be expressed to the second decimal figure. Sectional areas containing no error greater than .02, and weights per foot containing no error greater than .08, will be considered correct.

(845) Using a factor of safety of 4, what is the weight per foot required for each channel, in cases (*a*), (*b*), and (*c*) of the preceding question, in order that the column will be able to support safely a load of 40,000 lb.?

$$\text{Ans.}\begin{cases}(a) & 8.43 \text{ lb.}\\(b) & 9.32 \text{ lb.}\\(c) & 10.20 \text{ lb.}\end{cases}$$

NOTE.—In answering Questions 846 to 892, inclusive, the student will refer to the stresses shown on the incomplete stress sheet for a six-panel Pratt truss bridge of 90 feet span and 18 feet roadway, made in compliance with Question 718. The material for this structure will be wrought iron, proportioned for a pin-connected bridge, according to Cooper's Specifications for Highway Bridges.

(846) What is the sectional area required (*a*) for the hip vertical *B b*? (*b*) For the main tie *B c*? (*c*) For the tie bar *C d*? And (*d*) for the counter tie *D c'*?

$$\text{Ans.}\begin{cases}(a) & 1.54 \text{ sq. in.}\\(b) & 3.80 \text{ sq. in.}\\(c) & 2.12 \text{ sq. in.}\\(d) & .55 \text{ sq. in.}\end{cases}$$

(847) Give size of material suitable for each of these members, with the actual sectional area of same.

$$\text{Ans.}\begin{cases}\text{For } B\ b\text{, 2 bars } \frac{7}{8}'' \times \frac{7}{8}'' = 1.53 \text{ sq. in.}\\\text{For } B\ c\text{, 2 bars } 2\frac{1}{2}'' \times \frac{3}{4}'' = 3.75 \text{ sq. in.}\\\text{For } C\ d\text{, 2 bars } 2'' \times \frac{9}{16}'' = 2.25 \text{ sq. in.}\\\text{For } D\,c'\text{, 1 bar } \frac{7}{8}'' \times \frac{7}{8}'' = .76 \text{ sq. in.}\end{cases}$$

(848) By what amount is the (tensile or compressive) wind stress in each lower chord increased by the vertical reactions at the bottoms of the end posts, due to the wind pressure against the portal? Ans. 4,440 lb.

(849) (*a*) According to the requirements of the specifications, will it be necessary to make any provision for the wind stress in the panel *c d* of the lower chord? (*b*) What is the sectional area required for this member?

Ans. (*b*) 6.56 sq. in.

(850) Give size of material suitable for this section, with actual sectional area of the same.

Ans. 2 bars 4″ × $\frac{13}{16}$″ = 6.50 sq. in.

(851) Can the total compressive wind stress, due to the assumed wind pressure, neutralize or reverse the tension in any panel of the lower chord, under any condition of load? If so, in what panel?

(852) Will the total tensile wind stress in any panel of the lower chord exceed one-quarter of the stress in the same from dead and live load only? If so, in what panel?

(853) What is (*a*) the total net compressive wind stress and (*b*) the total tensile wind stress that can come upon the panel *b c* of the lower chord? (*c*) What is the total amount of tensile wind stress that must be *provided for* in determining the sectional area for this member?

$$\text{Ans.}\begin{cases}(a) & +\ 4{,}200\text{ lb.}\\(b) & -\ 13{,}300\text{ lb.}\\(c) & -\ 16{,}700\text{ lb.}\end{cases}$$

(854) (*a*) For this member, what is the sectional area required by the live and dead load stresses only? (*b*) What is the required section to provide for the tensile wind stress also?

$$\text{Ans.}\begin{cases}(a) & 4.10\text{ sq. in.}\\(b) & 4.41\text{ sq. in.}\end{cases}$$

(855) If the section is made up of four 3″ × 2½″ angles, what is the weight per foot required for each angle, allowing for holes for two ⅝″ rivets in each angle? Ans. 4.93 lb.

(856) Taking a mean between the allowed live load and dead load unit stresses as the unit stress for the wind load, what amount of compressive wind stress can be safely borne by this section? Ans. + 47,900 lb.

NOTE.—In through Pratt truss bridges with inclined end posts it is customary to give the member *a b* of the lower chord the same section as the member *b c*. This was not done, however, in the bridge which was worked out in the text as an illustration.

(857) Using four 3″ × 2″ angles in the form shown at *L*, Fig. 303, Art. **1407,** for the vertical post *C c*, what is (*a*)

the sectional area required for the post and (b) the weight per foot required for each angle ?

Ans. $\begin{cases} (a)\ 5.34 \text{ sq. in.} \\ (b)\ 4.45 \text{ lb.} \end{cases}$

(858) Using four $2\frac{1}{2}'' \times 2''$ angles in the same form for the intermediate post $D\,d$, what is (a) the sectional area required for the post and (b) the weight per foot required for each angle ?

Ans. $\begin{cases} (a)\ 2.51 \text{ sq. in.} \\ (b)\ 2.09 \text{ lb.} \end{cases}$

(859) Using no metal less than one-quarter inch in thickness, as required by the specifications, what weight of angle will be used (a) for the post $C\,c$ and (b) for the post $D\,d$?

Ans. $\begin{cases} (a)\ 4\frac{1}{2} \text{ lb. per ft.} \\ (b)\ 3\frac{3}{4} \text{ lb. per ft.} \end{cases}$

(860) Assuming the end post to be composed of two eight-inch channels connected by a twelve-inch cover-plate, what is the sectional area required for it by the live and dead load stresses ? Ans. 12.19 sq. in.

(861) If a $12'' \times \frac{5}{16}''$ cover-plate is used, what will be the weight per foot required for each channel ? Ans. 14.07 lb.

(862) Using the same external dimensions for the upper chord, what is the sectional area for the panel $B\,C$ required by the live and dead load stresses ? Ans. 8.46 sq. in.

(863) What is the sectional area given by a $12'' \times \frac{1}{4}''$ cover-plate and two eight-inch channels of the lightest weight rolled ? Ans. 9 sq. in.

(864) What is the sectional area in the panel $C\,D$ of the upper chord required by the live and dead load stresses ?

Ans. 9.52 sq. in.

(865) Using a $12'' \times \frac{1}{4}''$ cover-plate, what is the weight per foot required for each channel in this panel of the chord ?

Ans. 10.87 lb.

(866) What is the total eccentricity of this section, i. e., what is the distance of its center of gravity vertically above the centers of the channels ? Ans. 1.30 in.

(867) What is the bending moment at the center of this member due to its own weight, adding 15% for details ?

Ans. 4,106 in.-lb.

(868) (*a*) How much should the eccentricity be reduced, i. e., how far below the center of gravity of the section should the pins be located, in order that the maximum stress, when combined with the bending stress, shall be uniformly distributed over the section ? (*b*) What should be the net eccentricity of the pins for this member ?

Ans. $\begin{cases} (a) & .0473 \text{ in.} \\ (b) & 1.2527 \text{ in.} \end{cases}$

(869) (*a*) What is the maximum intensity of stress per square inch upon this member, due to the bending moment produced by its own weight ? (*b*) Will it be necessary to increase the sectional area of this member in order to provide for the bending stress ? Ans. (*a*) 124 lb.

(870) If the effective section of the upper chord member *B C* consists of two 8-inch channels, each weighing 10 lb. per foot, and a 12″ × ¼″ cover-plate (Questions 862 and 863), what is the total eccentricity of this member ?

Ans. 1.375 in.

(871) What is the bending moment in the central portion of this member due to its own weight, adding 15% for details ? Ans. 6,550 in.-lb.

(872) (*a*) How much should the eccentricity be reduced, i. e., at what distance below the center of gravity of this section should the pins be fixed, in order that the maximum stress, when combined with the bending stress, shall be uniformly distributed over the section ? (*b*) What should be the net eccentricity of the pins in this member ?

Ans. $\begin{cases} (a) & .085 \text{ in.} \\ (b) & 1.29 \text{ in.} \end{cases}$

(873) What is the total eccentricity of the end post, i. e., the perpendicular distance between a plane passing through the center of gravity of the section and a plane passing through the centers of the channels? (See Questions 860 and 861.) Ans. 1.2786 in.

(874) What is the bending moment at the center of this member, due to its own weight, adding 15 per cent. for details? Ans. 22,300 in.-lb.

(875) (*a*) How much should the eccentricity be reduced in fixing the positions of the pins, in order that the maximum stress, when combined with the bending stress, shall be uniformly distributed over the section? (*b*) What should be the eccentricity of the pins to obtain this condition?

Ans. { (*a*) .3275 in.
(*b*) .9511 in.

(876) If a 12″ × ⅜″ cover-plate were used for the end-post, what would be (*a*) the required weight per foot of each channel and (*b*) the total eccentricity of the member?

Ans. { (*a*) 12.82 lb.
(*b*) 1.5458 in.

(877) For this section what must be the eccentricity of the pins in order that the maximum compression, when combined with the bending stress, shall be uniformly distributed over the section? Ans. 1.2183 in.

(878) (*a*) What is the maximum intensity of stress per square inch upon this member due to the bending moment produced by its own weight? (*b*) Will it be necessary to increase the section of this member in order to provide for the bending stress? Ans. (*a*) 505 lb.

NOTE.—A 12″ × ⅜″ cover-plate will be used for this end post.

(879) What is the sectional area required by the stress upon the lateral rod in each panel of the lower lateral system?

Ans. { Panel *a b*, .95 sq. in.
Panel *b c*, .61 sq. in.
Panel *c d*, .29 sq. in.

(880) If round rods are used, and all conditions of the specifications are fulfilled, what will be the diameter and sectional area of each lateral rod in each panel of the lower lateral system?

Ans. { Panel *a b*, 1⅛″ ○ = .99 sq. in.
Panel *b c*, 1″ ○ = .79 sq. in.
Panel *c d*, 1″ ○ = .79 sq. in.

(881) What diameter of round rod must be used in each panel of the upper lateral system ?

(882) What is the resultant stress upon each lateral strut from the assumed initial stresses upon the lateral rods connecting at the end ? Ans. 12,400 lb.

(883) (*a*) If, for each intermediate lateral strut, two 5″ × 4″ angles are used with the longer legs back to back, what is the weight per foot of each angle required by the resultant stress ? (*b*) What is the minimum weight per foot of an angle of this size ? (*c*) What would be the weight per foot required for each angle if 5″ × 3½″ angles were used ?

Ans. (*a*) 8.43 lb.
(*b*) 10.8 lb.
(*c*) 16.80 lb.

(884) If each flange of the portal consists of two 3½″ × 2½″ angles with the shorter legs together, separated ½″ by the lattice bars, (*a*) what is the weight per foot required for each angle of the top flange ? (*b*) What is the sectional area given by the flange ? (See Question 710.)

Ans. (*a*) 5.75 lb.
(*b*) 3.45 sq. in.

(885) What is the resultant stress upon the shoe strut produced by the initial stress assumed to be upon the lower lateral rod connecting at the end ? Ans. 7,800 lb.

(886) If four 3″ × 2″ angles are used, making a section of the form shown at *L*, Fig. 303, Art. **1407**, what is the sectional area required for the shoe strut ?

Ans. 3.50 sq. in.

(887) As rolled in iron, a 3″ × 2″ angle having ¼ in. thickness of metal will weigh 4.0 lb. per foot. (*a*) Will four such angles give the required section for the shoe strut ? (*b*) What will be the section given ? (*c*) Will four angles of any other size having the required thickness afford better economy ? Ans. (*b*) 4.80 sq. in.

(888) (*a*) What is the net area of section required for the bottom flange of the floor-beam ? (*b*) Assuming ⅜ in.

thickness of metal and allowing for holes to contain ¾ in. rivets, what will be the weight per foot for each angle required to give the sectional area?

Ans. { (*a*) 4.06 sq. in.
(*b*) 7.87 lb.

(889) (*a*) Will the preceding conditions be fulfilled by two 3½″ × 3″ angles, each weighing 7.9 lb. per foot? (*b*) What weight of angles will be required for the upper flange?

(890) The maximum shear upon the floor-beam is $\frac{34{,}080}{2} = 17{,}040$ lb. If the dimensions of the web-plate be 22″ × $\frac{5}{16}$″, (*a*) will the shear be sufficiently resisted? (*b*) Will stiffeners be required?

(891) The floor of this bridge is to be of long leaf yellow pine with floor plank 3 in. in thickness. (*a*) How many lines of joist will be required? (*b*) What will be the dimensions required for each joist? (*c*) What are the minimum dimensions allowed for the wheel-guards?

Ans. { (*a*) 10 lines.
(*b*) 3″ × 12″.
(*c*) 4″ × 6″.

(892) Make, according to the instructions given in Art. **1452**, a complete stress sheet, similar to Fig. 306 of the same article, for the structure to which Questions 846 to 891, inclusive, relate. In writing sectional areas, two decimal figures are to be used, but only one decimal figure should be employed in writing the weights per foot. The principal data and the stresses may be obtained from the incomplete stress sheet made in compliance with Question 713.

NOTE.—In answering Questions 893 to 910, inclusive, the student will refer to the stresses shown on the incomplete stress sheet for a four-panel low-truss bridge of 60 feet span and 14 feet roadway, made in compliance with Question 745. The material for this structure will be medium steel proportioned for riveted connections (i.e., tension members will be of plates or shapes), according to Cooper's Specifications for Highway Bridges.

(893) (*a*) What is the net compression in the panel *a b* of the lower chord? (*b*) What is the total amount of

compressive stress, and (*c*) the total amount of tensile stress that must be provided for by the member *a b* ?

$$\text{Ans.}\begin{cases}(a) & +\ \ 900\text{ lb.}\\(b) & +\ 1{,}600\text{ lb.}\\(c) & -\ 20{,}700\text{ lb.}\end{cases}$$

NOTE.—According to item (*s*) of the specifications (Art. **1412**) both the tensile and compressive stresses must each be increased by $\frac{8}{16}$ of the smaller stress, or 720 pounds. The compressive wind stress is increased by this amount; but as this increment of stress represents *wind* and does not exceed one-quarter of the amount of stress due to dead and live load, it need not be considered in connection with the live and dead load *tensile* stresses.

(894) (*a*) What is the sectional area for this member required by the tensile stresses ? (*b*) If this member is to be composed of two bars ⅜ of an inch in thickness and the connections are to be riveted, so that the section cut out by two holes *drilled* for ⅝-inch rivets is to be deducted from the area of each bar, what will be the required (practical) dimensions of the bars ?

$$\text{Ans.}\begin{cases}(a) & 1.60\text{ sq. in.}\\(b) & 2\text{ bars }3\tfrac{1}{2}'' \times \tfrac{3}{8}''.\end{cases}$$

(895) These bars will be latticed to resist compression, in the manner shown in Mechanical Drawing Plate, Title: Highway Bridge: Details III, Fig. 3. Taking a mean between the live and dead load unit stresses allowed in compression as the unit stress for the wind load, what amount of compressive wind stress will this member safely bear ?

Ans. 13,620 lb.

(896) (*a*) What is the sectional area required for the member *b c* of the lower chord ? (*b*) If two bars, having riveted connections, are to be used, and the section cut out by two holes *drilled* for ⅝-inch rivets is to be deducted from the sectional area of each bar, what will be the required practical dimensions of each bar, assuming the metal to be ½ in. in thickness and taking the width to the nearest quarter inch above the required width ? (*c*) What is the net sectional area, available for tension, given by this section ?

$$\text{Ans.}\begin{cases}(a) & 3.73\text{ sq. in.}\\(b) & 2\text{ bars }5\tfrac{1}{4}'' \times \tfrac{1}{2}''.\\(c) & 3.88\text{ sq. in.}\end{cases}$$

(897) If the section of the top chord and end post is formed of two 6-inch channels latticed on both sides in the form shown at *F*, Fig. 303, Art. **1407,** (*a*) what is the sectional area required in the panel *B C* of the upper chord? (*b*) What is the weight per foot required for each channel? (*c*) What is the weight per foot which must be used for each channel, considering the minimum weight to be the same as for iron?

Ans. { (*a*) 4.34 sq. in.
(*b*) 7.38 lb.
(*c*) 7.50 lb.

(898) What is (*a*) the sectional area required for the panel *C C'* of the upper chord, and (*b*) the weight per foot for each channel necessary, in order to give the required area?

Ans. { (*a*) 5.80 sq. in.
(*b*) 9.86 lb.

(899) What is (*a*) the sectional area required for the end post *a B*? (*b*) The weight per foot of each channel necessary to give the area?

Ans. { (*a*) 4.40 sq. in.
(*b*) 7.48 lb.

(900) (*a*) What is the sectional area required for the web member *B b*? (*b*) If, for this member, two bars are used with riveted connections, what will be the practical dimensions required for the bars, using ⅜-inch thickness of metal, and deducting from the area of each bar the section cut out by two holes drilled for ⅝-inch rivets? (*c*) What is the net sectional area given by the bars?

Ans. { (*a*) 2.33 sq. in.
(*b*) 2 bars 4½" × ⅜".
(*c*) 2.34 sq. in.

(901) What is the total amount of (*a*) live load compressive stress, (*b*) live load tensile stress, and (*c*) dead load compressive stress, to be provided for in the member *b C*?

Ans. { (*a*) + 11,400 lb.
(*b*) − 1,300 lb.
(*c*) + 2,900 lb.

(902) If the section for this member is made up of four 2½" × 2" angles connected in the form shown at *L*, Fig. 303, Art. **1407,** what will be (*a*) the sectional area required by

the compressive stresses? (*b*) The weight per foot of each angle necessary to give the required section? (*c*) Deducting from the area of each angle the section cut out by two holes *punched* for ⅝-inch rivets, what amount of live load tensile stress will this section safely bear?

Ans. { (*a*) 4.28 sq. in.
(*b*) 3.64 lb.
(*c*) 30,860 lb.

(903) (*a*) What is the sectional area required to resist the tensile stresses upon the members *C c*? (*b*) What is the net sectional area given by four 2″ × 2″ angles, each weighing 3.2 lb. per foot, if the section cut out by two holes *punched* for ⅝″ rivets be deducted from the area of each angle? (*c*) If the angles are connected in the form shown at *L*, Fig. 303, what amount of live load compressive stress will be safely resisted by the section?

Ans. { (*a*) 1.16 sq. in.
(*b*) 2.26 sq. in.
(*c*) 5,410 lb.

(904) (*a*) What is the sectional area required for the diagonal member in the end panel *a b* of the lateral system? (*b*) What is the net sectional area given by a single 2″ × 2″ angle, weighing 3.2 lb. per foot, after deducting the section cut out by two holes punched for ⅝″ rivets?

Ans. { (*a*) .53 sq. in.
(*b*) .56 sq. in.

(905) If each diagonal member in the lateral system is composed of an angle of this size, (*a*) in what manner do the specifications require it to be connected? (*b*) Why is the section cut out by *two* rivet holes required to be deducted in the preceding question?

(906) Using the floor-beam stresses (maximum flange stress) as found in compliance with Question 740 for the floor-beam of a Howe truss bridge having the same dimensions and load as the present structure, (*a*) what is the net sectional area required for the bottom flange? (*b*) If the flange consists of two 3″ × 2½″ angles, what is the weight per foot of each angle, deducting the holes for ⅝″ rivets?

Ans. { (*a*) 2.61 sq. in.
(*b*) 5.24 lb.

(907) A web-plate $18'' \times \frac{5}{16}''$ is to be used. (*a*) Will this web-plate fulfil all requirements of the specifications? (*b*) If not, what requirement does it not fulfil?

(908) (*a*) What amount of resultant stress must the shoe strut be proportioned to resist? (*b*) Using for the shoe strut four $2\frac{1}{4}'' \times 1\frac{1}{2}''$ angles connected in the form shown at *L*, Fig. 303, Art. **1407,** with the unit stress allowed for wrought iron, what is the sectional area required to resist the resultant stress? (*c*) Steel angles of this size having $\frac{1}{4}$ inch thickness of metal weigh 3 lb. per foot; what will be the sectional area given by four angles of this weight?

Ans. { (*a*) 3,960 lb.
(*b*) 1.98 sq. in.
(*c*) 3.53 sq. in.

(909) The floor is of long leaf yellow pine, the floor plank being 3 inches in thickness and the wheel-guards being each 4″ × 6″. (*a*) If spaced 2 feet between centers, how many lines of joist will be required? (*b*) What are the required dimensions for each joist?

Ans. { (*a*) 8 lines.
(*b*) 3″ × 12″.

(910) Make, according to instructions given in Art. **1452** and similar to Fig. 306, a complete stress sheet for the structure to which Questions 893 to 909, inclusive, relate. For riveted tension members write the *net* sectional areas, placing the letters *N. S.* after each net area. The principal data and stresses may be obtained from the incomplete stress sheet made in compliance with Question 745.

NOTE.—In answering the following questions, the student will refer to the stresses shown on the incomplete stress sheet for a nine-panel Pratt truss bridge with curved upper chord, having a span of 162 feet and a roadway 20 feet in clear width, made in compliance with Question 770. Various members of this structure will be proportioned according to the requirements of two different sets of specifications, thus affording a comparison of results.

(911) If the material in the center panel *e e′* of the lower chord be proportioned for wrought iron in the form of forged eye-bars, what is the sectional area required (*a*) by Cooper's specifications? (*b*) By Thacher's?

Ans. { (*a*) 14.28 sq. in.
(*b*) 13.66 sq. in.

(912) If the material of the same member is proportioned for wrought iron in the form of plates or shapes, what is the sectional area required (*a*) by Cooper's specifications? (*b*) By Thacher's?

Ans. { (*a*) 15.43 sq. in.
(*b*) 15.10 sq. in.

(913) If the same member is proportioned in medium steel in the form of forged eye-bars, what is the sectional area required (*a*) by Cooper's specifications? (*b*) By Thacher's?

Ans. { (*a*) 11.90 sq. in.
(*b*) 10.97 sq. in.

(914) If the required section is made up by two chord bars, give suitable dimensions for each of the preceding questions, with the sectional area given by the same.

Ans. { (*a*) 2 bars 6″ × 1″ = 12.00 sq. in.
(*b*) 2 bars 5½″ × 1″ = 11.00 sq. in.

(915) If the section of the upper chord is made up by two 10-inch channels connected by a cover-plate, what would be the required width of the plate?

(916) (*a*) What metals, used in bridge construction, are injured by punching? (*b*) In what material and under what conditions should rivet holes be reamed?

DETAILS OF CONSTRUCTION.

(ARTS. 1508–1624.)

EXAMINATION QUESTIONS.

(917) (*a*) What is the minimum pitch to be used for rivets? (*b*) What is the minimum distance that should be allowed between rivet lines?

(918) (*a*) How near may the center of a rivet be to the edge of a plate? (*b*) What is the minimum thickness of metal in which a rivet may be countersunk?

(919) (*a*) What is meant by the terms *single shear* and *double shear*, as applied to rivets? (*b*) What amount of shearing stress may be allowed upon a rivet in double shear, as compared with what may be allowed upon the same size of rivet in single shear?

(920) (*a*) What is understood by the term *camber*, as applied to a truss? (*b*) What amount of camber is usually given to a bridge truss? (*c*) How is the camber produced in the truss of a through bridge?

NOTE.—Questions 921 to 937, inclusive, refer to the bridge shown on Mechanical Drawing Plates, Titles: Highway Bridge: Details I, II, III, IV, and Highway Bridge: General Drawing, and for which the complete stress sheet is shown in Fig. 306, Art. **1452.**

In obtaining bending moments on pins, the centers of actual bearings when known will be taken as the points of application of the stresses, and, in order that the effect of wind stress may be provided for, the total stress will in each case be made to correspond with the sectional area used. Bending moments will be obtained to the nearest 100 inch-pounds.

(921) (*a*) What is the distance between the centers of bearings of the pin upon the chord at the intermediate joint *C* of the upper chord, calling the web of each channel ¼ of an inch thick? (*b*) What is the distance between the centers of the bearings of the counter rods upon the same pin?

Ans. $\begin{cases} (a) \ 7\frac{3}{4} \text{ in.} \\ (b) \ 5\frac{1}{4} \text{ in.} \end{cases}$

For notice of copyright, see page immediately following the title page.

(922) (*a*) Construct the moment diagram determining the maximum bending moment upon the pin at the intermediate joint *C* of the upper chord. (*b*) If the pin is of wrought iron, what diameter is required to resist this bending moment?

Ans. { (*a*) 8,600 in.-lb.
(*b*) 1¾ in.

(923) With the truss fully loaded, what is (*a*) the horizontal, and (*b*) the vertical, component of the live load stress in the main tie *B c*? What is (*c*) the horizontal, and (*d*) the vertical, component of the dead load stress in the same?

Ans. { (*a*) 16,200 lb.
(*b*) 16,200 lb.
(*c*) 6,900 lb.
(*d*) 6,900 lb.

(924) With the same condition of load, what is (*a*) the dead load stress and (*b*) the live load stress in the intermediate post *C c*?

Ans. { (*a*) 2,300 lb.
(*b*) 0,000 lb.

(925) Assuming the entire amount of dead load, as well as live load, that comes upon the truss at the joint *c* to be delivered upon the pin through the beam hanger, what is the total amount of vertical stress in the latter?

Ans. 20,800 lb.

(926) If, in order to provide for the effect of wind stress, the live and dead load stress upon each member be increased sufficiently to correspond with the sectional area given by the member, what is the total amount of stress to be assumed (*a*) upon the lower chord member *b c*, and (*b*) upon the lower chord member *c c'*, in determining the bending moment upon the pin at joint *c*? (*c*) What is the resultant horizontal stress upon the beam hanger produced by this assumed increase of stresses?

Ans. { (*a*) 55,200 lb.
(*b*) 75,200 lb.
(*c*) 3,100 lb.

NOTE.—It will be noticed that in the present case the assumption of horizontal stress in the beam hanger, as made in the preceding question, is really inconsistent. For the greater increase of stress (or section) to provide for wind stress occurs in the panel *b c*, which increase of stress, according to the assumption, must be produced by the lateral rod in the center panel *c c'*, the occurrence of which condition is extremely improbable. But the assumption satisfactorily provides for the effect of the wind stress in the chord members according to the conditions of the specifications, and gives a condition of equilibrium at the joint.

(927) At the same joint of the lower chord, what is the distance apart of the centers of the bearings upon the pin (*a*) of the lower chord member *b c*? (*b*) Of the intermediate post *C c*? (*c*) Of the lower chord member *c c'*? (*d*) Of the main tie bars *B c*? (See plan of chord packing in Plate, Title: Highway Bridge: Details IV, Fig. 15.)

Ans. { (*a*) 8½ in.
(*b*) 7¼ in.
(*c*) 5⅞ in.
(*d*) 4 in.

(928) Construct the moment diagram determining the vertical bending moment upon the pin in the joint *c* of the lower chord, (*a*) at the center of the connection of the chord bar *c c'*; (*b*) at the center of the main tie bar *B c*; (*c*) at the center of the pin. (See plan of chord packing in Fig. 15 of plate referred to in preceding question.)

Ans. { (*a*) 800 in.-lb.
(*b*) 1,900 in.-lb.
(*c*) 18,900 in.-lb.

(929) Construct the moment diagram determining the horizontal bending moment upon the same pin (*a*) at the center of the connection of the chord bar *c c'*; (*b*) at the center of the main tie bar *B c*, and (*c*) at the center of the pin.

Ans. { (*a*) 36,200 in.-lb.
(*b*) 26,800 in.-lb.
(*c*) 30,000 in.-lb.

(930) What is the resultant bending moment upon this pin at the points at which the horizontal and vertical bending moments (*a*), (*b*), and (*c*) were found in compliance with the two preceding questions?

Ans. { (*a*) 36,200 in.-lb.
(*b*) 26,900 in.-lb.
(*c*) 35,500 in.-lb.

(931) If the pin is of wrought iron, what will be the diameter required to resist the maximum bending moment obtained in the solution of the preceding question?

Ans. 2¾ in.

(932) If the members connecting upon the pin at the lower chord joint *b* are packed upon the pin in the manner shown in Plate, Title : Highway Bridge : Details IV, Fig. 16, what is the distance between the centers of bearings (*a*) of the lower chord member *b c*? (*b*) Of the lower chord member *a b*? (*c*) Of the hip vertical bars *B b*?

Ans. (*a*) 8½ in.
(*b*) 6⅝ in.
(*c*) 2⅜ in.

(933) If, in order to provide for the effect of wind stress, the live and dead load stresses upon each member be increased sufficiently to correspond with the sectional area given by the member, what is the total amount of stress to be assumed (*a*) upon the lower chord member *b c*, and (*b*) upon the lower chord member *a b*, in determining the bending moment upon the pin at joint *b*? (*c*) What is the resultant horizontal stress upon the beam hanger produced by this increase of stress?

Ans. (*a*) 55,200 lb.
(*b*) 47,400 lb.
(*c*) 7,800 lb.

(934) Construct the moment diagram determining (*a*) the maximum horizontal bending moment upon the pin and (*b*) the horizontal bending moment at the center of the pin in the lower chord joint *b*. Ans. (*b*) 38,800 in.-lb.

(935) (*a*) At what point in the same pin does the maximum vertical bending moment occur? (*b*) By what formula may this bending moment be found? (*c*) What is the amount of the maximum vertical bending moment?

Ans. (*c*) 12,400 in.-lb.

(936) What is the resultant bending moment at the center of this pin? Ans. 40,700 in.-lb.

(937) What diameter of pin will be required to resist this resultant bending moment (*a*) if the pin is of wrought iron? (*b*) If the pin is of medium steel, allowing upon the latter a fiber stress of 20,000 lb. per square inch?

Ans. (*a*) 2⅞ in.
(*b*) 2¾ in.

NOTE.—Questions 938 to 1034, inclusive, refer to the complete stress sheet for a 6-panel Pratt truss through bridge of 90 feet span and 18 feet roadway, made in compliance with Question 892. Certain details for this structure will be proportioned in wrought iron according to Cooper's Specifications for Highway Bridges.

(938) (*a*) As required by the specifications, by what amount should the length of the upper chord be increased in each panel, in order to produce the required amount of camber? (*b*) If the length of the upper chord in each panel be increased $\frac{1}{4}$ of an inch, what will be the amount of camber given to the truss? (*c*) To what fractional part of the span will the camber be equal?

Ans. (*b*) $1\frac{1}{8}$ in.
(*c*) $\frac{1}{960}$.

(939) (*a*) With this amount of camber, what is the theoretical length of the diagonal in inches? Give answer to third decimal place. (*b*) What will be the practical length of the end post from center to center of pin holes? (*c*) What will be the practical length of the tension diagonals from center to center of pin holes?

Ans. (*a*) 254.647 in.
(*b*) 21′ $2\frac{1}{16}$″.
(*c*) 21′ $2\frac{5}{8}$″.

(940) What will be, approximately, the greatest bending moment upon a lower chord pin? Ans. 33,700 in.-lb.

(941) (*a*) If pins of medium steel are used, what diameter of pin will be required to resist this bending moment, allowing a fiber stress of 20,000 pounds per square inch? (*b*) Will this size of pin comply with the conditions of item (l_1) of the specifications? Ans. (*a*) $2\frac{5}{8}$ in.

(942) If this diameter of pin be used, what will be the practical diameter of the heads required for the chord bars in the panel *c d* of the lower chord, the heads being of the same thickness as the bar? Ans. 9 in.

(943) With the same size of pin, what will be the total practical thickness of bearing required for the end post?

Ans. $1\frac{3}{4}$ in.

(944) What is the estimated thickness of the web of each channel of the end post, the thickness of web for the minimum weight of the same being $\frac{7}{32}$(= .219) of an inch?

Ans. .324 in.

(945) Calling the thickness of the web of each channel of the end post $\frac{5}{16}$ (.3125) of an inch, what will be the required practical thickness of the reinforcing pin plate on each channel? Ans. $\frac{9}{16}$ in.

(946) Assuming the stress in the end post to be uniformly distributed upon its pin bearing, what amount of (*a*) live load stress and (*b*) dead load stress is taken by each pin plate?

$$\text{Ans.} \begin{cases} (a) & 15{,}300 \text{ lb.} \\ (b) & 6{,}600 \text{ lb.} \end{cases}$$

(947) If $\frac{5}{8}''$ rivets are used, how many will be required in each pin plate in order to properly provide for bearing?

Ans. 8 rivets.

(948) How many rivets will be required in each pin plate in order to provide for shearing? Ans. 10 rivets.

(949) What is the average net eccentricity of the end post and the end panel *B C* of the upper chord? (See Questions 872 and 877.)

(950) Giving the pin an eccentricity of 1 inch, make in pencil on drawing paper, to a scale of 3 inches to the foot, a side elevation of the end post at the shoe joint, locating the position of the pin hole and showing details of end post for slightly more than the lower two feet of its length. Make the pin plate 6½ inches wide and show three rows of rivets not staggered; i. e., show chain riveting. Use 15 rivets in each pin plate, making the pitch equal to four diameters. Show pitch of rivets connecting the cover-plate to the channel; start first rivet even with first rivet of pin plate and bring fifth rivet midway between two rivets in pin plate. Show outline of end of the lower chord member in position and mark rivets in pin plate of end post that are countersunk on inside. Show also in pencil the approximate position of batten plate, making the pitch of rivets in the latter approximately the same as in the cover-plate directly opposite. Show lower corners of channels cut away to a horizontal line about 3¾ inches below the center of the pin and

give dimensions for cutting. Mark all dimensions plainly and follow the rules of Arts. **1511** and **1512.**

NOTE.—In showing the approximate position of the batten plate, it is a good plan to locate some certain rivet in the batten plate directly opposite a rivet in the cover-plate, connecting the rivets by a dotted or broken line. This will indicate sufficiently the approximate position of the batten plate.

(951) By adding the increase in the thickness of the web, over the minimum thickness, to the width of flange given for the minimum weight of an 8″ channel in Mechanical Drawing, Fig. 79, what is the width of the flange of each channel of the end post, expressed to the nearest sixteenth of an inch ? Ans. $2\frac{3}{16}$ in.

(952) Allowing the cover-plate of the end post to project $\frac{1}{16}$ of an inch beyond the edge of the flange of each channel, what will be the distance between the backs of the channels ?
Ans. $7\frac{1}{2}$ in.

(953) Assuming the increase in the thickness of the web above the minimum thickness, in each channel of the end post, to be $\frac{1}{8}$ of an inch (it is really somewhat less than this), (*a*) how far from the back of the channel should the rivet holes through the flange be spaced ? (*b*) What will be the distance between the rivet lines in the cover-plate of the end post ?

Ans. (*a*) $1\frac{1}{4}$ in.
(*b*) 10 in.

(954) If the cover-plate of the upper chord is to project $\frac{1}{16}$ of an inch beyond the edge of the flange of each channel, what will be the distance between the channels, back to back, calling the width of the flange $2\frac{1}{16}$ inches ?
Ans. $7\frac{3}{4}$ in.

(955) (*a*) How far from the backs of the channels of the upper chord should the rivet holes through the flanges be spaced, considering the channels to be practically of the minimum weight ? (*b*) What will be the distance between the rivet lines in the cover-plate of the upper chord ?

Ans. (*a*) $1\frac{1}{8}$ in.
(*b*) 10 in.

(956) The diameter of the pin at the hip joint will be assumed to be 2⅝ inches; what is the total thickness of bearing upon it required for the upper chord member *B C*?

Ans. 1.96 in.

(957) What weight per foot of an 8″ channel, of the form shown for the minimum weight in Mechanical Drawing, Fig. 79, will give ¼ of an inch thickness of web, calling the thickness of web for the lightest weight .21 of an inch?

Ans. 11.07 lb.

(958) Assuming that, in order to comply with the specifications, channels weighing 11 lb. per foot are used throughout the upper chord, what is the practical thickness required at the hip joint for the pin plate on each channel of the upper chord? Ans. ¾ of an inch.

(959) Assuming the total stress upon the panel *B C* of the upper chord to be distributed evenly over the pin bearing, what is the amount (*a*) of live load stress and (*b*) of dead load stress taken by the pin plate on each channel?

Ans. { (*a*) 20,300 lb.
(*b*) 8,700 lb.

(960) If ⅝″ rivets are used, how many rivets will be required in the pin plate upon each channel?

Ans. 13 rivets.

(961) Make in pencil on drawing paper, to a scale of 3 inches to the foot, a side elevation of the hip joint, showing details, including about two feet of the length of the upper chord and slightly more than the same amount of the length of the end post. On the end post show a single pin plate 6¼ inches wide on the inside of the channel, but on the chord show double pin plates of the same width, placing both upon the outside of the channel. Use 12 rivets in the pin plate upon the end post and 15 rivets in the pin plates upon the channel of the chord, extending each pin plate 2⅛ inches beyond the center of the pin, measured in directions parallel to both members. Space the rivets in the pin plates the same as at the shoe joint, except that the rivets nearest to the pin in each pin plate must be at a

sufficient distance to clear the end of the opposite pin plate. Show the rivet spacing along the cover-plate of each member, locating the first rivet in each member opposite the center of the pin and neglecting any consideration of portal or lateral connections. Show a hip cover-plate ¼ of an inch thick, attaching to each member by two rivets (in each rivet line), for which the holes in the end post will be shown vacant. Give exact distance on *inside* of the bent plate between the bend and the center of the first rivet. Show the outlines of the ends of the main tie bar and hip vertical rod in position, and locate, approximately, the position of the batten plate on each member, making the pitch of rivets in the batten plate practically the same as in the cover-plate directly opposite. Follow Arts. **1511** and **1512.**

(962) What will be the maximum pitch allowed for the rivets in the cover-plate of the end post?

(963) Extend each portion of the end post in the side elevation made in compliance with Questions 950 and 961, showing upon each drawing the pitch of the rivets in the cover-plate of the end post, until it will be exactly 6 inches for the entire intervening portion of the end post.

(964) What length of batten plate will be required upon this member?

(965) (*a*) What uniform pitch of the rivets in each batten plate will correspond as nearly as possible with the pitch of the rivets in the cover-plate directly opposite, and still make the distance between each outer rivet and the end of the batten plate as nearly as possible to 1¼ inches? (*b*) What will be the distance between the center of each outer rivet and the end of the batten plate? Ans. $\begin{cases} (a)\ 2\frac{9}{16} \text{ in.} \\ (b)\ 1\frac{5}{16} \text{ in.} \end{cases}$

(966) If the lattice bars on the end post were placed at an angle of exactly 60 degrees with the axis of the member, what would be the exact length of a lattice bar from center to center of rivets? Ans. 11.547 in.

(967) If the pitch of the rivets connecting the lattice bars to the flange of each channel be made 11½ inches, can the

lattice bar at each end be made to connect upon the last (inner) rivet of each batten plate, by slightly adjusting the position of the latter ?

(968) (*a*) What will be the distance (parallel to the axis of the member) between the (inner) rivets in the two batten plates to which the lattice bars connect ? (*b*) How many spaces of 11½ inches will there be in each rivet line ?

Ans. { (*a*) 16′ 9¼″.
(*b*) 17½ spaces.

(969) What is the practical length of each lattice bar from center to center of rivet holes ? Ans. $11\frac{17}{32}$ in.

(970) In the side elevations made in compliance with Questions 950 and 961, show correctly the position of each batten plate upon the end post, accommodating the same to the above spacing of the lattice bars. If possible, fix the position of each batten plate by locating a rivet of the batten plate directly opposite a rivet in the cover-plate, connecting the two rivets by a dotted line. Having done this, ink in both these drawings.

(971) What is the greatest diameter of eye-bar head that can be used at the hip joint ?

(972) If the head of the main tie bar be made 5⅞ inches in diameter, what will be (*a*) the theoretical and (*b*) the practical thickness of the same necessary to give the required sectional area across the head, as calculated by taking a mean between formulas **192** and **193**, Art. **1595?**

Ans. { (*a*) .98 of an inch.
(*b*) 1 inch.

(973) What practical size can be used for the main tie bars, giving the same sectional area as two bars 2½″ × ¾″, and having a thickness corresponding more nearly to the required thickness of the head ?

NOTE.—Two bars 2″ × $\frac{15}{16}$″ will be substituted for the main tie bars.

(974) In what condition of load will the greatest bending moment upon the hip pin occur ?

(975) With the truss fully loaded, what is (*a*) the horizontal, and (*b*) the vertical, component of the total (live and dead load) stress in the main tie bar *B c*?

Ans. { (*a*) 28,900 lb.
(*b*) 28,900 lb.

(976) What is (*a*) the horizontal, and (*b*) the vertical, component of the total stress in the end post?

Ans. { (*a*) 48,200 lb.
(*b*) 48,200 lb.

(977) What amount of dead load may be assumed to come upon the hip pin through the bearing of the upper chord? Ans. 1,900 lb.

(978) Give the distance between the centers of the bearings of (*a*) the upper chord, (*b*) the end post, (*c*) the main tie bars, and (*d*) the hip vertical bars, upon the pin in the hip joint. Also, with the truss fully loaded, give the horizontal and vertical stress upon *each* bearing, designating those stresses which act upwards or to the left upon the pin by the + sign, and those which act downward or to the right by the – sign.

Ans.

Member.	Distance Between Centers of Bearings, In.	Horizontal Stress. Lb.	Vertical Stress. Lb.
(*a*) Upper Chord,	8¾	+ 38,550	– 950
(*b*) End Post,	7¼	– 24,100	+ 24,100
(*c*) Main Tie,	5⅛	– 14,450	– 14,450
(*d*) Hip Vertical,	3⅛		– 8,700

(979) Construct the moment diagram determining the vertical bending moment upon the hip pin at the center of the bearing (*a*) of the end post, (*b*) of the main tie bar, and (*c*) of the hip vertical bar.

Ans. { (*a*) 700 in.-lb.
(*b*) 23,800 in.-lb.
(*c*) 32,600 in.-lb.

(980) Construct the moment diagram determining the horizontal bending moment upon the hip pin at the center

of the bearing (*a*) of the end post, (*b*) of the main tie bar, and (*c*) of the hip vertical bar.

Ans. (*a*) 28,900 in.-lb.
(*b*) 44,300 in.-lb.
(*c*) 44,300 in.-lb.

(981) What is the maximum resultant bending moment upon the hip pin ? Ans. 55,000 in.-lb.

(982) If the pin at the hip joint is of medium steel, and a fiber stress of 22,500 lb. per square inch is allowed upon it, what will be the diameter of the pin required to resist the maximum resultant bending moment ? Ans. $2\frac{15}{16}$ in.

(983) If, at the hip joint, the pin plates upon the chord remain unchanged, but two pin plates are used upon each channel of the end post, the pin plate upon the outside of the channel being $\frac{1}{4}$ of an inch thick, and that upon the inside being $\frac{5}{16}$ of an inch thick, what is the distance between the centers of the bearings (*a*) of the upper chord, (*b*) of the end post, (*c*) of the main tie bars, and (*d*) of the hip vertical bars, upon the hip pin ?

Ans. (*a*) $8\frac{3}{4}$ in.
(*b*) $7\frac{3}{4}$ in.
(*c*) $5\frac{5}{8}$ in.
(*d*) $3\frac{5}{8}$ in.

NOTE.—With the arrangement of pin plates indicated in the preceding question, only the inner pin plates upon the end post and the outer pin plates upon the chord will extend beyond and around the pin.

(984) With the above arrangement of pin plates, construct the moment diagram determining the vertical bending moment upon the hip pin at the center of the bearing (*a*) of the end post, (*b*) of the main tie bar, and (*c*) of the hip vertical bar.

Ans. (*a*) 500 in.-lb.
(*b*) 24,100 in.-lb.
(*c*) 32,800 in.-lb.

(985) Construct the moment diagram determining the horizontal bending moment upon the hip pin at the center of the bearing (*a*) of the end post, (*b*) of the main tie bar, and (*c*) of the hip vertical bar.

Ans. (*a*) 19,300 in.-lb.
(*b*) 34,600 in.-lb.
(*c*) 34,600 in.-lb.

(986) (*a*) What is the maximum resultant bending moment upon the pin? (*b*) If the pin is of medium steel, using a fiber stress of 22,500 lb. per square inch, what diameter of pin will be required by this bending moment?

Ans. { (*a*) 47,700 in.-lb.
(*b*) $2\frac{13}{16}$ in.

(987) If a $2\frac{3}{4}''$ pin be used at the hip joint, what will be the total thickness of bearing required (*a*) for the upper chord? (*b*) For the end post?

Ans. { (*a*) 1.87 in.
(*b*) 1.65 in.

(988) What will be the required thickness of pin plate upon each channel (*a*) of the upper chord? (*b*) Of the end post?

Ans. { (*a*) $\frac{11}{16}$ of an inch.
(*b*) $\frac{1}{2}$ of an inch.

(989) If, at the hip joint, the diameter of the head of the main tie bar $B c$ be made $5\frac{7}{8}$ inches, the diameter of the pin being $2\frac{3}{4}$ inches, what will be (*a*) the theoretical, and (*b*) the practical, thickness required for the head of each tie bar? Use formula **190,** Art. **1594.**

Ans. { (*a*) 1.08 in.
(*b*) $1\frac{1}{16}$ in.

(990) If, at the hip joint, a pin plate $\frac{1}{4}$ of an inch thick be placed upon both the inside and outside of the web of each channel of the end post, and the pin plates of the chord are all placed upon the outside of the channels, what will be the distance between the bearings (*a*) of the upper chord? (*b*) Of the end post? (*c*) Of the main tie bars? (*d*) Of the hip vertical bars?

Ans. { (*a*) $8\frac{11}{16}$ in.
(*b*) $7\frac{13}{16}$ in.
(*c*) $5\frac{13}{16}$ in.
(*d*) $3\frac{3}{4}$ in.

NOTE.—It will be noticed that a pin plate $\frac{1}{4}$ of an inch thick upon the inside of the channel of the end post will be too thin for countersinking the rivets, Art. **1512** (VI). Consequently, the rivets nearest the pin must be placed at a sufficient distance to clear the head of the tie bar, without countersinking, and it will be necessary to allow only $\frac{1}{16}$ of an inch clearance between the pin plate and the head of the tie bar.

(991) With the preceding arrangement of pin plates, construct the moment diagram determining the vertical bending moment upon the hip pin at the center of the bearing (*a*) of the end post, (*b*) of the main tie bar, and (*c*) of the hip vertical bar.

Ans. $\begin{cases} (a) \text{ } 400 \text{ in.-lb.} \\ (b) \text{ } 22{,}700 \text{ in.-lb.} \\ (c) \text{ } 31{,}700 \text{ in.-lb.} \end{cases}$

(992) Construct the moment diagram determining the horizontal bending moment upon the hip pin at the center of the bearing (*a*) of the end post, (*b*) of the main tie bar, and (*c*) of the hip vertical bar.

Ans. $\begin{cases} (a) \text{ } 16{,}900 \text{ in.-lb.} \\ (b) \text{ } 31{,}300 \text{ in.-lb.} \\ (c) \text{ } 31{,}300 \text{ in.-lb.} \end{cases}$

(993) (*a*) What is the maximum resultant bending moment upon the pin? (*b*) If composed of medium steel, using a fiber stress of 22,500 lb. per square inch, what diameter of pin will be required by this bending moment? (*c*) With this size of pin, what will be the thickness of pin plate required on each channel of the upper chord?

Ans. $\begin{cases} (a) \text{ } 44{,}600 \text{ in.-lb.} \\ (b) \text{ } 2\frac{3}{4} \text{ in.} \\ (c) \text{ } \frac{11}{16} \text{ in.} \end{cases}$

NOTE.—The operations from Questions 978 to 993, inclusive, afford a very practical illustration of a method of effecting a material reduction of the bending moment upon a pin by a judicious arrangement of the bearings. The reduction in the amount of bending moment, as obtained in Questions 981 and 993 is $55{,}000 - 44{,}600 = 10{,}400$ inch-pounds; the reduction in the diameter of the pin is $2\frac{15}{16} - 2\frac{3}{4} = \frac{3}{16}$ of an inch. A pin $2\frac{15}{16}$ inches in diameter could not have been advantageously used at the hip joint.

(994) If the intermediate post *C c*, latticed in the form shown in Plate, Title: Highway Bridge: Details I, Fig. 3, connects upon the inside of $\frac{3}{8}''$ plates which are riveted upon the inside of the channel of the chord in the same general manner as shown for joint *C* in Fig. 2 of the same plate, what will be the distance out to out of the angles of the post, allowing $\frac{1}{16}$ of an inch clearance on each side?

Ans. $6\frac{3}{4}$ in.

(995) How many rivets will be required to connect the upper end of the post *C c* to the plates referred to in the preceding question, making proper allowance for field riveting ? Ans. 12 rivets.

(996) (*a*) Assuming the head of each tie bar *C d* to be $\frac{5}{8}$ of an inch thick and allowing for countersunk rivets upon the inside of the adjacent plates, what will be the maximum bending moment upon the pin at joint *C* of the upper chord ? (*b*) What diameter of pin will be required by this bending moment, using medium steel with a fiber stress of 22,500 pounds per square inch ?

Ans. { (*a*) 8,700 in.-lb.
(*b*) $1\frac{5}{8}$ in.

(997) If a $1\frac{3}{4}''$ pin is used, will the webs of the channels and the $\frac{3}{8}''$ plates riveted upon the inner sides of the same give sufficient bearing for the pin ?

(998) If the thickness of the head of the tie bar *C d* be made $\frac{5}{8}$ of an inch (Arts. **1597** and **1598**), what will be (*a*) the calculated, and (*b*) the practical, or nominal, diameter of each head ?

Ans. { (*a*) 4.63 in.
(*b*) 5.00 in.

(999) What is the total thickness of bearing required for the pin plates at the bottom of the intermediate post *C c*, assuming the diameters of all lower chord pins to be $2\frac{5}{8}$ inches ? Ans. .46 of an inch.

(1000) (*a*) What should be the clear distance between the center of the pin and the lower ends of the angles of the intermediate post *C c* ? (See Question 942.) (*b*) What may be considered to be the length of the column formed by each pin plate ?

Ans. { (*a*) $4\frac{3}{4}$ in.
(*b*) 6 in.

(1001) (*a*) How wide should the pin plates be upon the post *C c* ? If the total sectional area of the pin plates at the bottom of the post be made equal to the gross sectional area of the angles of the post, what will be (*b*) the calculated and (*c*) the practical thickness of each pin plate ?

Ans. { (*b*) .415 of an inch.
(*c*) $\frac{7}{16}$ of an inch.

(1002) If each pin plate at the bottom of the post *C c* is made $\frac{7}{16}$ of an inch thick, (*a*) what will be the live load unit stress allowed upon it, considered as a column? (*b*) What will be the total sectional area required for the two pin plates by the stresses upon the member? (*c*) What will be the sectional area given by the two pin plates?

Ans. $\begin{cases} (a) \text{ 6,370 lb.} \\ (b) \text{ 2.50 sq. in.} \\ (c) \text{ 5.69 sq. in.} \end{cases}$

(1003) (*a*) What amount of live load stress will these pin plates bear as columns? (*b*) If the rivets by which the pin plates are connected to the angles are to be as strong as the pin plates acting as columns, how many rivets will be required to connect *each* pin plate, allowing shearing and bearing values at 9,000 and 15,000 pounds per square inch, respectively?

Ans. $\begin{cases} (a) \text{ 36,200 lb.} \\ (b) \text{ 8 rivets.} \end{cases}$

(1004) What will be the distance between (*a*) the inner surfaces and (*b*) the outer surfaces of the pin plates at the bottom of the post *C c*?

Ans. $\begin{cases} (a) \text{ } 6\frac{7}{8} \text{ in.} \\ (b) \text{ } 7\frac{3}{4} \text{ in.} \end{cases}$

(1005) At the shoe connection (or joint *a*) of the lower chord member *a b* (which member is given the same section as the member *b c*), what is the sectional area of the pin plates required in the section on each line *a*, *b*, and *c*, Fig. 326, Art. **1601?**

Ans. $\begin{cases} (a) \text{ 2.89 sq. in.} \\ (b) \text{ 3.81 sq. in.} \\ (c) \text{ 3.50 sq. in.} \end{cases}$

(1006) (*a*) If the widths of these pin plates are made the same as the width of the member (perpendicular to the lattice bars), what will be the net width of the section upon the line *a*? (*b*) If $\frac{1}{16}$ of an inch clearance be allowed between the edge of the pin plate (as cut away) and the lower side of the cover-plate of the end post, what will be the net width of the section on the line *c*?

Ans. $\begin{cases} (a) \text{ } 1\frac{5}{16} \text{ in.} \\ (b) \text{ } 1\frac{5}{8} \text{ in.} \end{cases}$

(1007) (*a*) What will be the total thickness of the pin plates in order to give the required sections on the lines *a* and *c* (i. e., the greatest thickness required on either of the lines)? (*b*) What will be the practical thickness of pin plate required on each side of the member? (*c*) How far should the pin plates extend beyond the center of the pin? Give the practical dimension.

Ans. (*a*) 2.15 in.
(*b*) $1\frac{1}{16}$ in.
(*c*) $3\frac{1}{8}$ in.

NOTE.—In practice, these pin plates would probably be made to extend $3\frac{1}{4}$ inches beyond the center of the pin.

(1008) In determining the number of rivets required in the pin plates, what should be considered to be (*a*) the live load stress and (*b*) the dead load stress upon the member *a b* (corresponding with the sectional area of the member)? (*c*) How many rivets are required at the end *a* of the member *a b* to connect the pin plates to the angles composing the member?

Ans. (*a*) 36,100 lb.
(*b*) 15,400 lb.
(*c*) 22 rivets.

(1009) (*a*) What is the total thickness of bearing required for the pin plates at the end *b* of the same member, assuming the diameter of all lower chord pins to be $2\frac{5}{8}$ inches? (*b*) What is the total thickness of bearing required for the pin plates at each end of the member *b c*? Ans. (*a*) 1.31 in.

(1010) (*a*) If each pin plate at the end *b* of the member *a b* and at each end of the member *b c* is a single plate $\frac{11}{16}$ of an inch thick, how wide should each be? (*b*) How far should each of these pin plates extend beyond the center of the pin? (*c*) How many rivets will be required in each pin plate?

Ans. (*a*) $6\frac{7}{8}$ in.
(*b*) $4\frac{1}{8}$ in.

NOTE.—In practice, these pin plates would probably be made 7 inches wide and extend $4\frac{1}{4}$ inches beyond the center of the pin.

(1011) (*a*) If the pin plates upon the member *a b* at joint *a* are composed of two $6\frac{1}{2}'' \times \frac{9}{16}''$ plates and two $6\frac{1}{2}'' \times \frac{1}{2}''$

plates (Question 1007) arranged in the manner shown at *a* in Plate, Title: Highway Bridge: Details III, Fig. 1, the 9/16" plate being upon the outside of the angles, and 3/16 of an inch clearance being allowed on each side for countersunk rivets in both chord and end post, what will be the width of the member *a b*, out to out of angles? (*b*) What will be the distance out to out of the pin plates at the end *b* of the same member?

Ans. { (*a*) 6 in.
(*b*) 7⅝ in.

(1012) (*a*) If, at the lower chord joint *c*, the pin plates on the member *b c* are designed to pack outside of the pin plates on the intermediate post *C c*, what will be the width of the member *b c*, out to out of angles? (*b*) At the lower chord joint *b*, by how much will the pin plates on the member *b c* clear those on the member *a b*?

Ans. (*a*) 7⅞ in.

(1013) Make a sketch of lower chord joint *c*, showing in plan the best practical arrangement of the connecting members, having reference to the bending moment upon the pin. Show simply the ends of the members connecting upon the pin, omitting other details. Show each end of the pin broken off, omitting nut or thread. Consider the beam hanger to be 1⅛ inches square, the heads of the tie bars *B c* to be of the same size as at the hip joint, and the loop of the counter tie to be flattened to ¾ of an inch in thickness. Mark the distance from center to center of each pair of bearings.

(1014) In what condition of load will the greatest bending moment be produced upon the pin at the lower chord joint *c*?

(1015) In determining the bending moment upon the pin, what should be considered to be the total stress (*a*) in the member *b c*? (*b*) In the member *c d*? (*c*) What will be considered to be the horizontal stress in the beam hanger?

Ans. { (*a*) 51,500 lb.
(*c*) 3,300 lb.

(1016) With the truss fully loaded, what is (*a*) the horizontal, and (*b*) the vertical, component of the total stress in the main tie member *B c*? (*c*) In the same condition of load, what are the horizontal and vertical components of the live load stress in the diagonal *c D*? (*d*) In the same condition of load, what is the total stress in the intermediate post *C c*?

Ans. { (*b*) 28,900 lb.
(*d*) 11,550 lb.

(1017) Give in a tabular form the distance between the centers of the bearings upon the pin at joint *c* of (*a*) the member *b c* of the lower chord, (*b*) the intermediate post *C c*, (*c*) the lower chord bars *c d*, and (*d*) the main tie bars *B c*, together with the horizontal and vertical stress or pressure upon *each* bearing, with the truss fully loaded, designating those stresses which act upwards or to the left upon the pin by the + sign, and those which act downwards or to the right by the — sign. (*e*) Give also the horizontal and vertical stress in the beam hanger, using the same signs.

(1018) Construct the moment diagram determining the vertical bending moment upon the pin in the joint *c* of the lower chord (*a*) at the center of the bearing of the chord bar *c d*, (*b*) at the center of the bearing of the main tie bar *B c*, and (*c*) at the center of the pin.

Ans. { (*a*) 4,000 in.-lb.
(*b*) 9,800 in.-lb.
(*c*) 7,300 in.-lb.

(1019) Construct the moment diagram determining the horizontal bending moments at the same three points.

Ans. { (*a*) 33,800 in.-lb.
(*b*) 21,000 in.-lb.
(*c*) 24,300 in.-lb.

(1020) (*a*) What is the maximum resultant bending moment upon this pin? (*b*) Using medium steel with a fiber stress of 22,500 lb. per square inch, what diameter of pin is required by this bending moment?

Ans. { (*a*) 34,000 in.-lb.
(*b*) $2\frac{9}{16}$ in.

(1021) What is the total load supported by the floor-beam, as obtained from information given on the stress sheet? Ans. 34,100 lb.

(1022) What is the shear in the beam at the support and at points 1, 2, 3, 4, 5, and 6 ft. from the support?

Ans.
(*a*) At support, 17,050 lb.
(*b*) At 1 ft., 16,100 lb.
(*c*) At 2 ft., 14,210 lb.
(*d*) At 3 ft., 12,310 lb.
(*e*) At 4 ft., 10,420 lb.
(*f*) At 5 ft., 8,530 lb.
(*g*) At 6 ft., 6,630 lb.

(1023) (*a*) If the floor-beams be made 22½ in. deep, out to out of angles, what will be the distance between the lines of rivets connecting the flange angles to the web plate? (*b*) If ⅝″ rivets are used for this purpose, what is the least value of one rivet in either shearing or bearing?

Ans.
(*a*) 19 in.
(*b*) 2,930 lb.

(1024) What is the pitch of rivets in the flange at the support and at points 1, 2, 3, 4, 5, and 6 ft. from the support, as required by the increments of the flange stress only?

Ans.
(*a*) At support, 3.27 in.
(*b*) At 1 ft., 3.46 in.
(*c*) At 2 ft., 3.92 in.
(*d*) At 3 ft., 4.52 in.
(*e*) At 4 ft., 5.34 in.
(*f*) At 5 ft., 6.53 in.
(*g*) At 6 ft., 8.40 in.

(1025) Assuming the entire load upon the beam, exclusive of its own weight (which will be estimated by increasing the weight of web and flanges 10 per cent. to provide for weight of details), to be uniformly distributed along the upper flange in the loaded portion of the beam, what is the load per lineal inch upon the upper flange? Ans. 153 lb.

(1026) Make the graphical construction determining the final pitch of the rivets in the upper flange at points 1, 2, 3, 4, 5, and 6 ft. from the support.

Ans.
(*b*) At 1 ft., 3.40 in.
(*c*) At 2 ft., 3.85 in.
(*d*) At 3 ft., 4.41 in.
(*e*) At 4 ft., 5.15 in.
(*f*) At 5 ft., 6.17 in.
(*g*) At 6 ft., 7.69 in.

(1027) Make a sketch of the floor-beam, similar to Fig. 322, Art. **1588,** showing the pitch required and the pitch used for the rivets in the flanges. Show also a practical arrangement of stiffeners upon the beam, with pitch of rivets in same. Omit other details.

(1028) Assuming the lower lateral rods to be attached to the floor-beams in the same manner and at the same position as shown in Plate, Title: Highway Bridge: Details II, Fig. 4, what will be the length from center to center of pins of each lateral rod in the intermediate panels of the lower lateral system ? Ans. 22′ 9¼″.

(1029) Assuming, for example, that the lateral rods in the panel *b c* are round rods 1¼ inches in diameter and connect at each end upon a 2″ pin, make a sketch similar to plate, title as above, III, Fig. 5, giving dimensions of rod, loops, thread ends, and turnbuckle, corresponding to those there given, allowing 4 inches between the two thread ends. Omit the radii of curves

(1030) If beam hangers similar to that shown in Fig. 7 of the same plate are used, what will be the total sectional area required for each hanger ? Ans. 1.89 sq. in.

(1031) If rods 1⅛″ square are used for the hangers, (*a*) what will be the required diameter of the upset screw end, and (*b*) what will be the distance between the ends of the two branches, center to center, allowing ⅛ of an inch clearance over the pin ? Ans. (*b*) 4⅜ in.

(1032) (*a*) If the pin plates at the bottom of the intermediate post *C c* extend 4¾ inches below the center of the pin, how far should it be from the upper extremity of the parallel portion of the two branches of the hanger to the inner side of the bend? (*b*) If the upset begins $\frac{3}{4}$ of an inch above the bottom of the beam, how long is the parallel portion of the hanger above the upset?

Ans. (*a*) $6\frac{1}{16}$ in.
(*b*) $21\frac{3}{4}$ in.

(1033) Assuming the flange angles of beam to be so supported by stiffeners that the hanger plate may be considered as a uniformly loaded beam, (*a*) what is the estimated thickness and (*b*) the practical thickness required for the hanger plate, its width being 5 inches? (*c*) If $\frac{3}{4}$-inch check nuts are used, what will be the required length of each upset end of the hanger?

Ans. (*a*) .79 in.
(*b*) $\frac{3}{4}$ in.
(*c*) 5 in.

(1034) Make in ink a sketch of the beam hanger and hanger plate, similar to Fig. 7 of Plate, Title: Highway Bridge: Details IV, giving all necessary dimensions according to the directions given in the text.

DETAILS, BILLS, AND ESTIMATES.

(ARTS. 1625–1727.)

EXAMINATION QUESTIONS.

NOTE.—The following questions refer to the Pratt truss bridge of 90 ft. span and 18 ft. roadway, for which the complete stress sheet has been made in compliance with Question 892.

Questions 938 to 1034, inclusive, refer also to the same structure.

(1035) If, at joint *D* of the upper chord, pin plates ⅜ in. thick are riveted upon the inside of the channels of the chord for the purpose of connecting the intermediate post by field riveting, in the same manner as at joint *C* (see Question 994 and following), how many rivets will be required for connecting the intermediate post *D d*?

Ans. 4 rivets.

NOTE.—In order that this connection may possess considerable lateral stiffness, it will be well to use 8 rivets.

(1036) If the lateral strut $C\ C_1$ be designed to connect to the chord by being riveted upon the top of the cover-plate, how many field rivets will be required for the connection?

Ans. 8 rivets.

NOTE.—It will be noticed that 4 rivets in this connection, passing through both the cover-plate and the upper flanges of the channels, will have sufficient bearing, so that the shearing stress will be the critical condition, while for all other rivets in the connection, passing through the cover-plate only, the bearing stress will be the critical condition.

(1037) What is the amount of stress which must be resisted (*a*) by the connection for the upper flange of the portal, and (*b*) by the connection at the lower end of the portal bracket?

Ans. { (*a*) 16,800 lb.
(*b*) 15,100 lb.

(1038) If the portal connections are made by means of plates ¼ in. in thickness, how many rivets will be required for the connection (*a*) of the upper flange and (*b*) for the

For notice of copyright, see page immediately following the title page.

connection at the bottom of the portal bracket, the rivets being hand-driven ?

Ans. { (*a*) 11 rivets.
(*b*) 10 rivets.

(1039) If 5½″ × ⅜″ pin plates are used at the bottom of the intermediate post *D d*, what will be the total amount of live load compressive stress which they will safely bear as struts ? Ans. 24,700 lb.

(1040) If these pin plates are to be so connected to the angles of the post that the connecting rivets will have as great a resistance as the combined strut resistance of the pin plates, (*a*) how many rivets will be required to connect both plates ? (*b*) How many rivets, practically, should be used ?

Ans. { (*a*) 11 rivets.
(*b*) 12 rivets.

(1041) If plate hangers are riveted to the ends of the floor-beam, what should be the minimum permissible thickness of each hanger plate ? (See formula **196,** Art. **1598.**)

(1042) If each hanger plate is ⅝ in. thick, (*a*) how wide should each be in order to give sufficient material in the section through the pin hole, the diameter of the pin being 2⅝ in.? (*b*) Will this thickness give sufficient bearing upon the pin ? Ans. (*a*) 6½ in.

(1043) If each hanger plate is connected to two 3″ × 2½″ × ⅜″ end stiffener angles, riveted to the web of the floor-beam, (*a*) how many rivets joining each hanger plate to the connecting angles will be required by the stress ? (*b*) What is the smallest practical number of rivets that should be used for this purpose ?

Ans. { (*a*) 11 rivets.
(*b*) 12 rivets.

(1044) (*a*) If $\frac{1}{16}$ in. clearance is allowed between each hanger plate and the adjacent end of the flange angles of the floor-beam, what will be the length of the flange angles ? (*b*) If $\frac{5}{16}$ in. clearance be allowed between each hanger plate and the corresponding end of the web-plate of the floor-beam, what will be the length of the web-plate?

Ans. $\begin{cases} (a)\ 18'\ 11\frac{1}{4}''. \\ (b)\ 18'\ 10\frac{3}{4}''. \end{cases}$

(1045) What is the vertical pressure upon the leeward shoe, due to the wind force against the portal? (*b*) What is the total vertical load or pressure that must be resisted by the shoe?

Ans. $\begin{cases} (a)\ 4{,}440 \text{ lb.} \\ (b)\ 52{,}640 \text{ lb.} \end{cases}$

(1046) (*a*) If a $2\frac{5}{8}''$ pin be used at each shoe joint, what will be the total thickness of the shoe standards, as required by the bearing stress? (*b*) If the bearing is sustained entirely by two standards, what will be the practical thickness of each standard?

Ans. $\begin{cases} (a)\ 1.34 \text{ in.} \\ (b)\ \frac{11}{16} \text{ in.} \end{cases}$

(1047) What will be the distance between the outer surfaces of the pin plates at the shoe joint of the end post? (See Question 952 and following). Ans. $9\frac{1}{4}$ in.

(1048) If rivets are countersunk in the outer sides of these pin plates and also in the inner sides of the shoe standards, what will be the least practicable width between the inner surfaces of the shoe standards? Ans. $9\frac{3}{4}$ in.

(1049) For the structure to which the preceding questions refer, make complete detail drawings of all riveted work, including all members and parts which can not be sufficiently described in shop lists. Use a scale of $1\frac{1}{2}$ in. to the foot. The details of all riveted members should be thoroughly worked up and drawn out and completed in pencil before being inked in. In working up the details, the student will be greatly aided by the special information obtained in answering the questions relating to Details of Construction, most of which refer to the details of this structure.

Wherever the dimensions of members or material are so modified in the questions as to differ from those shown on the stress sheet, the modified dimensions will be used.

Make the details of the end post generally the same as in the drawings made in compliance with Questions 950 and 961. Make the details of the upper chord, including lateral hitches and connections for the lateral struts and for the upper flange of the portal, of the same general character as shown in Plate, Title: Highway Bridge: Details I, Fig. 2, but show only one splice in the chord, locating it near the center joint *D*. Provide sufficient connection for the lower end of the portal bracket, using the required number of rivets, but do not extend the connection upwards along the end post further than necessary. Use plate hangers for the floor-beams. Connect the lower lateral rods to the floor-beams, using a connection somewhat similar to that shown on plate, title as above, II. Make the general construction of the shoe similar to that shown on plate, same title, III, but show the small angles, which serve to keep the nest of rollers in position, to be riveted upon the bottom of the shoe plate of the roller shoe, somewhat as shown in Fig. 330, Art. **1632.**

The student will exercise judgment in designing the details, and will, for each case and to the best of his ability, study out the most simple, neat, and efficient detail suitable to the attending conditions. The drawings should be arranged neatly and compactly upon the sheets. A neat, brief title should be placed in the lower right-hand corner of each sheet, giving the contract number, shop number, length of span *c.* to *c.*, clear width of roadway, and number of the sheet. The contract number and shop number may be assumed to be 579 and 736, respectively.

(1050) Make shop lists for all forged and welded bars and for all pins required for this structure, as follows:

(*A*) List of all forged eye-bars.
(*B*) List of all plain bars with welded loops.
(*C*) List of all bars with welded loops and turnbuckles.
(*D*) List of all bars with upset screw ends.
(*E*) List of all chord pins.
(*F*) List of all cotter pins and anchor bolts.

The ordered material may be omitted from these **lists.**

(1051) Make order lists for all material required for this structure, except for rivets, as follows:

(*G*) List of all material for riveted members as shown on drawings.

(*H*) List of all material for forged members and pins, as given on shop lists.

On these lists, any fictitious name may be used for the location of the bridge.

(1052) If for this structure joists of medium steel were to be used, (*a*) how many lines of joists would be required? (*b*) What would be the unit stress allowed upon the joists?

Ans. { (*a*) 7 lines.
(*b*) 14,400 lb.

(1053) (*a*) For a uniform load of 100 lb. per square foot, what will be the bending moment upon each joist (except in two outer rows)? What will be (*b*) the depth and (*c*) the weight per foot required for each joist, in order to most economically give a resisting moment equal to this bending moment?

Ans. { (*a*) 101,250 in.-lb.
(*b*) 6 in.
(*c*) 13 lb. per foot.

(1054) Using timber joists, make the complete lumber bill for this structure according to the dimensions specified on the stress sheet.

(1055) Make the shipping bill for the metal work of this structure, omitting erection bolts and pilot nuts.

(1056) Make the erection diagram for the structure.

(1057) (*a*) Make an approximate estimate of the weight of all metal work in the structure, according to the instruction given in Art. **1673** and following, assuming the joists to be of wood. (*b*) What is the approximate weight of the metal per lineal foot of the bridge? Ans. (*b*) 355½ lb.

INDEX.

A. PAGE

Accuracy of dead load . . . 1060
Alternate stresses, Provision for by Cooper's specifications . 825
" " latticing of members subject to 857
Anchor bolts, Size and Position of 1016
" " Specifications relating to . . . 901
" " Various forms of . 1016
Anchors, Specifications relating to 901
Angles in tension, how connected . 815
" Maximum and minimum weights of 830
Annealing of steel 630
Area, Gross and net 854
" Increase of, to provide for bending stress . . . 887
" Increase of, to provide for wind stress 850
" Minimum, of lateral rods . 816
" of bridge member, how determined 815
" Reduction of, at fracture, required of steel 811
" " " by rivet holes 815
Arrangement of floor joists . 862, 1045
" " rivets . 924, 941, 946
Asphaltum paint 1077
Attachment of hand-rail posts . 1074
Axis, Neutral 661, 663

B. PAGE

Back wall of bridge 1045
Balanced section 841
Baltimore truss, Description of . 778
" " Stresses in, graphically determined 779, 782
Barbed bolt 1017
Batten-plates, Design of . 945, 950
" " Length of, how estimated . . . 1057
" " Pitch of rivets in . 946

PAGE

Batten plates, Specifications relating to . . . 899
Batter braces 687
Beam, Bending moment of . 649, 884
" Continuous 647
" Deflection of 668
" flanges, Pitch of rivets in 967-971
" Law of bending stresses in . 663
" overhanging both supports, uniform load . . . 1010
" Restrained 647
" Simple 647
" Strength and stiffness of . 671
" under uniform load, Force, moment, and shear diagrams for 651
" under vertical loads . . 653
Beam-hangers, Design of . . 998
" Double . 1067
" Ordered length of . 1039
" Plate . . . 1067
" Position of . . 973
" Single . . . 1066
" Specifications relating to . . . 899
" Stiff 1069
" Weights of, used in estimates . . 1059
Beams above chords . . 1066, 1069
" below chords . . . 1065
Bearing stress on pins . 897, 932, 933
" " " " Determination of . 915
" " " rivets . . 897, 921
" " " " Determinaation of . 920
" " " timber . . . 876
Bed-plates, Design of . . . 1014
" Specifications relating to 901
Bending moment 649
" " diagram . . 651
" " due to weight of beam . . . 883

PAGE
Bending moment graphically determined . . 650
" " in end post due to wind pressure . 851
" " " floor beam . . 728
" " " " " with sidewalks 799
" " " pins . . . 927
" " on shoe plate . 1009
" " Position of load for maximum 802, 803
" " Stresses due to . 735
Bending stress allowed on pins . 897
" " due to weight of member . . 880
" " how determined . 662
" " Law of . . . 663
" test 809
Bessemer steel 630
Bill of iron 1034
" " lumber 1044, 1053
" Shipping 1048
Blister steel 630
Bolts, Anchor 1016
" Erection 1052
" Forms of 1017
" for wheel guards, Number and size of 1049
" Shearing and bearing stresses allowed on 897
" Specifications relating to . 902
Braced portal, Stresses in . . 795
Braces of truss 687
Bracing, Specifications relating to 900
Brackets, Sidewalk 798
Brick, Ultimate bending stress of 665
" " compressive strength of 638
" Weight of 625
Bridge, Classification of loads on . 690
" Deck 687
" floor, Weight of . . . 706
" Half-deck 688
" High-truss 688
" Low-truss 688
" Pin-connected . . . 688
" Riveted 689
" seat 1045
" Through 687
" Truss, arch, and suspension distinguished . . 685

C. PAGE
Camber 903
" Effect of, on length of diagonals 905

PAGE
Camber, Increase of panel length to provide for . . . 904
" Specifications relating to 902, 905
" Usual amount of . . . 904
Camel-back truss 773
Cantilever 647
" truss 686
Capacity of bridge 690
Cast iron, Advantages and disadvantages of . . . 626
" Coefficient of elasticity of 636
" Factors of safety used for 641
" in compression, Elastic limit of 638
" " compression, Ultimate strength of . . 638
" " shear, Coefficient of elasticity of . 639
" " " Ultimate strength of . . . 639
" " tension, Elastic limit of . . . 636
" " " Ultimate strength of . . . 636
" little used in bridge work 807
" malleable . . . 627
" Properties of . . . 626
" Ultimate strength of, for flexure 638
" Uses of 626
" Varieties of . . . 625
" Weight of 625
Cast steel 630
Cementation process of making steel 630
Center of gravity found by moments 882
Chains, Open link 683
" Strength of 683
" Stud link 683
Channels, Distance between, in chord sections . . 827
" Formula for width of flange 833
" Maximum and minimum weights of . . . 830
Check nuts 999
Chord stresses, Formula for, in Howe and Pratt trusses . . 762
" " Formulas for, in inclined-chord and divided-panel trusses 768, 781
" " from wind pressure, when considered . 815

PAGE
Chords, Curved 750
" Effect of wind stresses upon 850
" Inclined . . . 750, 764
" Lower, design of . . 818, 979
" Upper, " " . . 833, 948
" Width of 760
Clearance between eye-bars . . 937
" General rules relating to 911
" of pin holes . . . 899
Clerk Maxwell method for the graphical determination of stresses 733
Coefficient of elasticity . . . 635
" " " of materials in compression . 638
" " " " materials in shear . 639
" " " " materials in tension 636
" " " " structural steel . 811
" " expansion of wrought iron and steel . . 1007
" " friction of wrought iron, steel, and masonry . . . 878
Cold bending test 809
Columns 672
" Failure of, by buckling . 673
" Fixed and hinged ends of 673
" Formulas for unit-stress allowed on . . . 824
" Length of 825
" Rankine's formula for resistance of . . . 823
" Resistance of, to flexure . 674
Comparison of strength and stiffness of beams 671
Compression, Laws of . . . 637
" members . . . 686
" " Length of . 825
" " Proportioning of 826, 833
" " Symmetrical and unsymmetrical . 1084
" " Unit stresses allowed on 824
" " Usual forms of . . 819
" " Curve and straight line, formulas for 823

PAGE
Compressive stresses, Formulas for, based on fatigue of metals . 873
" " Formulas for steel . . 869
" " Formulas for wrought iron . . 822
Concentrated loads 690
" wheel loads . . 801
Connections, Strength of . . . 897
Continuous beam 647
" truss 686
Cooper's specifications for highway bridges 869
Cotter pins 1023
" Dimensions of . . 1024
" Grip of 1024
Counter stresses, when induced . 709
Counterbraces 687
" Design of . . 987, 992
" when required . . 711
Countersunk rivets 1083
Covering capacity of paints . . 1079
Cover-plates, Maximum width and thickness of . . 825
" Pitch of rivets in . 941
" Width of . . . 833
Crucible steel 630
Curve formulas for columns . . 823
Curved chords, Graphical determination of stresses in 751, 753
" " when used . . 749
Cylinders, Strength of, under uniform pressure . . 645
" subjected to external pressure 645
" Thick 645

D. PAGE
Data of bridge 695
Dead load 690, 704
" Approximate formulas for calculating . . 705
" Difference between assumed and final . . 1060
" Distribution of, between upper and lower chords 707
" of bridge with sidewalks 798
" stresses 707
" " for wrought-iron compression members (Cooper) 824

PAGE
Dead load stresses, wrought-iron tension members (Cooper) . 814
" " Graphical determination of, in Baltimore truss 779
" " Graphical determination of, in Petit truss . 787
" " Graphical determination of, in truss with curved chords . 751
" " Graphical determination of, in truss with inclined chords 765
" " Graphical determination of, in truss with parallel chords . 707
" " Graphical determination of, in Whipple truss 775
Deck bridges 687
" Distribution of dead load between chords of 877
" Minimum stress in center strut of . 761
" Stresses in . . 711, 715
Deflection of beams 668
Depth of floor beam 730
" truss . . 749, 862, 871, 875
" " used for lateral system 712
Design of batten plates . . 945, 950
" beam hangers . . 998
" bed plates . . . 1014
" compression members 826, 833
" counters . . . 987, 992
" diagonal members . . 986
" end post . . . 833, 941
" floor beams . . 859, 966
" intermediate posts . . 951
" knee braces . . . 964
" lateral rods . . . 992
" " struts . . 838, 963
" lattice bars . . 845, 958
" lower chord . . 818, 979
" pin plates . . 914, 944, 980
" pins . . . 912, 931, 936
" portal bracing . . 842, 958
" shoe strut . . . 848, 965
" tension members . . 817
" " web members . 986
" ties 986

PAGE
Design of upper chord . . 833, 948
Design, Practical hints concerning 1087
Detail drawings, Nature and use of 895
Details of floor beams . . . 966
" Specifications relating to 897, 902
Diagonal members, Calculation of exact length of . . . 905
" " Design of . . 986
" " Economical inclination of . 749
Diagram, erection 1053
" of bending moments . 651
Dimensions of nuts 996
" " timber floor joists . 865
Divided panel 778
Double beam hanger . . . 1067
" intersection or double quadrangular truss . . . 774
Drawings, Detail and working . 895
" Shop, data given on . 1026

E. PAGE
Eccentricity, Negative . . . 887
" Net 887
" of pin plates . . 1084
" " pins 886
" " section . . . 883
" " stress, Effects of . 881
Economical form and height of bridge truss 749
Effect of wind on chords and end posts 815, 850
Effective depth of floor beam . . 730
Elastic limit 634
" of materials in compression . . . 638
" " materials in tension 636
" " structural steel in general . . . 811
" required by specifications for steel . . 811
" required by specifications for wrought iron 809
Elasticity, Coefficient of . . . 635
" Fundamental laws of . 634
Elongation required by specifications (steel) . 810, 811
" required by specifications (wrought iron) 809
" Ultimate . . . 636
End posts, Correction for wind stress 815, 850
" Design of . . . 833, 941
Erection bolts, Use and dimensions of 1052

PAGE
Erection diagram, Nature and uses of 1053
Error in determining maximum web stresses 793
Estimates of weight of material, how made 1054
Expansion, Coefficient of, for wrought iron and steel 1007
" in bridge work, how provided for . 901, 1007
" rollers, Design of . 1005
" " Specifications relating to . 901
External forces in truss . . . 685
Eye-bars, Clearance between . . 937
" Heads of . . . 899, 975

F. PAGE
Factors of safety 640
" " Table of . . 641
" " Use made of . 640
Fatigue of metals, Cause of . . 872
" " Formulas for unit stresses founded on . . . 873
" " Importance of in structural work 873
" " Requirements for 1085
" " Specifications founded on . 874
" " Woehler's law relating to . 872
Fiber stress 931
Field rivets 1048
Filler rings, Uses of 1025
Final pitch of rivets, how determined 971
Fixed load 690
Flange stresses in floor beam, Formula for . . 730
" " " portal, how determined . 721, 725
Flanges of channels, Width of . 833
" Pitch of rivets in . 967, 971
" Staying of 875
Flat-ended columns, Requirements for 1086
Flexure, Strength of beam in . 665
Floor beam hangers . . 998, 1066
" " stresses 726
" beams 726
" " Bending moment at any point of . . 728
" " Design of . . 859, 966
" " Effective and total depth of . . . 730
Floor beams, Formula for weight of 727
" " Maximum flange stress in 730
" " Maximum shear in . 731
" " Position and connections of . . . 1066
" " Shear at any point of 968
" " Weight of, used in estimates . . . 1058
" " with sidewalk cantilevers . . . 798
" Details of 966
" joists, Arrangement of 862, 1045
" " Dimensions of . . 865
" " Metal . . . 1062, 1064
" " Stresses allowed on 863, 864
" Weight of 706
Forces, External 685
" Gradually and suddenly applied 641
Forked ends, Specifications relating to 900
Friction, Coefficients of, for steel, wrought iron, and masonry . 878

G. PAGE
Girder, Latticed 689
" Plate 689
" " effective depth of . 859
" " minimum thickness of web of . . . 859
" " stiffening of . . 859
" " Thacher's specifications relating to 875, 876
" Riveted 689
Graphical determination of bending moment 650
" " of live load stresses in truss with parallel chords . 694
" " of maximum stresses in chords . 697
" " of maximum stresses in web members . . 700
" " of stresses in Baltimore truss 779, 782

PAGE
Graphical determination of stresses, Clerk Maxwell method 733
" " of stresses in curved chords 751, 753
" " of stresses in inclined chords . 765
" " of stresses in lower lateral system 711, 715
" " of stresses in Petit truss 787, 789
" " of stresses in Pratt truss 694, 707
" " of stresses in Warren girder . 745
Graphite paint 1078
Gray cast iron 626
Grip of cotter pins 1024
" pins, how determined . 1018
Gross section 854
Gyration, Radius of 664

H. PAGE
Half-deck bridges 688
Half-hitch truss 786
Handbooks 813
Hanger plates, Dimensions of . 1000
Hardening 629
" Effects of . . . 629
Head-room, Amount of, in bridges 862, 871, 875
Heads of eye-bars . . . 899, 975
Hemp ropes, Strength of . . 681
High steel, Properties of, required by specifications . 811
" " Weight and specific gravity of rolled sections of 878
" truss bridges 688
Horizontal component of chord stresses 763, 781
Howe truss 741
" Formula for chord stresses in . . . 762
" Stresses in . . . 743
" where advantageously used 741
Hub guards 1071, 1072

I. PAGE
I beams 1062
" Properties of Carnegie steel 1064
" Strength of 1063
Imperfect design 1082
Inclined chord 750
" chords, Formulas for stresses in 763, 781
" " Stresses in truss having . 765, 768
" " when used . . 749
Inertia, Moment of 664
Initial stresses in lateral rods, how treated 825, 838
Intensity of stress 632
Intermediate posts, Design of 833, 841, 951
Iron, Cast 625
" hub guards 1072
" Malleable 627
" order 1035
" " how made . . 1035, 1042
" oxide paint 1077
" Wrought 627

J. PAGE
Joints, Splicing 898, 950
" Strength of 897
Joists, Arrangement of . . 862, 1045
" Dimensions of . . . 865
" Metal 1062, 1064
" Number of lines of . . 865
" Stresses allowed on . 863, 864

K. PAGE
Knee-brace, Design of . . . 964
" Length of, used in estimates . . . 1057

L. PAGE
Lacing, Definition of . . . 946
Lateral rods, Connections of . 949, 973
" " Design of . . . 992
" " Initial strain on . . 825
" " Minimum area of . 816
" stresses 760
" struts, Design of . . 838, 963
" " Stress assumed on . 825
" " Stress in, due to initial stress in lateral rods . . . 839
" system, Attachment of, to chords . . 900
" " Depth of truss used for . . . 712

PAGE
Lateral system, Minimum stress in center strut of . 761
" " Panel length of, in truss with non-parallel chords . 761
" " Pratt truss type of 761
Lattice bars 946
" " Design of . . 845, 958
" " Dimensions and spacing of . 946, 948, 954
" " in tension members . 857
" " Length of, how estimated . . . 1057
" " Ordered length of . 1039
" " Specifications relating to 899
" " Stresses in bars of portal . . . 725
Latticed girder 689
Laws of elasticity 634
Length, how affected by pin holes . 907
" " changed by camber . 904
" Maximum, of compression members 825
" of pins 1037
" " various members, how estimated . . . 1055
" Theoretical, of bridge members 825
Lewis' specifications . . . 877
Light weights 1083
Limiting values of pitch of rivets 898, 964
Line, Neutral 662
" of action of stresses in members 687
Live chord stresses, Maximum . 694
Live load 690
" stresses allowed by Cooper's specifications for wrought-iron members . 814, 824
" " Graphical determination of, in truss with parallel chords . 694
" " Graphical determination of, short method . 737
" " in Baltimore truss 782
" " " chords, found from dead load stresses . . 753
" " " Petit truss . 789
" " " truss with curved or inclined chords . 753, 768

PAGE
Live load stresses in Whipple truss 776
" web stresses, Maximum . 699
Load, Concentrated 690
" Dead 690
" Effect of distribution of, on stresses in members . . 694
Loads, Moving 690
" Panel 691
" Uniform 690
" Wheel 690
" Wind 690
" Position of, for maximum bending moment . 802, 803
Lomas recessed nut 1021
Loops of bars and rods . . 987, 1037
Lower lateral system, Graphical determination of stresses in 711, 715
Low-truss bridges 688
Lumber, Bill of 1044, 1053
" Usual sizes carried in stock 1044

M. PAGE
Malleable cast iron 627
Materials, Ordering 1034
" Sizes carried in stock . 1041
" used in bridge construction 807
" used in bridge construction, Selection of . . 892
" used in engineering construction . . . 625
Maximum and minimum weights of channels . . . 830
" bending moment, Position of loads for . . 802, 803
" length of compression members . . . 825
" live chord stresses . . 694
" load web stresses . . 699
" shear in floor beam . . 731
" Error in, from assumption of panel concentrations 793
" stresses in chords, Condition of loading for . 694
" stresses in chords. Graphical determination of . 697
" stresses in deck bridges 715
" " " diagonal and vertical members of Pratt truss . . . 700
" stresses in Howe truss . 742
" " " web members, Graphical determination of 700

PAGE

Maximum stresses in web members, Short method for . . 737
Medium steel, Properties of, required by specifications . . 811
" " Unit stresses allowed on . 869, 870
Members, Convenient and limiting dimensions of . . 891
" Exact length of diagonal 905
" Line of action of stresses in 687
" Maximum and minimum dimensions of . . 825
" Stresses in, how affected by distribution of load 692, 694
" Tension and compression 686
" Theoretical length of . 825
Metal floor joists . . . 1062, 1064
Metals, Fatigue of 872
Minimum area of lateral rods . 816
" diameter of pins . 899, 1020
" thickness of web of girder 859
Modulus of elasticity . . . 811
" Section 1063
Moment, Bending 649
" diagram 651
" " Determination of stresses by . 733
" of inertia 664
" Resisting 664
Moments, Principle of, applied to determination of stresses in braced portal 796
Moving load 690

N. PAGE

Nails, Dimensions and weights of . 1050
Name plates 1701
Negative eccentricity . . . 887
Net eccentricity 887
" section of bridge members 815, 854
Neutral axis 661
" line 662
" surface 663
Nuts, Check 999
" Dimensions of . . . 994, 996
" Lomas recessed . . . 1021
" Pilot 1019
" Standard 999

O. PAGE

Open-hearth steel 630
Order list, how made . . 1034, 1040
Ordered lengths of material . . 1035

P. PAGE

Packing rings, Uses of . . . 1025
Paint, Asphaltum 1077
" Graphite 1078
" Iron oxide 1077
" Red and white lead . . 1076
Painted surface, Amount of . . 1080
Painting of bridges 1075
" Preparing iron for . . 1081
Paints, Covering capacity of . . 1079
Panel 686
" concentrations 691
" " Error arising from assumption of . . 793
" Divided 778
" Increased length of, to provide for camber . . . 904
" lengths 686
" load 691
" " how determined . . 691
Petit truss, Description of . . 786
" " Graphical determination of stresses in 787, 789
Pig iron 625
Pillar 672
Pilot nuts 1019
Pin holes, Clearance of . . . 899
" " Reinforcing plates in . 900
" pilot, Use of 1020
" plates, Design of . 914, 944, 980
" " Dimensions of . . 924
" " Eccentricity of . . 1084
" " for tension members, metal behind pin . 983
" " Strut resistance of . 956
" washers, Description and uses of 1025
Pin-connected bridges . . . 688
" " " when used . 877
Pins, Bearing stress allowed on 897, 932
" Bending moment upon . . 927
" " stress allowed on . 897
" Clearance of 899
" Computation of bending moment on 927
" Correction in size of, to provide for wind stress . . 929
" Determination of size of . 912, 932
" Dimensions of, used in estimates 1057
" Graphical determination of bending moment on . 936-939
" Grip of, how determined . 1018
" Hip, design of 936
" Length of 1024, 1037

PAGE
Pins, Maximum bending moment on 913
" Minimum diameter of . 899, 1020
" Position of 886
" " " bearing upon . 925
" Resisting moments of, how determined 931
" Screw ends of 1019
" Shearing stress allowed on . 897
" Table of resisting moments of 933
Pipes, Strength of, under uniform pressure 643
Pitch, Final or resultant, how determined 971
" of rivets in batten-plates . 946
" " " " beam flanges 967–971
" " " " cover-plates . 941
" " " " portal bracing . 959
" " " Limiting values of 898, 964
Plate beam hangers 1067
" girders 689
" " Allowed shearing stress in web of 859, 877
" " Assumed distribution of tension and compression in . . 859
" " Effective depth of . 859
" " Minimum thickness of web of . . . 859
" " Position of flanges on web 898
" " Stiffening of . . 859
" " Thacher's specifications relating to 875, 876
Plates, Maximum width of . . 825
" Design of 915
" Reenforcing, at pin holes . 900
Portal bracing, Design of . . 958
" Design of 842
" Functions of 718
" Stresses in latticed . . 718
" " " braced . . . 795
" Weight of, used in estimates 1058
Position of loads for maximum bending moment 802, 803
" " pins 886
Posts, End, design of . . 833, 941
" Intermediate, design of . 951
Pratt truss, Condition of loading for maximum stress 700
" " Description of . . 689
" " Formula for chord stresses in . . 762
" " Graphical determination of stresses in 694, 707
" " with curved or inclined chords . . 773

PAGE
Preparing iron for painting . . 1081
Proportioning material for tension members 817

R. PAGE
Radius of gyration 664
" " " Approximate formulas for value of . . 821
" " " Cases in which formulas give least . . 827
" " " in usual compression members . . 821
Railing for bridges . . . 1072, 1073
Rankine's formula for columns . 823
Reactions from wind pressure at foot of end posts . . 719
" in truss, how determined 696
Recessed nut 1021
Red lead paint 1076
" " " Proportions for mixing 1080
" " Use of 1079
Reenforcing plates at pin holes . 900
Resisting moment 664
" " of pins . . 931
Resultant pitch of rivets . . . 971
Rivet holes, Distance from, to back of angle . . . 955
" " Distance from, to back of channel . . . 941
" " Distance from, to edge of piece . . 898, 902
" " how considered, in determining net section 815
Riveted girders 689
" trusses 689
Rivets, Arrangement of, in batten-plates 946
" Arrangement of, in cover-plates 941
" Arrangement of, in pin-plates 924
" Bearing on, how determined 920
" " stress allowed on 897, 921
" Countersunk . . . 1083
" Dimensions of . . . 1038
" how ordered 1048
" Limiting values of pitch of 898, 964
" Machine and field driven . 898
" Number of, how determined 919, 935, 953, 982

PAGE
Rivets, Shearing on, how determined . . . 918
" " stresses allowed on . . 897, 921
" Specifications relating to . 902
" Usual sizes of . . . 898
" Weight of, used in estimates 1058
Rods, Sway 795
Rollers, Expansion . . . 901, 1005
Ropes, Strength of, and dimensions of 681
Rust, Protection of bridges from . 1075

S. PAGE
Screw ends of pins, Dimensions of 1020
" " Specifications relating to 899
" " upset, Dimensions of . 989
Section modulus 1063
Shafts 678
" Formulas for strength of 679, 680
Shear at any point of uniformly loaded beam . . 731, 968
" " point of maximum bending 802
" axis 649
" Definition of 639
" diagram 649
" " Stresses determined by 733
" Effect of negative dead load 710
" from live and dead load, how found 710
" Graphical representation of 648
" line 649
" Maximum, in floor beam . 731
" Single and double . . . 903
" Strength of various materials in 639
" Stresses due to . . . 735
" Value of, for maximum stresses in web members and vertical posts . . 704
" Vertical, in beam . . . 647
Shearing strengths of various materials 639
" stress allowed on pins . 897
" " " " rivets 897, 921
" " " " timber 639
" " " " web of girder 859, 877
" " on rivets, how determined . . 918
Shipping bill, how made . . . 1048
Shocks, Effects of 641

PAGE
Shoe-plates 1005
" " Bending moment on . 1009
" " Formula for thickness of 1011
" " Proportioning of . . 1005
Shoe-strut, Design of . . 848, 965
Shop drawings. Data given in . 1026
" lists, how made . . . 1027
Sidewalk brackets 798
Sidewalks, how supported . . 798
Simple truss 686
Single beam hanger 1066
" quadrangular, single intersection truss . . . 689
Soft steel, Properties of, required by specifications . 810
" " Weight and specific gravity of rolled shapes of . . . 878
Sole plate 1005
Spacing of lattice bars . . . 946
Span of a truss 686
Specifications 813
" for highway bridges, Cooper's . . . 869
" Lewis's . . . 877
" relating to anchor bolts . 901
" " " details 897, 902
" " " floor . 861
" " " rivets . 902
" Thacher's . . . 875
Spikes, Weight and dimensions of 1051
Splices, Use of, in joints . . 898, 950
Standards of shoe 1005
Static load on bridge . . . 690
Steel, Advantages of, over wrought iron 808
" Bessemer 630
" Blister 630
" Coefficient of elasticity of 636, 811
" " " expansion of . 1007
" Cooper's specifications relating to 869
" Crucible 630
" Different grades of . . 628, 810
" Elastic limit of . . 636, 811
" Factors of safety used for . 641
" General properties of . . 628
" Grades of, used in bridge work 810
" Growing use of . . . 807
" Open hearth 630
" Processes of manufacturing . 630
" Proportion of carbon contained in 631

PAGE

Steel, Structural 807
" Tempering 629
" Ultimate strength of, for compression 638
" Ultimate strength of, for flexure 665
" Ultimate strength of, for shearing 639
" Ultimate strength of, for tension 636
" Weight of 625, 879
Stiff suspenders, Compression in . 877
Stiffened chords 858
" " Compression allowed on . . 877
Stiffeners in floor beams 972
" " plate girders . 859, 876
Stiffness of beams 669
Stirrup 998
Stock list 1034
" Material carried in . . 1041
Stone, Coefficient of elasticity of . 638
" Ultimate bending strength of 665
" " compressive strength of . 638
" Weight of 625
Straight-line formula . . . 823
Strain 632
" and stress, Relation between 634
" sheet 733
" Unit 633
Strength and stiffness of beams, Comparison of . . 671
" of I beams . . . 1064
" Ultimate 636
" " for steel . . 810
" " " wrought iron 809
Stress 632
" and strain, Relation between 634
" suddenly and gradually applied 641
" distinguished from force . 635
" Intensity of 632
" Kinds of 633
" line of action in truss members 687
" sheet 732
" " Complete form of . 866
" Unit 632
Stresses allowed on floor joists . 863
" Bending, due to weight of member 880
" Counter 709
" Determination of, by sections and moments . 796

PAGE

Stresses, Determination of, by stress and shear diagrams . 733
" due to bending moments . 735
" " " shear . . . 735
" Floor beam 726
" from varying loads . . 692
" in braced portals . . 795
" " chords, Graphical determination of . . . 697
" " curved chords . 751, 753
" " Howe truss . . . 742
" " Petit truss . . 787, 789
" " Pratt truss . . 694, 707
" " truss, determined graphically 696
" " truss with inclined chords . . . 765, 768
" " Warren girder . . 745
" " web members . . 700
" " Whipple truss . . 775
Stringers, Maximum bending moment in 802
" Metal 1062
Structural steel 807
Strut resistance of pin plates . . 956
Struts 686
Sub-panel 779
Sub-strut 795
Sub-tie 786
Sudden stresses, Effect of . . 641
Surface, Neutral 663
Sway rods 795
Swedged bolt 1017
Symmetrical truss 685

T. PAGE

Temperature, Variations of, provided for in bridge work . 901, 1007
Tempering 629
Tensile strength of various materials 636
" stresses, Formulas for. founded on fatigue of metals 873
Tension 633
" members of truss . . 686
" " Proportioning of 817
" " subject to compression . 687, 858
" " Unit stresses allowed for . 814
" " Usual forms of . 816
" web members, Design of . 986
Thacher's specifications . . . 875
Theoretical length of bridge members 825

PAGE
Thickness, Limiting, in bridge members 825
Through bridges 687
Ties 686
" Design of 986
Timber, Bending strength of . . 665
" Coefficient of elasticity of 636
" Elastic limit of . . . 636
" Factors of safety used for 641
" Shearing strength of . 639
" Ultimate strength of, for compression . . . 638
" Ultimate strength of, for tension 636
" Unit-stress allowed on, for bending . . . 863, 876
" Unit-stress allowed on, for compression . . . 876
" Unit-stress allowed on, for shearing 876
" Usual sizes of, carried in stock 1044
" Varieties of, used for joists 864
" Weight of 625
Torsion 678
" Angle of 679
Truss 685
" Baltimore 778
" Camel back 773
" Cantilever 686
" Continuous 686
" Essential element of . . 685
" External forces of . . . 685
" Howe 741
" Members of 686
" Petit 786
" Pratt single quadrangular, single intersection . . 689
" Simple 686
" Span of 686
" Symmetrical 685
" Whipple 773
" with inclined chords . . 749
Turnbuckles, Dimensions of . . 991
" Position of, on counters 992

U. PAGE
Ultimate elongation 636
" strength 636
" " of materials for bending . . 665
" " of materials in compression . 638
" " of materials in shear . . 639
" " of materials in tension . . 636
Uniform load 690
Unit strain 633
" stress 632
" stresses allowed for tension members 814
" " " on medium steel 869, 870
" " " " timber . 876
Unsymmetrical sections . . . 1084
Upper lateral system, Graphical determination of stresses in . 717
Upset ends, Dimensions of . . 989
" " Length of bars having 1037
" " Specifications relating to 899

V. PAGE
Varying load, Stresses from . . 692
Vertical shear 647

W. PAGE
Warren girder 744
" " Graphical determination of stresses in . 745
Washers for bolts in wheel guards 1049
" " pins 1025
Web members, Design of . . 986
" " Economical inclination of . . . 749
" " Maximum stresses in 700
" plates, Maximum shearing stress in 859
" stresses, Error in the determination of maximum . . . 793
" " Short method for determining . . 737
Wedge bolt 1017
Weight, Maximum and minimum, of wrought-iron channels and angles . . 830
" of floor beam . . 727, 1058
" " materials, used in estimates . . . 1058, 1059
" " members, how estimated 1055
" " nails and spikes . . 1051
" " rolled sections of wrought iron, high and soft steel . . . 878
" " steel 878
" " timber bridge-floor . 706
" " various materials . . 625
" " wrought iron . . 828, 878
Wheel guards, Details of . . 1049

PAGE

Wheel loads 690

" " Concentrated . . 801

Whipple truss, Assumption regarding distribution of stresses in . . 775

" " Description of . . 773

" " Graphical determination of stresses in 775

White cast iron 626

" lead paint 1076

Width, Maximum, of plates in compression 825

" of flange of channel . . 833

" " truss 875

Wind load, Assumed amount of . 690

" " how stated . . . 691

" pressure, how transmitted to and by the portal 718

" " Provision for, in chords and end posts . . 815, 850

" " Provision for, in designing pins . 929

" " Reactions from, at feet of end posts . 720

" stresses, Graphical determination of, in lower lateral system 711, 714

" " Graphical determination of, in upper lateral system . 717

" " how transmitted to the anchorage . 718

Wire ropes, Strength of . . . 682

Woehler's law relating to fatigue of metals 872

Wooden hub guards 1071

Working drawings 896

" stresses 812

PAGE

Working stresses allowed on medium steel members 869, 874

" " allowed on wrought-iron compression members 824, 874

" " allowed on wrought-iron tension members . 814, 874

Wrought iron, Coefficient of elasticity of . . 636

" " Coefficient of expansion of . . 1007

" " Elastic limit of . 636

" " Factors of safety used with . . 641

" " how made . . 627

" " Properties of, required by specifications . . . 809

" " Supposed advantages of, over steel . . . 808

" " Ultimate strength of, for compression . . . 638

" " Ultimate strength of, for flexure . 665

" " Ultimate strength of, for shearing . 639

" " Ultimate strength of, for tension . 636

" " Use of, in bridge work . . . 807

" " Weight of . . 828

" " " " rolled sections of . 830

89057508475

b89057508475a

DEMCO

89057508475

b89057508475a